The thesis contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. I give consent to the final version of my thesis being made available worldwide when deposited in the University’s Digital Repository, subject to the provisions of the Copyright Act 1968.

Anastasia M. Suchowerska Iwanec
I hereby certify that the work embodied in this thesis contains published paper/s/scholarly work of which I am a joint author. I have included as part of the thesis a written statement, endorsed by my supervisor, attesting to my contribution to the joint publication/s/scholarly work.

Prof. John Carter
Supervisor

Dr. James Hambleton
Supervisor
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<th>Description</th>
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<tbody>
<tr>
<td>$A$</td>
<td>curve fitting coefficient</td>
</tr>
<tr>
<td>$a$</td>
<td>rock mass constant used in Hoek-Brown failure criterion</td>
</tr>
<tr>
<td>$b$</td>
<td>thickne of interburden between top seam and second seam, also referred to as IB</td>
</tr>
<tr>
<td>$b$</td>
<td>bulking factor</td>
</tr>
<tr>
<td>$c$</td>
<td>rock mass cohesion</td>
</tr>
<tr>
<td>$D$</td>
<td>thickness of bedding layers</td>
</tr>
<tr>
<td>$E$</td>
<td>Young’s Modulus of the strata</td>
</tr>
<tr>
<td>$E_c$</td>
<td>Young’s modulus of coal</td>
</tr>
<tr>
<td>$E_g$</td>
<td>Young’s modulus of the goaf</td>
</tr>
<tr>
<td>$E_{i}$</td>
<td>initial tangent Young’s modulus of the Terzaghi strain-stiffening material</td>
</tr>
<tr>
<td>$E_o$</td>
<td>Young’s modulus of overburden strata</td>
</tr>
<tr>
<td>$E_s$</td>
<td>secant Young’s modulus</td>
</tr>
<tr>
<td>$E_t$</td>
<td>tangent Young’s modulus</td>
</tr>
<tr>
<td>$G_{iso}$</td>
<td>isotropic shear modulus</td>
</tr>
<tr>
<td>$G'$</td>
<td>independent shear modulus</td>
</tr>
<tr>
<td>GSI</td>
<td>Geological strength index</td>
</tr>
<tr>
<td>$H$</td>
<td>depth of top seam, also referred to as OB</td>
</tr>
<tr>
<td>$h_c$</td>
<td>height of caving above the longwall panel roofline</td>
</tr>
<tr>
<td>$h_g$</td>
<td>height of caved zone above the longwall panel floor</td>
</tr>
<tr>
<td>$h_f$</td>
<td>maximum height to failing surface</td>
</tr>
<tr>
<td>$K$</td>
<td>ratio of horizontal in situ stress to vertical in situ stress</td>
</tr>
<tr>
<td>$K_s$</td>
<td>relative shear stiffness value of a joint set</td>
</tr>
<tr>
<td>$N$</td>
<td>stability parameter</td>
</tr>
<tr>
<td>$M$</td>
<td>stability parameter</td>
</tr>
<tr>
<td>$m_i$</td>
<td>material constant used in Hoek-Brown failure criterion</td>
</tr>
<tr>
<td>$m_{ib}$</td>
<td>reduced value of material constant used in Hoek-Brown failure criterion</td>
</tr>
<tr>
<td>$Q$</td>
<td>stability parameter</td>
</tr>
<tr>
<td>$R$</td>
<td>shear compliance of a joint set</td>
</tr>
</tbody>
</table>
\( s \)  
rock mass constant used in Hoek-Brown failure criterion

\( s_i \)  
convergence of the roof and the floor of a longwall panel at the time of contact with caved goaf

\( S \)  
percentage sandstone in the interburden

\( S_{\text{edge}} \)  
vertical subsidence above the edge of the longwall panel

\( S_{\text{max}} \)  
maximum vertical subsidence above the longwall panel

\( S_{\text{max1}} \)  
maximum incremental subsidence above a first seam longwall panel

\( S_{\text{max2}} \)  
maximum incremental subsidence above a second seam longwall panel

\( T \)  
exttracted coal seam thickness

\( T_x \)  
exttracted thickness of seam, where \( x \) is the order of seam extracted

\( W \)  
width of longwall panel from centreline of gateroad

\( W_{\text{eq}} \)  
equivalent extracted width for horizontal stress redistribution

\( (W/H)_{\text{crit}} \)  
critical width of cavity corresponding to the boundary of subcritical and supercritical failure of the overburden

\( w \)  
width of pillar (rib to rib)

\( \beta \)  
abutment or shear angle

\( \gamma \)  
unit weight of rock mass

\( \varepsilon \)  
strain

\( \theta \)  
the angle between the failure surface and vertical

\( \sigma_{ci} \)  
uniaxial compressive strength of intact rock

\( \sigma_n \)  
normal stress

\( \sigma_t \)  
tensile strength of the rock mass

\( \sigma_{t*} \)  
tension cut-off, i.e., prescribed maximum tensile strength of the rock mass

\( \sigma_h \)  
horizontal stress

\( \sigma_{hi} \)  
initial horizontal stress before mining of the first seam

\( \sigma_v \)  
vertical stress

\( \sigma_{vi} \)  
initial vertical stress before mining of the first seam

\( \sigma'_{ij} \)  
effective major principal stress

\( \sigma'_{ij} \)  
effective minor principal stress

\( \sigma'_{3\text{max}} \)  
upper limit of \( \sigma_j' \) for calculating equivalent Mohr-Coulomb parameters

\( \tau \)  
shear stress

\( \nu \)  
Poisson’s ratio
φ  

**Parameters for Wilson’s equation for vertical stress distribution around a longwall panel (Wilson, 1980)**

- \( F \) constant
- \( C \) stress dissipation constant
- \( k \) Rankine passive stress state constant
- \( M \) height of extraction
- \( p \) restraint on boundary of opening
- \( q \) overburden stress
- \( \sigma_0 \) unconfined compressive strength of coal
- \( \sigma'_0 \) unconfined compressive (residual) strength of failed coal
- \( \hat{\sigma} \) peak abutment stress

**Parameters for Gerrard’s equations for equivalent elastic moduli of a rock mass consisting of orthorhombic layers (Gerrard, 1982)**

- \( t \) thickness of a stratum layer with constant Young’s modulus
- \( \alpha \) constant
- \( \beta \) constant
- \( \gamma \) constant
- \( \zeta \) constant
- \( \lambda \) constant
- \( \chi \) constant
ABSTRACT

In coal mining, the most favourable and most easily won coal reserves are depleted first, typically from within a single coal seam. A recent trend in Australia and elsewhere in the world is to attempt to recover coal from multiple seams within a single site, a practice known as multi-seam mining. With longwall mining becoming one of the safest and most economical means of underground extraction of coal in Australia, we are likely to see an increase in the number of multi-seam longwall mining operations. Evidence thus far has indicated that the geomechanics of multi-seam longwall mining differs from that of single-seam longwall mining, especially with respect to variations in mine stability and subsidence.

The overarching aim of this Thesis is to critically compare predicted stresses and deformations for single-seam and multi-seam longwall mines based on commonly used constitutive laws and continuum-based modelling assumptions. The main approach used to predict stresses and deformations is the displacement finite element method. Finite element limit analyses of roof collapse are also considered. In all cases, two-dimensional (plane strain) conditions were assumed, and focus is on relatively wide longwall panels at shallow depth, known as supercritical longwall panels. Key objectives are to predict stress redistributions in multi-seam longwall mines, roof collapse in underground openings, and subsidence profiles above single-seam and multi-seam longwall mines.

