A Unified Approach to Linear Design and Predictive Control of Constrained Systems

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DECLARATION

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. I give consent to this copy of my thesis, when deposited in the University Library, being made available for loan and photocopying subject to the provisions of the Copyright Act 1968.

I hereby certify that the work embodied in this thesis is the result of original research, completed subsequent to admission to candidature for the degree. Note that, during the course of the candidature, several papers have been coauthored with my academic supervisors based on normal candidate-supervisor practice. I have included as part of the thesis a written statement, endorsed by my principal supervisor, attesting to my contribution to the joint publication/s/scholarly work.

He Kong
September, 2014
This thesis is the product of my Ph.D. studies at the University of Newcastle, Australia (UoN), since I came to Australia in August 2010. This journey would not have been possible without the company and support of several people.

First and foremost, I would like to express my heartfelt gratitude to my supervisors, Prof. Graham Goodwin and Dr. Maria Seron. Both of them are the best role models that a student can ever have. Their immense knowledge of system and control fields, great patience, relentless motivation and enormous enthusiasm have been instrumental in the establishment of the results of this thesis. They have taught me, both consciously and unconsciously, the importance to ask the right question and the significance of architectural issues, from a systemic perspective.

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---

1A couple of whispers. (a) I know it is weird but I have to confess: Although I was initially annoyed by their loud and unique ways of talking and laughing, later on I started feeling amused by just listening to the chats and laughter without understanding the language. Moreover, after a while, I assured myself
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that I was referred to as The Chinese Guy Sitting In The Corner in their chats sometimes, since I have developed some understanding of Spanish slowly! (b) Boris has a second good influence in bringing me to play soccer with the South American Community at campus, which I have really enjoyed on each Friday.
To my family
We make ourselves no promises, but we cherish the hope that the unobstructed pursuit of useless knowledge will prove to have consequences in the future as in the past.

_Abraham Flexner_

Our understanding of the world in which we live is inherently imperfect because we are part of the world we are seeking to understand. There may be other factors that interfere with our ability to acquire knowledge of the natural world, but the fact that we are part of the world poses a formidable obstacle to the understanding of human affairs.

_George Soros_

A noble ambition is among the most helpful influences of student life, and the higher this ambition is, the better. No man can work well unless he can speak as the Great Master did of the joy set before Him. And this leads to the greatest of all safeguards, and the most encouraging of all stimulating influences to a noble life, that is, the power of personal religion.

_Stanford Memorial Church Inscriptions (I was there in March 2010)_
Abstract

1 Introduction

1.1 Background and Motivation

1.1.1 Preliminaries on MPC

1.1.2 The Gap between Linear Control Methods and MPC

1.1.3 The Role of the Observer in Robust MPC

1.1.4 MPC for Achieving Trade-offs in Networked Control

1.2 Review of Some Aspects of Recent MPC Literature

1.2.1 Review of Inverse Optimal Control

1.2.2 Review of Pre-stabilizing Techniques in MPC

1.2.3 Review of Robust MPC

1.2.4 Review of Sparse MPC

1.3 Thesis Outline

1.4 Associated Publications

2 A Revisit to Inverse Optimality of Linear Systems

2.1 Introduction
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.3</td>
<td>Illustrative Examples</td>
<td>59</td>
</tr>
<tr>
<td>4.3.1</td>
<td>Second Order Plant with Constraints</td>
<td>59</td>
</tr>
<tr>
<td>4.3.2</td>
<td>The General Case with Constraints and Disturbances</td>
<td>63</td>
</tr>
<tr>
<td>4.4</td>
<td>Conclusions</td>
<td>63</td>
</tr>
<tr>
<td>5</td>
<td>Robust Model Predictive Control: The Role of the Observer</td>
<td>65</td>
</tr>
<tr>
<td>5.1</td>
<td>Introduction</td>
<td>65</td>
</tr>
<tr>
<td>5.2</td>
<td>The Role of the Observer in Robust MPC</td>
<td>66</td>
</tr>
<tr>
<td>5.2.1</td>
<td>Some Practical Reflections</td>
<td>66</td>
</tr>
<tr>
<td>5.2.2</td>
<td>Minimal Observer Requirements</td>
<td>67</td>
</tr>
<tr>
<td>5.3</td>
<td>Robust Analysis Incorporating Observer Dynamics</td>
<td>67</td>
</tr>
<tr>
<td>5.3.1</td>
<td>Problem Formulation</td>
<td>68</td>
</tr>
<tr>
<td>5.3.2</td>
<td>The LMI Condition on the Cost Function</td>
<td>71</td>
</tr>
<tr>
<td>5.3.3</td>
<td>Satisfaction of Constraints and Robust Stability</td>
<td>76</td>
</tr>
<tr>
<td>5.3.4</td>
<td>Upper Bound on the Nominal Cost</td>
<td>78</td>
</tr>
<tr>
<td>5.3.5</td>
<td>Illustrative Example</td>
<td>79</td>
</tr>
<tr>
<td>5.4</td>
<td>Beyond Luenberger Observers</td>
<td>81</td>
</tr>
<tr>
<td>5.5</td>
<td>Conclusions</td>
<td>83</td>
</tr>
<tr>
<td>6</td>
<td>An SVD-based Sparse Strategy for Networked Control</td>
<td>85</td>
</tr>
<tr>
<td>6.1</td>
<td>Introduction</td>
<td>85</td>
</tr>
</tbody>
</table>
6.2 SVD-based Sparse Strategy ......................................... 86
  6.2.1 SVD of the Optimization Problem .......................... 87
  6.2.2 SVD-based Sparse Algorithm: The Unconstrained Case .... 90
  6.2.3 Dealing with Convex Input Constraints ....................... 91
6.3 Stability Analysis .................................................... 94
  6.3.1 Perturbation Analysis ........................................... 94
  6.3.2 Trajectory-based Cost Reduction ............................. 98
6.4 Extensions to Reference Tracking ................................ 101
  6.4.1 Fundamentals of MPC for Reference Tracking ............... 101
  6.4.2 Embellishments on the SVD-based Sparse Strategy for Tracking .... 103
6.5 Realization of the Sparse Communication Strategy .............. 106
  6.5.1 Downlink Communications ................................... 106
  6.5.2 Uplink Communications ...................................... 106
6.6 Illustrative Examples ................................................. 107
  6.6.1 About the Model of Paper Machine Control ................ 107
  6.6.2 The Unconstrained Case ..................................... 110
  6.6.3 The Constrained Case ....................................... 111
  6.6.4 The Reference Tracking Case with Constraints ............ 114
6.7 Conclusions .......................................................... 116

