RELIABILITY ANALYSIS
OF DEGRADING
UNCERTAIN STRUCTURES

WITH APPLICATIONS TO FATIGUE AND
FRACTURE UNDER RANDOM LOADING

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I hereby certify that the work embodied in this thesis is the result of original research and has not been submitted for a higher degree to any other University or Institution.

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ABSTRACT

In the thesis, the reliability analysis of structural components and structural details subject to random loading and random resistance degradation is addressed. The study concerns evaluation of the probability of failure due to an overload of a component or structural detail, in consideration of random (environmental) loads and their combination, uncertain resistance parameters, statistical and phenomenological uncertainty and random resistance degradation mechanisms. Special attention is devoted to resistance degradation, as it introduces an additional level of difficulty in the solution of time variant reliability problems.

The importance of this study arises from the ageing of existing infrastructure in a world wide scale and from the lack of standards and codes for the ongoing safety management of general structures past their original design lives. In this context, probabilistic-based risk assessment and reliability analysis provide a framework for the safety management of ageing structures in consideration of inherent load and resistance uncertainty, current state of the structure, further resistance degradation, periodic inspections, in the absence of past experience and on an individual basis. In particular, the critical problem of resistance degradation due to fatigue is addressed.

The formal solution of time variant reliability problems involves integration of local crossing rates over a conditional failure domain boundary, over time and over random resistance variables. This solution becomes very difficult in the presence of resistance degradation, as crossing rates become time dependent, and the innermost integration over the failure domain boundary has to be repeated over time. Significant simplification is achieved when the order of integrations is changed, and crossing rates are first integrated over the random failure domain boundary and then over time. In the so-called ensemble crossing rate or Ensemble Up-crossing Rate (EUR) approximation, the arrival rate of the first crossing over a random barrier is approximated by the ensemble average of crossings. This approximation conflicts with the Poisson assumption of independence implied in the first passage failure model, making results unreliable and highly conservative.
Despite significant simplification of the solution, little was known to date about the quality of the EUR approximation. In this thesis, a simulation procedure to obtain Poissonian estimates of the arrival rate of the first up-crossing over a random barrier is introduced. The procedure is used to predict the error of the EUR approximation. An error parameter is identified and error functions are constructed. Error estimates are used to correct original EUR failure probability results and to compare the EUR with other common simplifications of time variant reliability problems. It is found that EUR errors can be quite large even when failure probabilities are small, a result that goes against previous ideas.

A barrier failure dominance concept is introduced, to characterize those problems where an up-crossing or overload failure is more likely to be caused by a small outcome of the resistance than by a large outcome of the load process. It is shown that large EUR errors are associated with barrier failure dominance, and that solutions which simplify the load part of the problem are more likely to be appropriate in this case. It is suggested that the notion of barrier failure dominance be used to identify the proper (simplified) solution method for a given problem. In this context, the EUR approximation is compared with Turkstra's load combination rule and with the point-crossing formula.

It is noted that in many practical structural engineering applications involving environmental loads like wind, waves or earthquakes, load process uncertainty is larger than resistance uncertainty. In these applications, barrier failure dominance in unlikely and EUR errors can be expected to be small.

The reliability problem of fatigue and fracture under random loading is addressed in the thesis. A solution to the problem, based on the EUR approximation, is constructed. The problem is formulated by combining stochastic models of crack propagation with the first passage failure model. The solution involves evaluation of the evolution in time of crack size and resistance distributions, and provides a fresh random process-based approach to the problem. It also simplifies the optimization and planning of non-destructive periodic inspection strategies, which play a major role in the ongoing safety management of fatigue affected structures.

It is shown how sensitivity coefficients of a simplified preliminary First Order Reliability solution can be used to characterize barrier failure dominance. In the fatigue and fracture reliability problem, barrier failure dominance can be caused by large variances of resistance or crack growth parameters. Barrier failure dominance caused by resistance parameters leads to problems where overload failure is an issue and where the simplified preliminary solution is likely to be accurate enough. Barrier failure dominance caused by crack growth parameters leads to highly non-linear problems, where critical crack growth dominates failure probabilities. Finally, in the absence of barrier failure dominance, overload failure is again the issue and the EUR approximation becomes not just appropriate but also accurate.