The changes in the vertical and horizontal stress distribution due to the extraction of a series of parallel longwall panels were predicted using isotropic and anisotropic linear elastic constitutive laws to represent the coal measure strata. The key finding from the study of vertical stress redistribution is that the abutment angle, the overburden depth, the pillar width and the anisotropic behaviour most influence the change in the in situ vertical stress in the lower seam. The redistribution of horizontal stress originally transmitted through the overburden generates smaller changes to the in situ stresses in the rock strata below the first mined seam than is predicted for the vertical stress. Transversely isotropic material causes the vertical stresses imposed onto the chain pillars to be transferred deeper into the underlying strata. The implications of the findings are that the predicted rapid changes in vertical stress with horizontal distance in transverse isotropic strata behaviour are likely to be reflected in more sudden changes in...
The differences occurring in predictions of roof collapse in underground rectangular cavities using the Hoek-Brown and Mohr-Coulomb failure criteria were evaluated. The predicted shape of the failure surface is shown to be governed by the friction angle of the rock mass. The friction angle also governs the so-called critical width, which corresponds to the boundary between subcritical and supercritical failure of the overburden. The predictions of the critical width matches best field observations in the New South Wales coalfields when the linear Mohr-Coulomb failure criterion is used with a friction angle of approximately 30 degrees. The prediction of the critical width when using the Hoek-Brown failure criterion overestimates the value observed in the field. This is because the Hoek-Brown failure criterion corresponds to effectively high friction angles in the range of tensile and very low confining stresses encountered in the strata above underground openings. Stability charts for rectangular cavities using the Hoek-Brown failure criterion and two forms of the Mohr-Coulomb failure criterion are presented to enable designers of underground openings to predict rapidly the safe widths of underground cavities.

Predictions of vertical subsidence profiles above single-seam and multi-seam longwall panels are compared using various constitutive laws to represent the coal measure strata and goaf. A key finding is that the best agreement between the numerical predictions and the field observations, for both the single-seam and multi-seam supercritical longwall cases, is when the coal measure strata is represented as an elastic material with closely spaced frictionless interfaces representing bedding planes. Representing the coal measure strata as a bedded material also allows for the vertical stresses to return to the level of the original overburden stress in the caved goaf material within the first seam, prior to extraction of the second seam. The results show that more sophisticated and numerically taxing constitutive laws do not necessarily lead to more accurate results when compared to field measurements. A case study based on a multi-seam mine in the Hunter Valley assists in validating the conclusions made in the comparative study.

The findings presented in this Thesis will enable engineers to design economically viable multi-seam longwall mines, while still meeting legislative needs in terms of the environment and safety of personnel.
CHAPTER 1. INTRODUCTION

There is an ever growing trend, both in Australia and overseas, where coal is being extracted in the vicinity of previously extracted seams of coal. This is in response to a situation where the areas of easily obtainable coal having already been extracted. Multi-seam mining, shortened from multiple seam mining, is the term used when coal is extracted above or below a previously extracted coal seam. Technological advances have enabled the longwall method of mining to become the most economical and safe means of extracting underground coal in the last couple of decades. The occurrence of multi-seam mines where each seam is extracted using the longwall method is only a recent phenomenon. Predicting failures and subsidence associated with multi-seam longwall mining has been identified to be more complex than when mining the first seam.

Multi-seam longwall mines are expected to become pervasive in the future as the demand for coal continues (Howarth et al., 2009). Worldwide coal production has doubled in the last two decades with demands predicted to continue to rise (US Energy Information Administration, 2012). Coal currently generates 41% of the world’s electricity (World Coal Association, 2013) and constitutes a large percentage of trading commodities for many countries. Australia, as one of the world’s largest coal exporters, traded $55 billion in 2008-09 (Australian Bureau of Agricultural and Resource Economics and Sciences, 2011), which represented around 23% of Australia’s total exports of goods and services. If we wish to continue meeting overseas demand there will be a need to pursue coal from seams above or below those already mined.

Coal mines are dangerous places to work with many potential hazards, e.g., roof collapse, gas explosions, gas poisoning and suffocation. The implementation of work and safety legislation has placed pressure on mining practices to take a greater interest in ensuring that miners are safe in their work environment. Similarly, the introduction of Environmental Planning Acts has required an assessment to be conducted of the impact on the environment of any new mining project.

It is the role of the geotechnical engineer to propose a mine design that ensures safe working conditions as well as the most economic means of extracting the coal. However, presently, there are no adequate geotechnical engineering guidelines for multi-seam longwall planning, approval or design processes in Australia, or worldwide.
1.1. Longwall mining

Longwall coal mining involves removing long panels off a coal face and allowing the roof to collapse once the mining face has progressed sufficiently (Figure 1.1). It is usually limited to horizontal or sub-horizontal seams. The face is mined by shearsers, which continually traverse the longwall panel removing coal from the face. The coal is then loaded onto a conveyor belt and taken for crushing and transport. A line of hydraulic-powered shields is set back from the coal face supporting the roof to protect the miners and mining equipment. When the shearer advances, the shields are brought forward allowing for the unsupported roof to collapse. This worked-out area of a mine is known as the goaf (also referred to as a gob).

![Figure 1.1 - Schematic diagram of the cut-away view of typical panels and gate roads during longwall mining (adapted from Infomine Inc., 2011).](image-url)

In an underground longwall operation, development roadways (or openings) are firstly excavated through the coal using roadheader machines. This is followed by the extraction of the coal via longwall mining methods. These roadways are generally referred to as gateroads (Figure 1.1). Chain pillars of coal are left to support these gateroads, which may in fact consist of two or three roadways on either side of a longwall panel. The gateroads adjacent to an unmined longwall panel are called the maingate and they are usually used for movements of material and intake air in one of
the individual roadways, and personnel in another. The gateroads adjacent to an already mined longwall panel is called the tailgate and these usually carry return air away from the longwall face. This arrangement means that the maingate of the current panel being mined will eventually become the tailgate when the adjacent panel is mined. Barrier pillars are left at either end of the longwall panel to support the roadways excavated at the ends of the panel.

Goaf formation occurs in progressive stages behind the hydraulic shields. When a new panel is extracted, the immediate longwall roof does not collapse until the face advances enough to reach a certain unsupported span. The volume of bulked caved material increases on the longwall floor, while the overlying overburden sags and the longwall floor heaves. When enough bulked caved material forms on the longwall floor, the sagged overburden can then contact the caved material (Peng et al., 1984; Salamon, 1990; Thin et al., 1993). This idealised description of goaf formation is for a bulking controlled goaf and is dependent on the in situ stresses and geology of the strata.

