Appendix A

Solution of the integral in
Section 4.3

We were not able to find a full analytical solution of the integral in (4.42) and (4.51) in standard references [Zwi96, Bur73, Jef94]. Therefore, we attempt to obtain the solution of the integral here.

The integral can be divided into two parts:

\[ L = \int_{-\omega_c}^{\omega_c} \frac{\omega^2 - \omega^2}{(\omega_i^2 - \omega^2)^2 + 4\zeta_i^2 \omega_i^2 \omega^2} d\omega \]
\[ = \omega_i^2 L_1 - L_2 \]  
(A.1)

where

\[ L_1 = 2 \int_0^{\omega_c} \frac{1}{(\omega_i^2 - \omega^2)^2 + 4\zeta_i^2 \omega_i^2 \omega^2} d\omega \]
\[ L_2 = 2 \int_0^{\omega_c} \frac{\omega^2}{(\omega_i^2 - \omega^2)^2 + 4\zeta_i^2 \omega_i^2 \omega^2} d\omega. \]  
(A.2)

Since both integrands are even functions of \( \omega \), it is sufficient to integrate them only from 0 to \( \omega_c \) as in (A.2).

The denominator can be written as:

\[ (\omega_i^2 - \omega^2)^2 + 4\zeta_i^2 \omega_i^2 \omega^2 = \omega^4 + 2\omega_i^2(2\zeta_i^2 - 1)\omega^2 + \omega_i^4 \]
\[ = (\omega^2 - \omega_{r1}^2)(\omega^2 - \omega_{r2}^2) \]  
(A.3)
where

\[
\begin{align*}
\omega_{r_1}^2 &= \omega_i^2 \left( 1 - 2\zeta_i^2 - 2\zeta_i \sqrt{1 - \zeta_i^2 j} \right) \\
\omega_{r_2}^2 &= \omega_i^2 \left( 1 - 2\zeta_i^2 + 2\zeta_i \sqrt{1 - \zeta_i^2 j} \right)
\end{align*}
\] (A.4)

and \(j = \sqrt{-1}\).

We consider only the under damped case \((\zeta_i < 1)\) since the case captures the majority of resonant systems of interest. However, the solution for critically damped and over damped cases can also be obtained in a more straightforward manner since \(\omega_{r_1}^2\) and \(\omega_{r_2}^2\) will be real numbers.

### A.1 First integral, \(L_1\)

The integrand of \(L_1\) can be written as partial fractions:

\[
\frac{1}{(\omega_i^2 - \omega^2)^2 + 4\zeta_i^2 \omega_i^2 \omega^2} = \frac{1}{\Omega_r} \left( \frac{1}{\omega^2 - \omega_{r_1}^2} - \frac{1}{\omega^2 - \omega_{r_2}^2} \right)
\] (A.5)

where

\[
\Omega_r = \omega_{r_1}^2 - \omega_{r_2}^2 = -4\zeta_i \sqrt{1 - \zeta_i^2 \omega_i^2 j}.
\] (A.6)

Consider the following indefinite integral with a complex constant \(a\). The solution to the indefinite integral is [Spi81]

\[
\int \frac{dz}{z^2 - a^2} = \frac{1}{2a} \ln \left( \frac{z - a}{z + a} \right) + c
\] (A.7)

where \(c\) is a constant. The integral \(L_1\) (A.2) can be solved by incorporating (A.5) and (A.7) to give

\[
L_1 = L_1^{\omega_i} - L_1^0
\] (A.8)

