Chapter 9

Conclusions

This thesis has investigated several issues in vibration analysis and control of smart structures. The main contributions of the thesis are as follows:

(i) We have considered a new approach for model correction of pointwise models of resonant systems that include damping. A convex numerical optimization procedure was developed to calculate the optimal feedthrough term that minimizes either the $\mathcal{H}_2$ norm or $\mathcal{H}_\infty$ norm of the error system.

(ii) We extended the previous model correction approaches for pointwise and spatial models of resonant systems that include damping. We included the effect of damping in the systems and obtained analytical solutions for the optimal feedthrough terms. The feedthrough terms minimize $\mathcal{H}_2$ norm and spatial $\mathcal{H}_2$ norm of the error systems for pointwise and spatial models respectively. There is no limitation on the number of out-of-bandwidth modes that can be considered in the calculations of the optimal feedthrough terms. In addition, we also extended the model correction approaches for models that are obtained from approximate methods, such as from the FE method. Hence, model correction for various systems with considerable damping can also be dealt with.

(iii) We extended the methodology for optimal placement of actuators by including an extra constraint to reduce the effect of control spillover.

Furthermore, we proposed a new approach for optimal placement of sensors. For this purpose, we introduced a new spatial measure, that is the spatial observability measure. We used this concept to develop an optimization methodology for optimal placement of sensors. We generalized the methodology for models that are obtained from approximate methods, not just from modal analysis. Further, since the experimental modal analysis method can be incorporated with the FE model, the optimal placement can be done for models obtained from system identification.

(iv) The optimal placement of collocated piezoelectric actuator/sensor pair over a plate structure was considered. We designed and built the experimental apparatus of a plate and its frame supports. The experimental results confirmed the simulations.

(v) We proposed a class of MIMO resonant controllers for vibration control of smart structures. We also introduced a class of resonant controllers that are robust against parametric uncertainties and unmodelled dynamics in the model. The optimization procedure to determine the controller damping ratios was also developed. We designed and implemented the resonant controller on a piezoelectric laminate beam for minimizing vibrations in spatially-averaged sense. It was observed that the controller succeeded in reducing the second and third resonances by about 20 dB and 15 dB respectively.

(vi) We extended the concept of spatial $\mathcal{H}_2$ norm for vibration control of smart structures. We designed the controller and implemented it on a piezoelectric laminate beam. Experiments showed that the implemented controller managed to reduce the beam vibrations due to the first six modes. A significant vibration reduction of over 20 dB was obtained for some modes. We showed that, compared to the pointwise $\mathcal{H}_2$ controller, the spatial $\mathcal{H}_2$ controller minimized the $\mathcal{H}_2$ norm of the closed-loop system more uniformly over the entire structure.

(vii) We also used the concept of spatial $\mathcal{H}_\infty$ norm for vibration control of smart structures. We designed and implemented the controller to minimize vibrations due to the first six modes of a piezoelectric laminate beam. The controller had sufficient robustness properties and succeeded in reducing vibrations due to the first six modes. The controller achieved considerable vibration reduction of over 20 dB for some modes. We also demonstrated experimentally that, compared to the pointwise $\mathcal{H}_\infty$ controller, the spatial controller minimized the $\mathcal{H}_\infty$ norm of the closed-loop system in a more uniform manner.

The experiments demonstrated that the developed resonant, spatial $\mathcal{H}_2$ and $\mathcal{H}_\infty$ controllers can be implemented successfully on real systems for minimizing structural vibrations.

Some recommendations for future research are given below.

(i) A different model correction approach can be considered. In this case, the optimal feedthrough terms can be found by minimizing either the $\mathcal{H}_\infty$ norm or spatial $\mathcal{H}_\infty$ norm of the error system. Pointwise and spatial models of multivariable systems with damping can be considered. A simpler case of the model correction for multivariable pointwise models without damping effect can be considered first.

(ii) The proposed optimal placement methodology can be applied to more complicated structures. For example, the optimal placement of piezoelectric actuators and sensors over cylindrical structures can be investigated. Multi-body systems such as structures with multiple flexible links can also be investigated. Those structures can be modelled with approximate methods such as the FE method, and the proposed optimal placement methodology can be used. Experimental apparatus for such structures can be built for this purpose. The methodology can be extended to include the optimal sizing and orientation of piezoelectric patches.

(iii) The feasibility of a more general class of multivariable resonant controllers can be explored. The passivity theory may be used to define a particular case of robust resonant controllers. An optimization procedure can be developed to choose the controller parameters for efficient structural vibration reduction. This procedure may include the determination of the resonance frequencies of the controller to reduce the structural resonances more effectively. The controllers can be implemented on piezoelectric laminate structures to demonstrate their effectiveness.

(iv) Implementation of spatial $\mathcal{H}_2$ and spatial $\mathcal{H}_\infty$ controllers for more complicated systems such as structures mentioned in (ii). Experiments can be done to test the effectiveness of the controllers in minimizing structural vibrations.

(v) The issue of spatial robustness can be analyzed thoroughly. This allows spatial uncertainties in the model to be incorporated in the design of a robust controller. The effect of the spatial uncertainties on the robustness of the closed-loop system can be analyzed. Experimental implementation of the spatial robust controller can also be done. For instance, the performance of the developed controller can be investigated by introducing some spatial uncertainties to the structure. The boundary conditions can be varied or several extra masses can be introduced at some locations over the structure. Types of spatial uncertainties that are physically representative of real structures can also be investigated.