MATHEMATICAL THINKING AND MATHEMATICS ACHIEVEMENT OF STUDENTS IN THE YEAR 11 SCIENTIFIC STREAM IN JORDAN

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the degree of Doctor of philosophy
DECLARATION

I hereby certify that the work embodied in this thesis is the result of original research and has not been submitted for a higher degree to any other University or Institution.

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Abstract

Mathematical Thinking and Mathematics Achievement of Students in the Year 11 Scientific Stream in Jordan

The first aim of this study was to identify important aspects of mathematical thinking, and to investigate the relationships between the different aspects of mathematical thinking and mathematics achievement. The second aim was to examine possible gender and school location (urban, suburban, and rural) differences related to aspects of mathematical thinking and mathematics achievement.

Two assessments were developed that were suitable for students in the Year 11 scientific stream in Jordan. One test was for aspects of mathematical thinking and the other for mathematics achievement, the latter being consistent with typical school achievement tests for these students in Jordan. The researcher chose and developed items to test mathematical thinking and mathematics achievement from the Third International Mathematics and Science Study (TIMSS), the internet, research literature, specialist books in mathematics and his own experience.

The data were collected in the 2003-2004 academic year from over 500 Year 11 scientific stream students (both male and female) at 20 randomly selected schools from six directorates in the Irbid Governorate, Jordan. In addition, 13 teachers were individually interviewed, and four groups of students were interviewed in focus groups to obtain information about their opinions and about different methods of thinking in mathematics.

The teacher interviews were used to identify consistencies and inconsistencies between the test results and the respondents’ opinions of difficulty and importance. In addition, information was obtained about the classroom time teachers devoted to
the different aspects of mathematical thinking and the teaching strategies they employed.

Six aspects of mathematical thinking were identified by the study: Generalization, Induction, Deduction, Use of Symbols, Logical thinking and Mathematical proof. Mathematical proof was also the most difficult aspect, while Logical thinking was the least difficult. Female students had significantly higher mean scores than males on three of the six aspects of mathematical thinking and on the total test scores. Students attending suburban schools had significantly higher mean scores than students at urban and rural schools on four aspects, and on the total scores. Using multiple regression analysis, all six aspects were found to be important for mathematics achievement. Mathematical proof and Generalization were the most important aspects, Use of symbols and Logical thinking were next in importance, and Deduction and Induction were the least important aspects. Approximately 70 per cent of the variance in mathematics achievement was explained by the six aspects of mathematical thinking, gender, and school location.

There was a high level of consistency between teacher opinions of the relative importance of aspects of mathematical thinking and the test results. However, there were some inconsistencies between the teacher opinions and test results with respect to relative difficulty levels of the six aspects.

By clarifying the importance for mathematics achievement of the six aspects of mathematical thinking identified, this study has relevance for the teaching of mathematics to Year 11, scientific stream students in Jordan.