where

\[
L_1^{\omega_i} = \frac{1}{\Omega_r} \left\{ \frac{1}{\omega_{r_1}} \ln \left( \frac{\omega - \omega_{r_1}}{\omega + \omega_{r_1}} \right) - \frac{1}{\omega_{r_2}} \ln \left( \frac{\omega - \omega_{r_2}}{\omega + \omega_{r_2}} \right) \right\}.
\] (A.9)
Define:
\[
\cos \alpha = 1 - 2\zeta_i^2. \tag{A.10}
\]
Consequently,
\[
\sin \alpha = 2\zeta_i \sqrt{1 - \zeta_i^2}. \tag{A.11}
\]
From (A.10) and (A.11), the expressions in (A.4) and (A.6) can be re-written as:
\[
\begin{align*}
\omega_{r1} &= \omega_i e^{-j\frac{\alpha}{2}} \\
\omega_{r2} &= \omega_i e^{j\frac{\alpha}{2}} \\
\Omega_r &= -2\sin \alpha \omega_i^2 j. \tag{A.12}
\end{align*}
\]
Using the previous expressions, we obtain
\[
\begin{align*}
\omega - \omega_{r1} &= r_a e^{j\theta_a} \\
\omega + \omega_{r1} &= r_b e^{-j\theta_b} \\
\omega - \omega_{r2} &= r_a e^{-j\theta_a} \\
\omega + \omega_{r2} &= r_b e^{j\theta_b} \tag{A.13}
\end{align*}
\]
where
\[
\begin{align*}
\omega &= \sqrt{\omega^2 - 2\omega \omega_i \cos \frac{\alpha}{2} + \omega_i^2} \\
\omega &= \sqrt{\omega^2 + 2\omega \omega_i \cos \frac{\alpha}{2} + \omega_i^2} \\
\theta_a &= \cot^{-1} \left( \frac{\omega - \omega_i \cos \frac{\alpha}{2}}{\omega_i \sin \frac{\alpha}{2}} \right) \\
\theta_b &= \cot^{-1} \left( \frac{\omega + \omega_i \cos \frac{\alpha}{2}}{\omega_i \sin \frac{\alpha}{2}} \right) \tag{A.14}
\end{align*}
\]
and \(\cot^{-1}(\Gamma)\) denotes the inverse cotangent of \(\Gamma\).

The following expressions can be obtained from (A.13):
\[
\begin{align*}
\ln \left( \frac{\omega - \omega_{r1}}{\omega + \omega_{r1}} \right) &= \ln \left( \frac{r_a}{r_b} \right) + j(\theta_a + \theta_b) \\
\ln \left( \frac{\omega - \omega_{r2}}{\omega + \omega_{r2}} \right) &= \ln \left( \frac{r_a}{r_b} \right) - j(\theta_a + \theta_b). \tag{A.15}
\end{align*}
\]
After some algebraic manipulation, it can be shown that $L_1^\omega$ (A.9) is real valued. This is as expected since the optimal feedthrough term will be real valued:

$$L_1^\omega = \frac{-1}{\omega_c^3 \sin \alpha} \left\{ \sin \frac{\alpha}{2} \ln \left( \frac{r_a}{r_b} \right) + \cos \frac{\alpha}{2} (\theta_a + \theta_b) \right\}. \quad (A.16)$$

The next task is to change the variables to the original variables. For this case, $\theta_a + \theta_b$ can be found from trigonometric identities [Zwi96]:

$$\cot (\theta_a + \theta_b) = \frac{\cot \theta_a \cot \theta_b - 1}{\cot \theta_a + \cot \theta_b} = \frac{\omega^2 - \omega_1^2}{2 \omega_1 \sin \frac{\alpha}{2}}. \quad (A.17)$$

Writing $\ln (r_a/r_b)$ as $-0.5 \ln (r_b^2/r_a^2)$ and using (A.17), the indefinite integral (A.9) becomes

$$L_1^\omega = \frac{1}{2 \omega_1^3 \sin \alpha} \left\{ \sin \frac{\alpha}{2} \ln \left( \frac{r_b^2}{r_a^2} \right) - 2 \cos \frac{\alpha}{2} \cot^{-1} \left( \frac{\omega^2 - \omega_1^2}{2 \omega_1 \sin \frac{\alpha}{2}} \right) \right\}. \quad (A.18)$$