The random process-based EUR solution of time-variant reliability problems developed and the concept of barrier failure dominance introduced in the thesis have broad applications in problems involving general forms of resistance degradation as well as in problems of random vibration of uncertain structures.
ANÁLISE DE CONFIABILIDADE ESTRUTURAL SOB DEGRADAÇÃO ALEATÓRIA DE RESISTÊNCIA - COM APLICAÇÕES A FADIGA E FRATURA SOB CARREGAMENTOS ESTOCÁSTICOS

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RESUMO

Esta tese aborda o cálculo de confiabilidade de componentes e detalhes estruturais sujeitos a carregamentos estocásticos e a degradação da resistência ao longo do tempo. Neste típico problema de confiabilidade estrutural dependente do tempo, a falha da estrutura ou componente fica caracterizada na primeira sobrecarga. O problema de confiabilidade consiste em prever a probabilidade de uma falha por sobrecarga, considerando incertezas nos carregamentos e nas suas possíveis combinações, nos parâmetros da resistência, incertezas estatísticas e fenomenológicas, bem como nos mecanismos aleatórios de degradação da resistência. Em especial, a tese aborda o problema de degradação da resistência de estruturas, que introduz sérias dificuldades no cálculo de confiabilidade.

A importância de se considerar degradação de resistência na análise de confiabilidade estrutural está no envelhecimento crescente da infraestrutura instalada, a nível mundial, e na inexistência de normas técnicas genéricas para a análise de segurança e para a extensão da vida útil de estruturas existentes. A análise probabilística de risco e, como parte dela, a análise de confiabilidade estrutural, permite uma análise quantitativa da segurança de estruturas existentes, considerando incertezas em carregamentos e resistência, condição atual da estrutura, inspeções periódicas não-destrutivas, progressiva redução da resistência, na falta de experiência anterior e em uma base individual para cada estrutura. Como exemplo de aplicação, o problema de fadiga e fratura sob carregamentos estocásticos é estudado na tese.

A solução usual de problemas de confiabilidade estrutural dependente do tempo envolve integrais sobre a fronteira do domínio de falha, sobre o tempo e sobre os parâmetros aleatórios de resistência. Sob redução da resistência, a solução usual se torna muito difícil, porque a taxa de falhas se torna dependente do tempo. Nesta tese, uma solução alternativa é estudada. Esta solução, chamada de “Ensemble Up-crossing Rate approximation” e abreviada como EUR, consiste em aproximar a taxa de chegada da primeira passagem por uma barreira aleatória pela taxa de passagens média sobre o envelope. Apesar de simplificar muito a solução do problema, esta aproximação dá origem a um erro admitidamente grande, mas que ainda não havia sido estudado em detalhe.

Uma metodologia é introduzida e utilizada para estimar o erro da aproximação EUR. Esta metodologia, baseada em simulação de Monte Carlo, consiste em uma série de experimentos numéricos realizados em computador. Através dela, parâmetros de erro são identificados e funções de erro são construídas. O estudo mostra que o erro da aproximação EUR pode ser grande mesmo em problemas onde as probabilidades de falha são pequenas, um resultado que se contrapõe a suposições anteriores. As estimativas de erro são utilizadas para corrigir resultados da solução EUR original e para comparar esta aproximação com outras soluções simplificadas do problema de confiabilidade estrutural dependente do tempo.
O presente estudo introduz o conceito de falha dominada pela barreira. Este conceito é utilizado para descrever problemas de confiabilidade onde a probabilidade de ocorrência de uma falha por sobrecarga é dominada pela realização da barreira e não pela realização do carregamento. Em outras palavras, a probabilidade de que uma pequena realização da resistência da estrutura cause uma falha por sobrecarga é maior do que a probabilidade de que a mesma falha seja causada por uma infeliz combinação de extremos de carregamento.