After extraction of the longwall panel, the deformed overlying strata are often subdivided into three zones (Figure 1.2) (Peng et al., 1984; Haycocks et al., 1990; McNally et al., 1996): caved goaf, fractured zone and deformed zone. The caved goaf is highly disturbed strata consisting of the immediate longwall roof, which has broken up and fallen onto the mine floor. The fractured zone is defined as strata broken up by vertical and horizontal cracks creating a network of blocks in full or partial contact across fractures. The deformed zone consists of strata deformed by gradually bending and sagging over large horizontal distances, with no major cracking cutting through the thickness of the strata. This zoning system is only qualitative because of the difficulty in subdividing a graded medium into distinct zones, but also due to the lack of detailed field information on post mining strata.
1.2. Geomechanics of longwall mining

Design of a safe and economical mine requires accurate prediction of the deformations and failure (collapse) that may occur during and after mining operations. Failure of the strata around any underground openings is one of the primary safety hazards in any longwall mine and has the potential to significant delay the mining schedule, which impacts the profitability of the mine. Apart from the possibility of collapse, subsidence is another key issue that geotechnical engineers need to predict, as it is important to both mining companies and the wider community. Subsidence is the term used to describe deformations of the ground surface. Subsidence is often the main source of criticism from the general public, because it can cause damage to infrastructure at the ground surface and negatively impact the natural environment.

Deformation and the potential for collapse of the coal measure strata, which consist of the coal seams and the geological layers above and below, depends on a number of influencing factors. The major factors include in situ stress (especially the vertical and horizontal stress), deformation and strength of the strata, geological configuration of the coal measure strata (e.g., depth of overlying strata) and the configuration of the extracted region of coal (e.g., longwall panel width). In single-seam mining, the in situ stresses arise due to gravity and the geological conditions. In multi-seam mining, the original in situ stresses are affected by extraction of coal in adjacent seams.
1.2.1. Instabilities

Failure of strata can be loosely subdivided into two categories: local failures around roadways and total collapse of the strata overlying the longwall panel. Local failures include roof collapse, significant floor heave and pillar failure. Total collapse of the overlying strata is inherent to the longwall mining process and can be further subdivided into two groups. So-called subcritical failures are contained entirely within the overburden height, whereas supercritical failures extend through the entire overburden. In Australian coalfields, coal seams are located at relatively shallow depths. Typically longwall panels are designed as wide as possible as this is most economical. Wide longwall panels at a shallow depth lead to supercritical failures. Focus in this Thesis is therefore on supercritical longwall panels.

Preventing failures around roadways involves implementation of ground control. Ground control is the term used to define all measures installed in an underground coal mine to prevent failure or excessive deformations of the roof, sides or floor. It primarily includes the installation of rock bolts, mesh and shotcrete. The phrase “adverse mining conditions” is used when the installed ground control does not prevent excessive deformations or ultimately allows for failure. These adverse mining conditions can include floor heave, pillar spalling (also known as rib sloughage), roof falls and gate road entry closure (Morsy et al., 2006; Mark et al., 2007). Design of ground control measures requires an assessment of potential failures that may occur during the mining processes in order to develop a support system that can prevent collapse from occurring. Effective ground control measures have allowed for an increase in the rate of coal production recorded over the last decade (Mitchell, 2009).

The potential for collapse is often ascertained in two steps. First, the in situ stresses are surmised from field data or theoretical predictions. Second, regions where the stress state satisfies or violates an assumed failure criterion are identified. It is common practice to disregard shear stresses (or otherwise treat them separately) and predict regions of failure based on the distributions of vertical and horizontal normal stress (e.g., Gadde et al., 2004). Such a procedure is in fact approximate, since the stresses are redistributed when failure of the material occurs. To define the exact location and shape of the failure surface, an analysis incorporating the failure criterion from the outset (e.g., elastoplastic analysis) is required.
1.2.2. Subsidence

Ground surface displacements above an extracted longwall panel form a three-dimensional feature called a subsidence trough, as schematically shown in Figure 1.3. Although the true deformation is three-dimensional, two-dimensional cross sections of the subsidence trough, oriented either parallel or perpendicular to the length of the longwall panel, are typically used in geotechnical reports. This is because it is easier to analyse and compare the shapes of two-dimensional subsidence profiles. A schematic representation of a subsidence profile is presented in Figure 1.4. Although the term subsidence refers to both vertical and horizontal ground movements, it is commonly used for only the vertical movements, a convention retained in this Thesis. Other key parameters used to describe ground surface movements above extracted coal mining works are shown in Figure 1.4: horizontal displacement, tilt, curvature and strain (Mine Subsidence Engineering Consultants, 2007). The tilt is the gradient or steepness of the subsidence bowl. The curvature is the second derivative of the subsidence profile and is the rate of change of the tilt. The strain is the change in horizontal distance between two points as a result of either bending or extension of the ground.

The complex nature of subsidence has led to the development of numerous methods for the prediction of magnitudes and profiles of subsidence (Whittaker et al., 1989; Bahuguna et al., 1991). Currently in Australia, both empirically derived relationships and theoretical models are used in subsidence predictions.

![Figure 1.3 - Schematic diagram of the subsidence trough that forms behind the longwall face.](image-url)
1.3. Multi-seam longwall mining

Multi-seam longwall panels can be positioned in a seam above or below a previously mined seam. The following four cases exhibit distinguishably different conditions: (Mark et al., 2007): undermining, where the proposed seam is below a previously mined seam (Figure 1.5(a)); overmining, where the proposed seam is above a previously mined seam (Figure 1.5(b)); simultaneous mining where the proposed seam is above or below another active seam; and ultra-close mining where the proposed seam is so close to a previously mined seam that the primary concern is interburden failure (Figure 1.5(c)). This Thesis only considers the undermining form of multi-seam mining, as knowledge of possible collapse and deformations for this case can be extended to the behaviour observed in overmining, simultaneous mining and ultra-close mining. In the case of undermining, the strata between the ground surface and the first mined seam are called the overburden (OB) and the strata between the first mined seam and the second mined seam are referred to as the interburden (IB) (Figure 1.5(a)) (Chekan et al., 1992; Ellenberger et al., 2003).
Extraction of a longwall panel disrupts the initial equilibrium of the strata. The in situ stresses in the strata prior to any mining activity are generally relatively homogeneous. When the roof is left un-supported behind the hydraulic shields, the strata seek a new equilibrium by redistributing the stresses previously supported by coal. For supercritical longwall panels, the caved goaf is compressed enough to allow it to support most of the overburden load. However, the chain pillars still carry additional

*Figure 1.5 - Schematic diagrams of types of multi-seam mining: (a) undermining, (b) overmining and (c) ultra-close mining (adapted from Mark et al., 2007).*
load that is not supported by the edges of the caved goaf. Therefore, the stress field becomes irregular in the surrounding strata, and these irregularities are thought to be the source of adverse mining conditions when multi-seam mining. Ground instabilities generated by stress irregularities caused by previous mining in strata are referred to as ‘multi-seam interactions’.

1.4. Uncertainties in single-seam and multi-seam longwall mining

Empirical and theoretical models are used to predict collapse and deformation, yet they differ widely in terms of how they consider the influencing factors discussed in Section 1.2. Empirical methods are usually based on direct relationships determined through limited observations or measurements, which restricts the development of our understanding of how all the influencing factors interact to govern the mechanics of the system. The site specific nature of empirical data also makes it difficult to transfer available empirical relationships to new mining environments. In the case of multi-seam longwall mining, the limited number of multi-seam mining cases means that the empirical method is not yet available to use for this new mining environment.