Now, $L_1$ (A.8) can be evaluated from (A.18) by substituting $\omega$ with $\omega_c$ and $0$ respectively:

$$L_1 = \frac{1}{2 \omega_1^3 \sin \alpha} \left\{ \sin \frac{\alpha}{2} \ln \left( \frac{\omega_c^2 + 2 \omega_1 \omega_c \cos \frac{\alpha}{2} + \omega_1^2}{\omega_c^2 - 2 \omega_1 \omega_c \cos \frac{\alpha}{2} + \omega_1^2} \right) - 2 \cos \frac{\alpha}{2} \cot^{-1} \left( \frac{\omega_c^2 - \omega_1^2}{2 \omega_1 \sin \frac{\alpha}{2}} \right) + 2 \pi \cos \frac{\alpha}{2} \right\}. \quad (A.19)$$

By considering the trigonometric identities $\cos \alpha = 2 \cos^2 \frac{\alpha}{2} - 1 = 1 - 2 \sin^2 \frac{\alpha}{2}$ [Bur73] and (A.10):

$$\cos \frac{\alpha}{2} = \sqrt{1 - \zeta_i^2}$$

$$\sin \frac{\alpha}{2} = \zeta_i. \quad (A.20)$$

### A.2 Second integral, $L_2$

We consider the second integral $L_2$ (A.2). The integrand can again be written as partial fractions:

$$\frac{\omega^2}{(\omega_1^2 - \omega^2)^2 + 4 \zeta_i^2 \omega_1^2 \omega^2} = \frac{1}{\Omega_r} \left( \frac{\omega_{r_1}^2}{\omega^2 - \omega_{r_1}^2} - \frac{\omega_{r_2}^2}{\omega^2 - \omega_{r_2}^2} \right). \quad (A.21)$$
Using (A.7), the integral is

\[ L_2 = L_2^\omega - L_2^0 \]  \hspace{1cm} (A.22)

where

\[ L_2^\omega = \frac{1}{\Omega_r} \left\{ \omega_{r1} \ln \left( \frac{\omega - \omega_{r1}}{\omega + \omega_{r1}} \right) - \omega_{r2} \ln \left( \frac{\omega - \omega_{r2}}{\omega + \omega_{r2}} \right) \right\}. \]  \hspace{1cm} (A.23)

\( L_2^\omega \) (A.23) can be shown, after some algebraic simplification, to be real valued as expected:

\[ L_2^\omega = \frac{1}{\Omega_r} \left\{ \omega_{r1} \ln \left( \frac{r_a}{r_b} \right) + \cos \frac{\alpha}{2} \beta \right\} \]

\[ = \frac{1}{2\omega_1 \sin \alpha} \left\{ - \sin \frac{\alpha}{2} \ln \left( \frac{r_a}{r_b} \right) - 2 \cos \frac{\alpha}{2} \cot^{-1} \left( \frac{\omega^2 - \omega^2}{2\omega_1 \sin \frac{\alpha}{2}} \right) \right\}. \]  \hspace{1cm} (A.24)

Then (A.22) can be evaluated using (A.24) after substitution of \( \omega \) with \( \omega_c \) and 0 respectively:

\[ L_2 = \frac{1}{2\omega_1 \sin \alpha} \left\{ - \sin \frac{\alpha}{2} \ln \left( \frac{\omega^2 + 2\omega_1 \omega_1 \cos \frac{\alpha}{2} + \omega^2}{\omega^2 - 2\omega_1 \omega_1 \cos \frac{\alpha}{2} + \omega^2} \right) \right. \]

\[ - 2 \cos \frac{\alpha}{2} \cot^{-1} \left( \frac{\omega^2 - \omega^2}{2\omega_1 \omega_1 \sin \frac{\alpha}{2}} \right) + 2 \pi \cos \frac{\alpha}{2} \]. \hspace{1cm} (A.25)