O estudo mostra ainda que problemas com falha dominada pela barreira geram os maiores erros quando utilizada a aproximação EUR. Assim, os maiores erros da solução EUR podem ser associados com falha dominada pela barreira. Nesta mesma situação de falha dominada pela barreira, outras soluções aproximadas, envolvendo simplificações na parte dos carregamentos, são mais apropriadas. Como resultado, propôs-se que o conceito de falha dominada pela barreira seja utilizado como orientação para estabelecer que tipo de solução (aproximada) é adequada para que tipo de problema. Neste contexto e a título de exemplo, a aproximação EUR é comparada com os métodos conhecidos como “Turkstra’s Load Combination Rule” e “Point-Crossing Formula”.

Em problemas práticos de análise estrutural, incertezas referentes a carregamentos ambientais como ondas, vento e terremotos, são tipicamente maiores do que incertezas em parâmetros de resistência. Neste tipo de problema, a falha dominada pela barreira dificilmente fica caracterizada. Assim, a despeito da grande possibilidade de erro da solução EUR, esta aproximação ainda tem uma importante gama de aplicação.

A tese aborda a análise de confiabilidade a fadiga e fratura de componentes e detalhes estruturais sujeitos a carregamentos estocásticos. Uma solução para este problema, baseada na aproximação EUR, é construída combinando-se modelos estocásticos de propagação de trincas com o modelo de falha à primeira sobrecarga. Esta solução envolve um cálculo da evolução no tempo das funções de distribuição de probabilidade das variáveis (aleatórias) tamanho da trinca e resistência da estrutura, numa abordagem baseada em processos estocásticos. Esta solução é muito apropriada em problemas envolvendo otimização e planejamento de estratégias de inspeções periódicas, as quais são fundamentais no problema de extensão de vida útil de estruturas.

O estudo mostra como os coeficientes de sensibilidade de uma solução preliminar baseada no método de aproximação de primeira ordem (FORM) podem ser utilizados para identificar ou caracterizar falha dominada pela barreira. O estudo mostra que, no problema de fadiga e fratura, falha dominada pela barreira pode ser causada por parâmetros de resistência ou por parâmetros da propagação de trincas. Quando causada por parâmetros de resistência, leva a problemas onde a probabilidade de falha devido a uma sobrecarga pode ser significativa, e para os quais a solução preliminar simplificada pode ser suficiente. Quando incertezas referentes aos parâmetros de propagação de trincas dominam o problema, a falha passa a ser não somente dominada mas determinada pela barreira. Neste caso, a falha ocorre devido ao crescimento crítico da trinca (também conhecida como falha por acúmulo de dano) e não devido a uma sobrecarga. Finalmente, quando a probabilidade de falha não é dominada pela barreira, o modo de falha por sobrecarga volta a ser crítico, e a aproximação EUR torna-se não só adequada como prescissa.

A solução do problema de sobrecarga sob carregamentos aleatórios elaborada nesta tese, que modelo a degradação da resistência como um processo estocástico e que está baseada na aproximação EUR, tem ampla aplicação a problemas envolvendo outras formas de redução da resistência, como corrosão, desgaste, fluência, etc, bem como em problemas de vibrações aleatórias em estruturas.
ANÁLISE DE CONFIABILIDADE ESTRUTURAL SOB DEGRADAÇÃO ALEATÓRIA DE RESISTÊNCIA - COM APLICAÇÕES A FADIGA E FRATURA SOB CARREGAMENTOS ESTOCÁSTICOS

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### 0.1 List of Abbreviations