Theoretical models in principal can account for the effect of each of the influencing factors, even though they differ considerably in implementation. Whether analytical or numerical, theoretical models can be broadly classified by how the material is represented, namely whether the material is treated as a continuum or a collection of discrete particles. In both cases, faithfully capturing the in situ stresses is difficult. Modelling coal measure strata as a continuum is difficult because the mechanics of the discontinuous rock mass needs to be smeared into a homogeneous material and represented by an appropriate constitutive law. In discrete methods, on the other hand, the difficulty in modelling coal measure strata arises in selecting the shape of the discrete particles and the associated interaction between them. Both the locations of discontinuities delimiting the particles and the contact interactions are site specific and still poorly understood.

While the incremental nature of the longwall mining process is three-dimensional, there are advantages in studying longwall mining assuming plane strain (two-dimensional) conditions, in both the transverse and longitudinal directions. Models of the transverse cross sections, parallel to the advancing face, can be used to study the behaviour of maingates, tailgates, pillars, longwall widths and side abutments. Models of the
longitudinal cross-sections, as a slice through the centre of the longwall, provide insight into the stress distributions local to the face. In either case, it is generally recognised that modelling in three dimensions should result in more realistic results. However, three-dimensional models can be prohibitively difficult to construct, and three-dimensional analyses require substantially longer run times compared to two-dimensional models.

Currently the understanding of instabilities is somewhat lacking in the multi-seam mining environment. Ground instabilities local to the roadways has been reported to be more pronounced when multi-seam mining than when mining in virgin ground. Pronounced ground instabilities when multi-seam mining have been attributed to irregularities in the in situ stresses present in the strata caused by the mining of the first seam. Further, prediction of the collapse of the whole interburden or overburden has also posed to be more complex than when single-seam mining. The severe consequences of not identifying the potential instabilities is highlighted in the example of Liddell Underground mine, Australia, where complete interburden collapse resulted in abandonment of the mine (Holt, 2001).

Apart from predicting collapse in multi-seam longwall panels, subsidence is the other key geotechnical issue that is proving to be difficult to predict for multi-seam longwall panels. The subsidence profile recorded above multi-seam longwalls differs with respect to the shape and magnitude compared to those observed in single-seam mining. In the few accounts of subsidence observed in multi-seam mining in New South Wales, Australia, it has been noted that subsidence above multi-seam mining cannot be predicted using the methods currently used for single-seam mining (Li et al., 2007; Mine Subsidence Engineering Consultants, 2007). The lack of a comprehensive database is the current limitation for the use of empirically derived relationships.

Prediction of in situ stresses and deformations using continuum-based models requires specification of a constitutive law. Currently, there is no consensus on the most appropriate constitutive law for representing coal measure strata. Numerical modellers have used a sweep of constitutive laws that vary in complexity, ranging from isotropic linear elasticity to strain-softening elastoplasticity. No comparative study to ascertain which models are most appropriate for predicting collapse and subsidence has been undertaken.
1.5. Objectives and scope

The overarching aim of this Thesis is to critically compare predicted stresses and deformations for single-seam and multi-seam longwall mines based on commonly used constitutive laws and continuum-based modelling assumptions. The major objectives are to:

- Compare the changes in the vertical and horizontal stress distribution due to the extraction of a series of parallel longwall panels as predicted using isotropic and anisotropic linear elastic constitutive laws to represent the coal measure strata.
- Evaluate the differences occurring in predictions of roof collapse in underground rectangular cavities using the Hoek-Brown and Mohr-Coulomb failure criteria, both of which are commonly used to represent strength of the coal measure strata.
- Compare predicted vertical subsidence profiles above single-seam and multi-seam longwall panels using various constitutive laws to represent the coal measure strata and goaf.

The main approach used to predict stresses and deformations is the displacement finite element method. Finite element limit analysis is also used for prediction of roof collapse mechanisms and corresponding stability numbers. Results are assessed with respect to analytical solutions and existing field data, which enables identification of approaches that are capable of providing realistic predictions.

The analyses conducted as part of this Thesis are two-dimensional and consider the cross section transverse to the length of the longwall panels. This simplifying assumption, discussed in Section 1.4, is made in light of not only the inherent difficulty in developing three-dimensional models but also the prevailing methods used by practitioners. Furthermore, the geometry of the longwall panels is limited to only consider supercritical longwall panels.

1.6. Thesis outline

The Thesis is organised as follows. First, the current knowledge and understanding of aspects of geomechanics of single-seam and multi-seam supercritical longwall mining is presented in Chapter 2. The effect of extraction of a longwall panel on the in situ stress and associated strains in the surrounding strata are described. The evidence indicating
that the geomechanics differs around multi-seam longwall panels than for single-seam longwall panels is detailed.

The research described in Chapter 3 and 4 attempts to identify which variables contribute to a redistribution of the in situ vertical and horizontal stresses, respectively, in strata underlying a coal seam extracted using the longwall method. In both cases, isotropic and anisotropic linear elastic constitutive laws are used to represent the coal measure strata. A simplified approach for predicting the vertical stress redistribution in the strata below a series of parallel supercritical longwall panels (where the second seam longwalls may be positioned) is considered. The approach uses an analytically derived equation for the vertical stress distribution at the level of the first seam rather than attempt to model the coal measure strata in their entirety. For the horizontal stress redistribution, the simplifying assumption is made that the goaf material has no stiffness in the horizontal direction, which predicts the maximum possible horizontal stress that might occur in the strata below the first seam.

The prediction of the roof collapse of an underground cavity assuming the rock mass obeys the Hoek-Brown failure criterion and Mohr-Coulomb failure criterion is presented in Chapters 5 and 6, respectively. Three methods are used in the analyses: a pre-existing analytical upper bound method; a finite element upper and lower bound formulation; and a displacement finite element model. The results of all three methods are compared in a series of stability charts. The results are also compared to trends inferred from field measurements.

The shape and magnitude of subsidence profiles predicted using various assumed constitutive laws for the coal measure strata and goaf are detailed in Chapter 7 and 8 for single-seam and multi-seam longwall panels, respectively. For the single-seam case, a comparison is presented of the predicted subsidence profiles when the overburden is represented as a purely isotropic linear elastic material, an elastoplastic material and a jointed material. Chapter 8 compares the subsidence profiles predicted above multi-seam supercritical longwall panels for both stacked and staggered arrangements for the constitutive laws identified in Chapter 7. Chapter 9 provides a case study, where the predicted subsidence profiles developed in Chapter 7 and 8 are compared to field measurements from a multi-seam longwall mine located in the Hunter Valley, Australia.

As a result of the research work undertaken during the course of this Thesis, five journal
articles and conference papers have been published, as detailed below.


CHAPTER 2. LITERATURE REVIEW

This Chapter reviews the literature pertaining to general aspects of multi-seam mining and the three key areas identified in the objectives (Section 1.5): stress distributions, roof collapse and subsidence. In a later part of the Chapter, the constitutive laws that have been used to represent the coal measure strata and examples of previous numerical modelling of longwall panels are presented. It should be noted that the section on constitutive assumptions only covers material common the subsequent Chapters. Some constitutive laws are limited to specific Chapters, and in these cases, details are provided in the Chapter in which the laws are utilised.

