Theory and Applications of Weighted Least Squares Surface Matching for Accurate Spatial Data Registration

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I hereby certify that the work embodied in this thesis is the result of original research and has not been submitted for a higher degree to any other University or Institution.

Robert Pâquet
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A Emilienne Léontine et Robert Léon.
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Abstract

This thesis discusses matching of 3D surfaces, in particular, their registration in a common coordinate system. This differs from object recognition in the sense that the surfaces are generally close to registration, sometimes so close that the mismatch cannot be detected on visual inspection. The surface matching algorithm, based on least squares theory, is therefore an estimation of the matching parameters, sometimes very small, which provides the most statistically accurate registration.

High redundancy is achieved with the algorithm, as each point of one surface can potentially participate in the formation of an observation equation for the least squares adjustment. The algorithm minimises the separation between the surfaces. The surfaces are defined by sets of points represented by their cartesian coordinates in \( \mathbb{R}^3 \) space, without restrictions on the mode of sampling used in the capture of the data. The registration is executed without control points. Modern non-thematic sampling methods, for instance airborne laser scanning, can benefit from such an algorithm. Other applications include processes where permanent control markers cannot be used, for example, medical applications or coastal erosion.

Surface matching has been used previously by a small number of people. The particular interest of this thesis, however, has been to test the accuracy and other characteristics of the matching, especially when weighting is used with the surface separations. This thesis presents and compares several weighting techniques including one technique based on the covariance function.

In addition, a statistical method to model matching accuracy as a function of the density of the control surface is formulated. The method is useful to ascertain the interpolation component of the matching error. The remaining component of the error can be deducted and analysed according to the project under consideration. Examples of project might be filtering in data fusion assessment, or volume displacement in landslide analysis.

The theory is developed using artificial data. This helps to isolate and analyse in turn the various characteristics of the surface matching. The thesis is then illustrated with examples involving real data sampled in Newcastle, NSW, Australia, using methods such as ALS, photogrammetry and GPS.