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<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tr>
<td>ANAL</td>
<td>Analytical - refers to distribution-model based solutions</td>
</tr>
<tr>
<td>BB</td>
<td>Broad band</td>
</tr>
<tr>
<td>BFD</td>
<td>Barrier failure dominance</td>
</tr>
<tr>
<td>CDF</td>
<td>Cumulative density function</td>
</tr>
<tr>
<td>C.O.V.</td>
<td>Coefficient of variation</td>
</tr>
<tr>
<td>CONV</td>
<td>Convolution Integration</td>
</tr>
<tr>
<td>EPFM</td>
<td>Elasto-plastic Fracture Mechanics</td>
</tr>
<tr>
<td>EUR</td>
<td>Ensemble Up-crossing Rate</td>
</tr>
<tr>
<td>FE</td>
<td>Finite Element (analysis)</td>
</tr>
<tr>
<td>FORM</td>
<td>First Order Reliability Method</td>
</tr>
<tr>
<td>FPI</td>
<td>Fast Probability Integration</td>
</tr>
<tr>
<td>HLRF</td>
<td>Hassofer-Lind-Rackwitz-Fiessler optimization algorithm</td>
</tr>
<tr>
<td>KS</td>
<td>Kolmogorov-Smirnoff goodness-of-fit test</td>
</tr>
<tr>
<td>LEFM</td>
<td>Linear Elastic Fracture Mechanics</td>
</tr>
<tr>
<td>LOGN</td>
<td>Log-normal crack propagation rate model</td>
</tr>
<tr>
<td>MCS</td>
<td>Monte Carlo Simulation</td>
</tr>
<tr>
<td>MKV</td>
<td>Diffusive Markov crack propagation model</td>
</tr>
<tr>
<td>NB</td>
<td>Narrow band</td>
</tr>
<tr>
<td>NUMER</td>
<td>Numerical - refers to fully numerical solutions</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability density function</td>
</tr>
<tr>
<td>POD</td>
<td>Probability of detection (of an inspection method)</td>
</tr>
<tr>
<td>PS</td>
<td>Pulse-sequence (load process)</td>
</tr>
<tr>
<td>PSD</td>
<td>Power Spectrum Density</td>
</tr>
<tr>
<td>RP</td>
<td>Random Process</td>
</tr>
<tr>
<td>RV</td>
<td>Random Variable</td>
</tr>
<tr>
<td>R6</td>
<td>R6 failure assessment diagram</td>
</tr>
<tr>
<td>SIF</td>
<td>Stress Intensity Factor</td>
</tr>
<tr>
<td>SM</td>
<td>Second-order second moment approximation</td>
</tr>
<tr>
<td>SORM</td>
<td>Second Order Reliability Method</td>
</tr>
<tr>
<td>TI</td>
<td>Time-integrated (extreme value) analysis</td>
</tr>
<tr>
<td>TH</td>
<td>Time-history</td>
</tr>
<tr>
<td>TPD</td>
<td>Transition Probability Density</td>
</tr>
<tr>
<td>TTCI</td>
<td>Time to crack initiation</td>
</tr>
</tbody>
</table>
0.2 List of Symbols

Random Variables - Random Processes:

\[ R \] (Resistance) Random Variable
\[ r \] particular outcome of \( R \)
\[ \mu_R \] Mean of \( R \)
\[ \sigma_R \] Standard Deviation of \( R \)
\[ \sigma^2_R \] Variance of \( R \)
\[ f_R(r) \] Marginal probability density function of variable \( R \)
\[ F_R(r) \] Marginal cumulative distribution of variable \( R \)

\[ \mathbf{R} \] Vector of (resistance) Random Variables
\[ \mathbf{r} \] Particular outcome of \( \mathbf{R} \) (vector)
\[ \mu_{\mathbf{R}} \] Vector of means
\[ \sigma_{\mathbf{R}} \] Vector of standard deviations
\[ \sigma^2_{\mathbf{R}} \] Vector of variances
\[ f_{\mathbf{R}}(\mathbf{r}) \] Joint probability density function of \( \mathbf{R} \)
\[ F_{\mathbf{R}}(\mathbf{r}) \] Joint cumulative distribution of \( \mathbf{R} \)
\[ \text{cov} \] covariance matrix