2.1. General aspects of ground instability in multi-seam mining

Ground instability reported in case studies (Su et al., 1986; Chase et al., 2005) and correlations in empirical data (Peng et al., 1980; Su et al., 1984; Wu et al., 1986; Ellenberger et al., 2003) have identified that the geomechanics around multi-seam mining works is complex. These case studies and analyses have identified a large number of variables that appear to influence the generation of multi-seam interactions, which are presented in Table 2.1 (Haycocks et al., 1990). These variables have been grouped into fixed and controllable mining variables. The fixed variables are predetermined by geological processes and cannot be altered. The controllable variables are chosen by mine designers after consideration of how they will interact with the fixed variables and possibly affect the mining of any subsequent seams.

Table 2.1 - Variables that contribute to multi-seam interactions, with critical variables as identified by Haycocks and Zhou (1990)

<table>
<thead>
<tr>
<th>Fixed Variables</th>
<th>Controllable Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Location of mine property</td>
<td>- Mining sequence</td>
</tr>
<tr>
<td>- Overburden thickness and rock type</td>
<td>- Mining methods</td>
</tr>
<tr>
<td>- Upper and lower seam thickness</td>
<td>- Time delay</td>
</tr>
<tr>
<td>- Upper and lower seam immediate roof and floor type</td>
<td>- Relative location and orientation of workings between seams</td>
</tr>
<tr>
<td>- Interburden thickness, layering and rock type</td>
<td>- Extraction percentage</td>
</tr>
<tr>
<td>- Existence of ground water</td>
<td>- Mining layouts and entry sizes</td>
</tr>
<tr>
<td>- Surface subsidence problems</td>
<td>- Method of support</td>
</tr>
<tr>
<td>- Seam inclination</td>
<td></td>
</tr>
<tr>
<td>- Horizontal stress</td>
<td></td>
</tr>
</tbody>
</table>
Correlations in empirical data have successfully identified broad trends between the fixed and controllable variables and adverse mining conditions. However, it is not possible to obtain specific relationships from the empirical databases because of the large scatter that is present in the results. The large scatter in the empirical data is to be expected as there is normally more than one variable changing between each mine site. A summary of the broad trends between controllable and fixed variables and multi-seam interactions is discussed in the following sections.

**Controllable variables**

As identified in Section 1.3, this Thesis only considers undermining, where each seam is extracted using longwall panels. Therefore, in this summary the effect of the mining sequence, mining method and time are not discussed. Only the longwall mining method is considered, and it is assumed that there is a significant time between mining each of the seams such that the strata have settled and found a new equilibrium. The known contributions of the identified controllable factors to multi-seam interactions when undermining are expanded on below.

The optimum coal mine design would be to leave no pillars, also referred to as full extraction, because pillars are the major source of multi-seam mining interactions (Whittaker et al., 1971; Peng et al., 1980; Chekan et al., 1993). However, the main disadvantage of full extraction is that it creates gate roadway closures and creates a significantly higher mining risk environment. Chain pillars between multi-seam longwall mines can be arranged in either stacked, pyramid or offset layout (Figure 2.1). Initially the stacked layout was the preferred layout as it was hypothesised that this minimised ground control problems due to multi-seam interactions (Peng et al., 1980; Peng et al., 1984). However, it is often difficult to ensure exact stacking layouts, as this often relies on surveying skills and the accuracy of previous mine maps. More recently, the offset arrangement of pillars in multiple seams has been regarded as the more favourable layout in practice (Haycocks et al., 1990; Gale, 2004). This arrangement does mean that the pillars induce high vertical stresses on the longwall face of the second seam. However, experience has shown that stability overall is easier to manage than when the pillars are placed over roadways (Gale, 2004).
Figure 2.1 - Pillar arrangement options for multi-seam mining: (a) stacked, (b) pyramid with increasing pillar size with depth (c) off-set (adapted from Haycocks and Zhou, 1990).

In single-seam mining, small pillar widths are preferred as more coal can be removed, so the pillars are designed to the minimum size such that they will not fail during the life of the mine. These pillars are referred to as critical width pillars. In the case of multi-seam mining, critical width pillars generate the most severe multi-seam interactions as they concentrate load to the highest degree (Su et al., 1986; Chase et al., 2005). Designing wide or yield chain pillars reduces these interactions (Figure 2.2). Wide pillars always ensure a safer design, as they are less likely to yield and ultimately collapse under the redistributed load applied by the overlying strata. However, this is at the expense of reaping less coal from the seam. Wide pillars may carry the same load as a smaller pillar, but because their load is distributed over a much larger area the stress concentration is noticeably less in seams above or below. Yield pillars are designed to do just that, i.e., yield under the load transferred from the adjacent goaf. Wide pillars, typically between 35 and 55 metres, are primarily used in Australian longwall mines for adequate ground control in the presence of significant horizontal stress fields.
Ground control conditions are affected by the relative orientation of overlapping longwall panels located in different seams. Chekan (1993) concluded that it is best to approach the goaf-solid boundary at 30 degrees to the rib-edge of previous workings, but this should be considered in conjunction with in situ stresses and associated ground control concerns (see Section 2.2). The rib-edge is the term used to describe the side of a coal pillar (i.e., the sides of an underground opening which consist of coal). Multi-seam interactions are greatest when the longwall face travels perpendicular to the old rib-edges in other seams (Whittaker, 1974; Mark et al., 2007). Further, more severe mining conditions are observed when the mining face is designed to travel from solid to goaf of previous workings, rather than goaf to the solid coal (Hsiung et al., 1987; Chekan et al., 1993; Mark et al., 2007).

**Fixed variables**

The thickness of the interburden, the structure within a stratum and its rock type, and the depth of mining have been found to influence multi-seam interactions. The broad trends of how these fixed variables affect multi-seam interactions are explained here.

Empirical studies have attempted to derive a relationship between the severity of multi-seam interaction and the overburden to interburden ratio (OB/IB) (Wu et al., 1986; Ellenberger et al., 2003). Analyses of case histories found that the interaction was minimal when the ratio OB/IB was generally less than 7, while extreme interactions...
were observed when ratio OB/IB was generally greater than 16. However, there is a large degree of scatter from the case history data used to determine these relationships, as shown in Figure 2.3. In order to refine this trend, Zipf (2005) considered more variables in his analysis and concluded that stable situations should be defined as strata with OB/IB ratios less than 5, while extreme interactions can be expected when the ratio OB/IB is greater than 50. The ratios in between would require further consideration of all other variables that could influence mine stability.

The percentage of sandstone present in the interburden has been shown to minimise the possibility of adverse mining conditions due to multi-seam interactions. If the percentage of sandstone in the interburden \( S \) is less than \( S = (33.5 - B)/0.128 \), where \( B \) is the interburden thickness, then the probability for multi-seam interactions is increased (Haycocks et al., 1990; Morsy et al., 2006). However, as can be seen in Figure 2.4 there is a lot of scatter in the empirical data from which the relationship was derived. Moreover, there are not a lot of data points around the specified equation, so it would be desirable to collect more data points to confirm the proposed relationship.

The density of layering in the interburden also affects the overall potential for multi-seam interactions. Interburdens with increased layering will generally transfer pillar stresses over a larger area (Figure 2.5(a)) (Wu et al., 1987). The stress influence factor, presented in Figure 2.5(b), is a ratio of stress at a particular distance below the pillar divided by the maximum stress within the pillar.