The integral \( L \) (A.1) can be solved using (A.19) and (A.25) as follows:

\[ L = \frac{1}{\omega_1 \sin \alpha} \sin \frac{\alpha}{2} \ln \left( \frac{\omega_c^2 + 2\omega_1 \omega_1 \cos \frac{\alpha}{2} + \omega^2}{\omega_c^2 - 2\omega_1 \omega_1 \cos \frac{\alpha}{2} + \omega^2} \right) \]

\[ = \frac{1}{2\omega_1 \cos \frac{\alpha}{2}} \ln \left( \frac{\omega_c^2 + 2\omega_1 \omega_1 \cos \frac{\alpha}{2} + \omega^2}{\omega_c^2 - 2\omega_1 \omega_1 \cos \frac{\alpha}{2} + \omega^2} \right). \]  \hspace{1cm} (A.26)

where \( \cos \frac{\alpha}{2} = \sqrt{1 - \zeta_r^2} \) (A.20).
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<td>Hermite cubic polynomial</td>
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<td>$I$</td>
<td>second moment of area; unit matrix; number of modes; number of transducers</td>
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</table>
$j$ imaginary number

$J$ cost function; number of transducers

$J_p$ polar moment of inertia

$J_r, J_2, J_\infty$ cost function

$J_t$ torsional parameter

$k$ proportional term in a feedthrough term; number of modes in an FE model

$k_{31}, k_{32}$ electromechanical coupling factor

$K$ stiffness matrix; constant related to piezoelectric actuator moment; feedthrough term; controller transfer function

$\dot{K}$ global stiffness matrix

$\mathcal{K}$ modal observability

$L$ length; integral; Lagrangian

$L_o$ observability Gramian matrix

$L$ differential operator: stiffness

$m$ distributed mass

$M$ bending moment; number of modes

$M$ global mass matrix

$M$ differential operator: mass; modal controllability

$N$ number of admissible functions; number of elements in an FE model; number of modes

$N_c$ number of controlled modes

$p$ perimeter of a section; pressure

$P$ transfer function gain; solution of Lyapunov inequality

$q$ generalized coordinate; electric charge

$Q$ shear force; generalized force; spatial weighting function

$r$ point coordinate; radius of a circle

$R$ control weight; controller feedthrough term

$\mathbb{R}$ set of real numbers

$S$ strain vector; controllability Gramian matrix

$\mathcal{S}_c$ spatial controllability

$\mathcal{S}_o$ spatial observability

$\mathcal{R}$ Rayleigh’s quotient; spatial domain

$T$ tension; transfer function

$T$ kinetic energy

$T^*$ reference kinetic energy

$t$ time

$u$ displacement; system input

$v$ displacement; sensor signal

$V$ transducer voltage

$\mathcal{V}$ potential/strain energy

$V_{\text{max}}$ maximum potential energy
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<td>$w$</td>
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<td>$z$</td>
<td>point coordinate; system output</td>
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<td>$\beta$</td>
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<td>$\Omega$</td>
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subscripts
\(a\)  actuator
\(b\)  beam
\(e\)  elemental (local)
\(i\)  beam mode number
\(m, n\) plate mode numbers
\(n\)  \(n^{th}\) element
\(p\)  piezoelectric patch
\(s\)  sensor

acronyms
FE  finite element
LTI  linear time-invariant
ODE  ordinary differential equation
MIIO  multiple-input, infinite-output
MIMO  multiple-input, multiple-output
PDE  partial differential equation
SISO  single-input, single-output
\(\text{Re}(F)\)  real part of \(F\)
\(\text{tr}\{M\}\)  trace of the matrix \(M\)
\(M^*\)  conjugate transpose of the matrix \(M\)
\(M^T\)  transpose of the matrix \(M\)
\(\lambda_{\text{max}}(M)\)  maximum eigenvalue of the matrix \(M\)
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