\[ S(t) \] (Load) Random Process
\[ R_{SS}(t) \] Correlation function of process \( S(t) \)
\[ \lambda_S \] Correlation length of process \( S(t) \)
\[ \alpha \] irregularity factor
\[ G(w) \] Power Spectrum Density function
Distributions:

\[ \phi(x) \] probability density of standard normal distribution
\[ \Phi(x) \] cumulative distribution of standard normal distribution
\[ N(\mu, \sigma) \] Normal distribution with moments \( \mu \) and \( \sigma \)
\[ LN(\mu, \sigma) \] Log-normal distribution
\[ iLN(\mu, \sigma) \] Inverted log-normal distribution
\[ RL(p) \] Raleigh distribution with parameter \( p \)

\[ E[.]. \] Expected value operator
\[ P[.]. \] Probability of the event in brackets

FORM analysis:

\[ T(z) \] Transformation to reduced space
\[ u = T(z) \] Standard normal variable in reduced space
\[ u^* \] Design Point in reduced space
\[ u_{n+1} \] additional variable in FPI solution
\[ \beta \] Reliability Index
\[ \alpha^2 \] sensitivity coefficients
\[ n_{conv} \] number of iterations for convergence
\[ n_{rv} \] number of random variables of the problem
\[ n_{si} \] number of simulations
Time variant reliability:

\[ v^+_S(r,t) \] Rate at which load process \( S(t) \) crosses barrier level \( r \) from below (up-crossing rate)

\[ v^+(r,t) \] Corrected up-crossing rate

\[ v^+_A(r,t) \] Up-crossing rate of amplitude (envelope) process

\[ v^+_D(g,t) \] Failure domain out-crossing rate

\[ v^+_I(r,t) \] Conditional up-crossing rate for which the assumption of independence is adequate

\[ \eta^+(r,t) \] Hypothetical function for which Poisson assumption is exact

\[ P_f(t) \] Failure probability

\[ P_S(t) \] Probability of survival

\[ P_{f_0} \] Initial failure probability \( (t = 0) \)

\[ P_f(t|r) \] Conditional failure probability

\[ \mu \] Mean of normalized random barrier

\[ \sigma \] Standard Deviation of normalized random barrier

\[ v^+_E D(\mu, \sigma, t) \] Ensemble up-crossing rate

\[ v^+_E I(\mu, \sigma, t) \] Conditional ensemble up-crossing rate for which the assumption of independence is exact or....

...Poissonian arrival rate of first crossing over random barrier.

\[ E_p(\mu, \sigma, t) \] EUR error parameter

\[ E_D(\mu, \sigma, t) \] Dependency EUR error

\[ \overline{E}_D(\mu, \sigma, t) \] Approximated EUR error

\[ v^+_E C(\mu, \sigma, t) \] Corrected ensemble up-crossing rate

\[ \overline{E}^v_D(\mu, \sigma, t) \] Averaged error estimate for time-variant barriers

\[ \overline{E}^r_D(\mu, \sigma, t) \] Reduced error estimate for time-variant barriers
Crack Propagation:

\[ A(t) \] Crack size random process
\[ A_0 \] Initial crack size
\[ A_{\text{crit}} \] Critical crack size
\[ A_{\text{th}} \] Threshold crack size
\[ c \] Crack propagation rate (deterministic)
\[ X(t) \] Unitary crack propagation rate random process
\[ K_{IC} \] Critical stress intensity factor
\[ S_y \] Yielding stress
\[ \Delta S \] Stress ranges
\[ m \] Crack propagation exponent
\[ \Delta K \] Stress intensity factor
\[ Y(a) \] Geometry function
\[ Q(.) \] Generic crack propagation function
\[ m(A, t) \] Drift coefficient
\[ \sigma(A, t) \] Diffusion coefficient
\[ T \] Design life or reference time
\[ T_{\mu}(a) \] Mean time to grow function
\[ TTTG(a_0, a) \] Total time to grow function
0.3 Motivation