It is still not conclusive whether the depth of mining has a direct effect on multi-seam interactions. The case study conducted on Harris number 1 Mines, in West Virginia, USA (Chase et al., 2005), indicated that multi-seam mining interaction is more probable when the depth of mining is greater than 3,000 m (10,000 ft). However, it has been disputed whether mining depth or low values of the horizontal to vertical stress ratio \( K \) govern multi-seam mining interactions (Seedsman, 2003).
Figure 2.3 - Severity of multi-seam interactions for undermining was classed as extreme, moderate and none (from Ellenberger et al., 2003).

Figure 2.4 - Influence of percentage of sandstone in the interburden on the overall stability of the interburden (Wu et al., 1987).
Each of the factors listed above has been identified as contributing to the formation of multi-seam interactions. Since the findings have been primarily deduced from field observations and correlations in empirical data they provide a true representation of the behaviour of the coal measure strata around an extracted longwall panel. However, to be able to design effective ground control it is necessary for mine designers to...
be able to identify the explicit quantitative contributions of fixed and controllable variables to the in situ stresses. The findings from the empirical data at best provide a general trend that cannot be transferred into accurate predictions of the stress environment when multi-seam mining. To do this, the stress distribution around the whole longwall panel must be considered.

2.2. Stress redistribution

To understand what variables govern the geomechanics of multi-seam interactions it is necessary to assess how stresses are redistributed as a result of mining the first seam. Stress redistribution occurs in several stages during the mining of a single-seam longwall panel: the initial excavation of roadways, the longwall retreat, the formation of the goaf and finally the compaction of the goaf. With a sound understanding of the original in situ stress field and how the stress is redistributed around the longwall panel, it is possible to predict the initial stress field when mining of the second seam begins and potential adverse mining conditions due to multi-seam interactions.

2.2.1. Virgin in situ stress

The in situ stresses, which are the stresses present within the strata prior to any mining activity, need to be determined prior to the design of any longwall panels. Unlike the design of structures in rock masses at the ground surface, the state of stress after an excavation of rock mass underground is primarily dependent on the initial state of stress in the strata. Knowledge of the initial state of stress in the strata allows mine designers to predict potential block failures or larger areas of instability (Brady et al., 1992). In the coalfields of Australia, the major principal stress axes usually lie in the vertical and horizontal planes.

The horizontal to vertical stress ratio, represented by the parameter $K$, is often considered as an important characteristic of the stress field environment (Seedsman, 2011). Australian mines typically operate at depths of 200-400 m below the ground surface with horizontal stress up to 1.5 to 4 times the vertical stresses. The in situ stresses arise from the weight of overburden strata as well as stresses of tectonic origin. The magnitude of in situ vertical stress correlates well with the product of the depth below the surface and the unit weight of overlying rock mass (Hoek et al., 1997). However, the origin of horizontal in situ stress is not completely understood.
Horizontal stresses in coal measure strata have been reported to vary significantly among coalfields around the world (Mark et al., 1994; Cartwright, 1997; Nemcik et al., 2005). Significant effort has been placed into understanding how horizontal stresses are generated in underground strata in order to shed light on horizontal stress redistribution.

It is generally recognised that the in situ horizontal stress depends on the depth below ground surface, stratum stiffness, current tectonic state of stress, or a combination of all three. Originally, it was expected that the magnitude of horizontal stress would correspond with depth below the ground surface only. This gave rise to horizontal stress ($\sigma_h$) typically being reported according to the parameter $K$, as defined as follows:

$$\sigma_h = K\sigma_v = K\gamma z$$

(2.1)

where $\sigma_v$ is the vertical stress, $\gamma$ is the unit weight of the rock mass and $z$ is the depth below the surface. Terzaghi and Richart (1952) initially suggested that the value of $K$ was independent of depth and depended on Poisson’s ratio ($\nu$) of the rock mass, according to $K = \nu/(1-\nu)$. However, measurements of horizontal stresses from around the world showed that $K$ tended to be higher, often much higher, than predicted by the Poisson effect (Brown et al., 1978). Stress measurements revealed that the magnitude of $K$ is typically higher at shallow depths and may decrease with depth. In contrast, stress measurements in United Kingdom coal measure strata did not correlate with depth but with stratum stiffness (Bigby et al., 1992). When the horizontal stress results from North America and Australia were analysed, they typically showed a correlation with both stratum stiffness as well as depth (Dolinar, 2003; Nemcik et al., 2005). Therefore, there appears to be inconclusive evidence that depth alone affects the horizontal stress magnitudes.

There have been several postulates for why the horizontal stresses would exceed the value determined by the Poisson effect. The commonly accepted reason is that stresses have been locked into the stratum from a time and environment of high compression, which could have arisen from either the strata having experienced deep burial or high tectonic compression. However, current-day deep-seated continental crust stresses have also been suggested as generators of high in situ horizontal stresses at shallower depths (Zoback et al., 2002; Zoback et al., 2007). The second proposed reason for the generation of significant in situ horizontal stress arose from analysis of the World Stress
Map. The World Stress Map is the result of a large collaborative effort of data compilation by over 40 scientists from 30 different countries (Müller et al., 2000). The World Stress Map shows that the orientation and relative magnitude of horizontal stress is quite uniform over large areas of the plate interiors. Stress measurements from coalfields from around the world conform with the tectonic stress field measurements detailed in the World Stress Map.

Although vertical in situ stress is generally well understood, there are many disputed theories as to what generates in situ horizontal stress. The main purpose of understanding how in situ stresses arise is to be able to predict better the stability of underground openings. However, if there is a further disruption to these in situ stresses, such as due to mining activity, it is important to be able to quantify how the in situ stresses will be redistributed.

### 2.2.2. Stress redistribution during roadway development

During roadway development, overburden pressures are redistributed when the gateroad is initially excavated. Pressure arches develop around the roadways to redistribute the vertical stresses around the traction-free openings (Figure 2.6(a)). This significantly reduces the vertical stress in the immediate roof of the gateroad (Seedsman, 2009). The redistribution of the vertical stress and the formation of an underground opening allows for the rock mass in the roof of the roadway to relax into the roadways excavation. Ground control measures, such as installation of rock bolts and cables, are used to support the roof from collapsing into the roadway. A structurally stable roadway roof can form if the blocks of rock present in the roof engage to form a Voussoir beam (Seedsman, 2009).

The size of chain pillars between the gateroads needs to be designed so that the pillars carry the load imposed firstly by the stress redistribution during roadways development and secondly by the additional much larger load imposed when extracting the longwall. The magnitude of the load applied onto the chain pillars as a result of the extraction of the longwall is explained further below. If pillars are not large enough to support all load applied to them, they yield. The yielding of pillars can cause even larger pressure arches to form around multiple roadways (Figure 2.6(b)).
Chapter 2 – Literature review

Figure 2.6 - Pressure arches of vertical stress formed around: (a) single roadways, and (b) around multiple roadways if individual pillars yield (adapted from Chekan et al., 1993).

The transfer of pillar stress into the underlying strata has been attributed to their size, shape, extraction ratio, degree of inhomogeneity and mechanical properties of the underlying strata, overburden thickness and horizontal stresses (Hsiung et al., 1987). Isolated remanent pillars concentrate stress more than goaf-solid boundaries in longwall mining (Mark et al., 2007). The pressure bulb theory presents an idealised stress distribution in homogeneous strata below an isolated pillar (Figure 2.7(a)) (Chekan et al., 1993).