7 Conclusions ................................................................ 119
This thesis studies the use of model predictive control (MPC) for linear systems from a design perspective. Our focus is on establishing a unified approach of linear design methods and MPC so that the benefits of the two can be obtained in a well-defined way for practical applications.

For this purpose, we have considered the design question from both the control and communication point of view.

On the control side, starting from a basic assumption that an unconstrained pre-stabilizing controller is available, we have considered several closely-related issues. Specifically, we have proposed novel tuning techniques so that an MPC controller can either replace an existing controller or gradually improve the control performance based on the latter. We have presented a detailed stability proof for these techniques. In this thesis we have also discussed the role of the observer in robust MPC. As such, we have considered systems with unstructured uncertainty and presented some design methods of robust policies. We have shown via theoretical analysis and numerical simulation that the choice of the observer makes a key difference in the resulting closed-loop performance.

On the communication side, we have presented a sparse communication strategy for networked control systems (NCS) based on the singular value decomposition (SVD) of the Hessian of the quadratic performance index generally considered in MPC and the unconstrained optimal controller. The singular vectors are employed to generate an orthonormal basis function expansion of the unconstrained solution to the finite horizon optimal control problem. The proposed control law is deduced from the former unconstrained controller based on cost reduction consideration. We have presented a thorough study of the associated stability analysis and have shown the advantages of the proposed method via simulation studies.
Nomenclature

Sets

\( \mathbb{C} \) the complex numbers
\( \mathbb{Z}^+ \) the set of positive integers
\( \mathbb{R}^n \) the \( n \) dimensional Euclidean space
\( \mathbb{R}^{n \times m} \) the \( n \times m \) matrices of real numbers
\( \mathbb{R}^{n \times m}[s] \) the \( n \times m \) polynomial matrices expressed as a function of \( s \) with \( s \in \mathbb{C} \)
\( \mathbb{U} \oplus \mathbb{V} \) Minkowski set sum of the sets \( \mathbb{U} \) and \( \mathbb{V} \), i.e., \( \mathbb{U} \oplus \mathbb{V} = \{ u + v \mid u \in \mathbb{U}, v \in \mathbb{V} \} \)
\( \mathbb{U} \ominus \mathbb{V} \) Pontryagin difference of the sets \( \mathbb{U} \) and \( \mathbb{V} \), i.e., \( \mathbb{U} \ominus \mathbb{V} = \{ x \mid x \oplus \mathbb{V} \subseteq \mathbb{U} \} \)
\( A\mathbb{U} \) image of the set \( \mathbb{U} \subseteq \mathbb{R}^m \) under the linear map \( A \in \mathbb{R}^{n \times m} \), i.e., \( A\mathbb{U} = \{ Ax \mid x \in \mathbb{U} \} \)

Matrices and Inequalities

\( \rho(A) \) the maximum magnitude of the eigenvalues of \( A \)
\( \lambda(A) \) the eigenvalues of \( A \)
\( I_n \) (or \( I \)) identity matrix of dimension \( n \times n \) matrices (or of appropriate dimension)
\( 0_{n \times m} \) (or \( 0 \)) matrix of zeros of dimension \( n \times m \) matrices (or of appropriate dimension)
\( A^T \) transpose of \( A \in \mathbb{R}^{n \times m} \)
\( A^{1/2} \) the square root of \( A > 0 \) (or \( A \geq 0 \))
\( \text{diag}[X, \ldots, Y] \) block diagonal matrix with \( X, \ldots, Y \) as its diagonal blocks
\( A > B \) \( A - B \) is positive definite (where \( A \) and \( B \) are symmetric matrices)
\( A \geq B \) \( A - B \) is positive semi-definite (where \( A \) and \( B \) are symmetric matrices)
Norms

\[ \| \cdot \| \] 2-norm of a vector or vector sequence in \( \ell_2 \), or \( H_2 \) norm of a transfer function

\[ \| \cdot \|_\infty \] 2-norm of a vector or vector sequence in \( \ell_\infty \), or \( H_\infty \) norm of a transfer function

Acronyms

MPC Model Predictive Control
NCS Networked Control Systems
SVD Singular Value Decomposition
DU Disturbances/Uncertainties
LTI Linear Time Invariant
ARE Algebraic Riccati Equation
IOCP Inverse Optimal Control Problem
CMFD Coprime Matrix Fraction Descriptions
LFT Linear Fractional Transformation
LMIs Linear Matrix Inequalities
LQR Linear Quadratic Regulator
PMC Predictive Metamorphic Control
CD Cross Direction
NMSS Non-minimal State Space
EBRL Extended Bounded Real Lemma
BRL Bounded Real Lemma
SDP Semi-definite Program
RHC Receding Horizon Control
MD Machine Direction
MHE Moving Horizon Estimation