The technological development experienced in most countries in the post second world war years has been accompanied by a massive build-up of civil infrastructure. This infrastructure includes large fleets of commercial airliners, military aircraft, ships, oil tankers, offshore platforms, pipeline systems, railways, road and railway bridges, re-entry spacecraft, mining equipment, nuclear and conventional power generators, transmission towers, dams, buildings and so many others. This huge amount of structural systems is in the process of ageing, and an increasing number of these structures is facing retirement of service. Perception that the investment in existing facilities and their replacement costs are extremely high are driving governments, regulating agencies, companies, engineers and decision makers alike to face a very concerning question:

"How can the life of ageing infrastructure be safely extended?"

In addition to the life extension problem, it is recognized that in many cases the level of loading of existing structures has exceeded expected design loads, due to increasing demands placed on these structures. Hence, there are potential serviceability and safety concerns about existing structures (Melchers, 2001).

There are significant differences between the design of an yet-to-be-built structure and the assessment of the same structure after many years in service. Whereas design codes tend to be conservative, excess of conservativeness in the assessment of existing structures may predict imminent failure and lead to unnecessary inspections, repairs or condemnation, at very high costs. Probabilistic-based risk assessment can be used to guide the development of codes for the on-going safety management of ageing infrastructure.

Probabilistic-based risk assessment provides a framework to manage the operation of existing structures in consideration of inherent uncertainty, in the absence of past experience, and on an individual basis. Moreover, it provides defensible estimates of the safety of individuals and of the safety and performance of structural systems in accordance to specified minimum safety levels. In probabilistic-based risk assessment, or more specifically in structural reliability analysis, account can be given to the uncertain initial state of a structure, to random or varying structural parameters, to the action and combination of random environmental loads, to operational uncertainty and to further resistance degradation of ageing structures.

The general problem of time-variant structural reliability analysis under resistance degradation has not received as much attention from the structural engineering community as
might be desired. In the chapters that follow, it will be shown that the handling of significant resistance degradation under random loading through existing time-variant reliability techniques poses severe difficulties. These difficulties concern the excessive number of numerical out-crossing rate calculations that are required when limit state functions are not given in closed form, load processes are non-stationary, resistance parameters are uncertain and when there is significant resistance degradation. This thesis addresses and makes some contributions towards time-variant reliability analysis of degrading uncertain structures.

Fatigue and fracture are one of the most important forms of structural degradation, accounting directly or indirectly for 10 to 40% of the US GDP (Fuchs and Stephens, 1977). Fatigue and fracture are the single most common “predictable” cause of failure of engineering structures. It is a particularly important failure mode of metal structures subject to random environmental loading. In the study of fatigue and crack propagation many early researches adopted a deterministic point of view, despite the inherent scatter observed in fatigue experiments and in the field. Today, fatigue and crack propagation are widely accepted to be random phenomena, and adopting a probabilistic standpoint helps understanding of the problem.

In this thesis, contributions are made towards probabilistic-based risk assessment procedures for structures subject to fatigue and fracture under random loading. More specifically, stochastic models of crack propagation are combined with time variant reliability models, providing a framework for risk assessment which accounts for critical crack growth and overload failure modes, and also allows the consideration of non-destructive inspections and repairs.

0.4 Outline

This thesis is divided in two complementary parts. The first half of the thesis is dedicated to time-variant reliability analysis under general forms of resistance degradation, and addresses the ensemble up-crossing rate approximation. In the second half, this approximation is applied to the problem of fatigue and fracture reliability analysis under random loading. The literature review sections are divided accordingly. Literature Review Part I (chapter 1) addresses general time-variant reliability analysis. Literature Review Part II (chapter 7) addresses random fatigue and crack propagation. It is considered that this disposition enhances the clarity of the exposition. At the end of each chapter, a Concluding Remarks section summarizes the most important results of that chapter and introduces the topics to be addressed in the following chapter. Figures are grouped consecutively at the end of each chapter.