The pillar zone of influence has been reported to extend two to three times the pillar width in a homogeneous interburden. Therefore, larger pillars in general will induce changes in the in situ stress deeper into the underlying strata. However, these changes in in situ stress will be smaller in magnitude because the load applied to the pillar is distributed over a larger area (Hsiung et al., 1987). Interburden inhomogeneity has been reported to extend the zone of influence (Figure 2.7(b)) (Su et al., 1986), which explains why zones of influence have been reported to extend as far as four pillar widths below and above a pillar (Gale, 2004) and up to eight pillar widths depending on the degree of interbedding (Figure 2.5) (Wu et al., 1987). Other geological variations (e.g., varying material strengths and stiffness, discontinuities, anisotropy) also affect the pressure redistribution (Su et al., 1986).
2.2.3. **Stress redistribution during longwall extraction**

Vertical stress redistribution around an active longwall panel was first detailed by Whittaker (1974), with the magnitude of the resulting stresses primarily dependent on the depth of mining (Isaac et al., 1988). The vertical stress that is induced in the abutments due to the extraction of a longwall panel is shown schematically in Figure 2.8. The maximum induced stress occurs 1 to 5 m into the face line and would equate to about 4 to 5 times the overburden pressure (Peng et al., 1984). The maximum induced vertical stress once the longwall has passed occurs in the ribs about 1 to 3 m from the rib-edge as is shown schematically in Section AA’ of Figure 2.8. The large width of supercritical longwall panels forces the roof to collapse after a longwall face has passed at a given location, and the mechanism of collapse was described previously in Section 1.1. The deformed and fractured zones of the overburden, as defined in Figure 1.2, can sag onto the caved goaf unless there is a thick strong geological layer that can bridge the overburden load (e.g., as found in the Newcastle coalfields (Holla, 1989; McNally et al., 1996)). When the deformed and fractured zones of the overburden sag and apply pressure onto the caved goaf, this compacts the caved goaf such that it is able to carry load once more. The vertical stress distribution after this has happened is shown schematically in Section BB’ of Figure 2.8.
It has long been considered that the caved goaf is compacted enough to be able to support the original overburden stress at the centre of the longwall panel (King et al., 1971; Wilson, 1983; Smart et al., 1987; Thin et al., 1993). Due to difficulty in instrumenting and accessing the caved goaf after it has formed, there has been no empirical data collected to verify this proposal. Field measurements have confirmed that the goaf formation process changes stresses around the gateroads (Seedsman, 2009). Many authors (Wilson, 1983; Smart et al., 1987; Trueman, 1990) have then made hypotheses as to the distance from the rib-edge for the vertical stress in the goaf to return to the overburden stress. However, similarly the lack of empirical data has prevented verification of the hypothesis.

The redistribution of horizontal stresses around longwall workings has been previously considered, as shown in Figure 2.9(a), but primarily only in the horizontal plane. The horizontal stress effects are most destructive in laminates (highly interbedded sedimentary rock), while massive rocks are relatively resistant to the effect of high

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Figure 2.8 – Vertical stress redistribution in the plane of the seam around a longwall (Whittaker, 1974)
horizontal stresses (Mark et al., 1994). The relative orientations of longwall panels to the maximum horizontal stress direction has been shown to affect ground control conditions experienced during longwall mining. Adverse ground control conditions due to relatively large horizontal stresses are typically observed at the maingate (Mark et al., 1994). The adverse ground control conditions include persistent compressive type roof failures, roof guttering and roof kinking. The horizontal stress redistribution around the goaf arises as a result of the goaf material being softer than the surrounding intact strata (Hasenfus et al., 2006). It has since been widely accepted that horizontal stress can concentrate at the tailgate or maingate corner of a longwall, as schematically shown in Figure 2.9(a). There are also tailgate and maingate corners that experience a relief of horizontal stress. The areas of relief in horizontal stress are referred to as being located in a stress shadow. Whether located in a gateroad experiencing horizontal stress concentration or stress relief, a change in mining conditions can be expected in these regions.

The consideration of the horizontal stress distribution around an extracted longwall panel has identified that adverse mining conditions can be avoided if the gateroads are orientated at an angle less than 30 degrees to direction of maximum horizontal stress (Mark et al., 1994; Kent, 1996; Meyer et al., 1999). The most adverse mining conditions are generated if the longwall panel is orientated so that the maximum horizontal stress direction acts across the roadways, i.e., the maximum horizontal stress direction is 90 degrees to the longer plan dimension of the longwall panel. If the gateroads are orientated at an angle that is larger than 60 degrees to the maximum horizontal stress, adverse conditions can be expected. This has been validated with field observations as shown in Figure 2.9(b).
Although extensive studies have been conducted on the multi-seam interactions arising from vertical stress redistribution when mining of the first seam (as summarised in...
Section 2.1), very little information has been compiled for the effect of horizontal stress redistribution on the stability when mining the second seam. Occasionally analyses have been conducted in three dimensions to assess the horizontal stress redistribution. However, the findings from these studies provide very little information about the horizontal stress distribution in the vertical plane. The in situ stress environment of Australian mines can mean that effects of horizontal stress redistribution are not easily distinguishable from those caused by the vertical stress redistribution. Early reports mention that high horizontal in situ stress conditions may be favourable when multi-seam mining as the horizontal stress may act as a confining stress and increase the strength of the interburden rocks (Su et al., 1986; Hill, 1994). However, the question of why high horizontal stress fields have not been considered a critical variable in multi-seam interactions has also come under scrutiny (Mark et al., 1994).

The redistribution of the original in situ stresses due to the extraction of the first seam plays a governing role in the severity of the multi-seam interactions that are experienced when longwall mining an underlying coal seam. There has not been a detailed study conducted to understand the effects of the initial in situ stresses when extracting a coal seam below a previously extracted longwall panel. This shall be an area of focus in the research presented in Chapters 3 and 4.

2.3. Strata deformations

Effective ground control designs and accurate subsidence predictions can only be achieved if there is an understanding on how strata will respond to induced stresses as a result of mining. The response to applied stresses is different for different rock types and geological structures (Whittaker, 1974). The strength of a rock mass governs whether the redistributed stresses can be sustained. If the rock mass is able to support the additional redistributed stresses generated by longwall mining, then the strata will remain stable. If the rock mass is not able to support these redistributed stresses then yielding and possible failure of the rock mass may occur. Such failure may manifest as caving, fracturing, slipping of discontinuities, heaving and subsidence.

There have been numerous attempts to categorise the material immediately above an extracted seam with respect to its degree of distortion and displacements. An example of such a categorisation above an extracted longwall panel is shown in Figure 2.10 (Haycocks et al., 1990; McNally et al., 1996). The caved goaf is defined as a chaotic
pile of thoroughly fragmented rock blocks that have bulked in volume from their original state as a result of falling onto the longwall floor. The fractured strata maintain their original in situ continuity, but have been extensively fractured and de-stressed by shearing along bedding, joint extension and tensile breakage through previously intact rock. The deformed zone remains relatively unfractured but has still been displaced by sagging downwards.

It is questionable whether it is appropriate to categorise the deformed overburden above an extracted longwall panel into discrete zones as there are probably no distinct boundaries. The deformed overburden likely consists of a progressive transition from a highly distorted rock mass at the longwall floor through to a rock mass with very little distortion closer to the ground surface. However, for the purpose of easy reference to the varying degrees of distortion and associated mechanical behaviour, the zone names used in Figure 2.10 are used in this Thesis.

The geological environment is understood to influence the mechanics of sub-surface strata deformations. This has been observed for the deformations around mining roadways (e.g., Holla et al., 1990; Gale et al., 1997; Seedsman, 2003). However, field investigations to characterize the distorted overburden above an extracted longwall panel are very rarely conducted. Testing the deformed overburden is difficult due to inaccessibility for sampling and an awkward drilling environment. The goaf material cannot be accessed until the entire longwall has been excavated, which prevents easy sampling of the goaf material. Drilling of the goaf poses many complications such as potentially high levels of explosive gases and water loss.
A field investigation in the Western Coalfields of NSW used mechanical anchors to successfully record how the overburden deformed as the longwall panel was extracted (Holla et al., 1990). The findings from the field measurements showed that tensile strains in the overburden were closely related to stratigraphy and proximity to the extracted seam. Larger strains were recorded in massive units (i.e., sandstone, conglomerate and siltstone). This observation correlated with the usually deeper subsidence bowls measured above longwalls where the overburden primarily comprises mudstones (e.g., United Kingdom). A physical model was also used to study the fracturing in an overburden (Sun et al., 1992). Fracture formation and propagation was found to depend on the strength of stratified rock, presence and position of a weak layer and the extraction thickness.

There is sparse information available on the fractured goaf and deformed zone, but it all suggests that the overburden strata are only a slightly distorted form of the original material prior to mining. The distortions are primarily in the form of shearing and displacements along planes of weakness, which are a function of the stratigraphy. For this reason, in this Thesis the fractured goaf and deformed zones of the overburden will be treated as if the discontinuities and the intact rock in the fractured rock mass have been smeared into a homogeneous material. The deformation and strength characteristics used to represent the smeared homogeneous material are presented in Sections 2.5.1 and 2.5.2.

### 2.3.1. Caved goaf

Several field investigations have been conducted to identify how high the caved goaf extends above the longwall floor. In a colliery located in the Hunter coalfield, the height of the caved goaf was recorded to extend twice the extracted seam height (Holla et al., 1986), while in a colliery in the Western coalfield, the height of the caved goaf was recorded to extend nine times the extracted seam height above the longwall floor (Holla et al., 1990). A summary of other field investigations used to determine the height of caving was compiled by Forster (1995). The summary highlighted that there is significant variation in the heights of the caved goaf. It is thought to be a result of the qualitative definition of the caving height which leads to some ambiguity when making assessments of field results (Holla et al., 1986).

Many engineers estimate the height of the caved goaf above the longwall floor by
assuming that the height of the caving is controlled by the bulking nature of the immediate roof of the longwall panel. This is referred to as bulking-controlled caving, as shown schematically in Figure 2.10. Bulking-controlled caving would only apply if the overburden does not contain very strong layers that bridge large distance (Salamon, 1990; Hill, 1995; Yavuz, 2004). The height of the caving above the longwall roof \( (h_c) \) in bulking-controlled caving is calculated using Equation (2.2) (Salamon, 1990):

\[
h_c = \frac{T - s_i}{b - 1}
\]

where \( T \) is the extracted seam height, \( s_i \) is the convergence of the longwall roof and floor and \( b \) is the bulking factor. Typically the convergence of the longwall roof and floor \( (s_i) \) is much smaller than the extracted seam height \( (T) \). Therefore, the total height of the caved goaf above the longwall floor can be approximated using Equation (2.3):

\[
h_g = T \left[ \frac{1}{b - 1} + 1 \right]
\]

Although it has been claimed that the geology governs the degree of bulking of the caved goaf, there appear to be discrepancies concerning its relative effect. The bulking factor \( (b) \) is defined as the total bulked volume of the caved material divided by the original volume of the intact rock prior to bulking (Pappas et al., 1993). It is equivalent to the void ratio plus 1. A bulking factor of 1.5 was found to be representative of the UK coal mining conditions which consist primarily of mudstones and shales (Whittaker et al., 1989; Trueman, 1990). A bulking factor of 1.5 indicates that the total height of caved goaf is three times the seam thickness above the longwall floor. A bulking factor of 1.2 has been typically used in numerical analyses when the overburden consists primarily of sandstone (Peng et al., 1980; Morsy et al., 2002; Morsy et al., 2006). However, these magnitudes for the bulking factors contradict the proposal that the bulking factor should be higher for sandstones than for shales (Hill, 1995; McNally et al., 1996). This is because sandstones break predominantly as larger blocks and shale fails mostly in thin slabs. Predominantly massive overburdens would have a bulking factor of 1.3-1.5 with a caved goaf height of \( (1-2)T \), while for predominantly thin-bedded overburden a bulking factor of 1.2-1.3 with a caved goaf height of \( (3-5)T \) is more appropriate (McNally et al., 1996).

Laboratory tests have indicated that the bulking factor of a caved goaf depends on the
rock strength, overburden pressure and the thickness to width ratio of the goaf particles (Pappas et al., 1993). Stronger rocks and rocks at a lower confining stress gave rise to a larger bulking factor. Also, goaf particles with a larger thickness to width ratio (e.g., a cube) gave rise to a larger bulking factor. In general, massive rocks are usually strong and break into cubes, which would correspond to a larger bulking factor. The inconsistency in the correlations of magnitudes of bulking factors with the geology highlighted here indicates that further study is required on this topic. The studies presented in this Thesis consider a bulking factor of 1.2, as this is the magnitude typically used in numerical analyses with overburden strata consisting of sandstone.

2.3.2. Field measurements of subsidence

Subsidence above single-seam panels

Field measurements of subsidence above single-seam longwall extractions have been recorded and studied for many decades. The key factors that have been found to govern the shape and magnitude of subsidence include the geometry of the extracted area of coal, the overburden geology and the surface topography (McNally et al., 1996; Mills et al., 2009).

The width \( W \) and depth \( H \) of an extracted longwall panel influence whether a subsidence profile is subcritical or supercritical. The term critical width is used to define the boundary between subcritical and supercritical subsidence profiles. A critical width is the ratio of width of a longwall panel \( W \) to overburden depth \( H \) at which maximum possible subsidence is reached (Mills et al., 2009). In many of the coalfields in Australia, critical widths of longwall panels correspond to a ratio \( (W/H)_{crit} \) of 1.0 to 1.6 (McNally et al., 1996; Mine Subsidence Engineering Consultants, 2007; Mills et al., 2009).

An indication that the overburden geology governs the magnitude of subsidence is reflected in the variation of maximum possible subsidence above supercritical longwall panels from different coalfields. In the Southern and Western Coalfields of NSW in Australia, the maximum subsidence above a supercritical longwall panel is approximately 55-65% of the seam’s extracted thickness (Mine Subsidence Engineering Consultants, 2007; Mills et al., 2009). In the Newcastle coalfields, the maximum subsidence above a supercritical longwall panel is approximately 56% of the seam’s extracted thickness. The reduced subsidence observed in the Newcastle coalfields is
attributed to the thick deposit of strong conglomerate located in the overburden above the mined seam (McNally et al., 1996; Holla, 1998).

In coalfields around the world, including the United States of America, South Africa and Germany, the magnitude of maximum subsidence above single-seam supercritical longwalls can range from 45 to 70% of the seam’s extracted thickness (Hall et al., 1981; Su, 1991; Wagner et al., 1991; Waddington et al., 1998; te Kook et al., 2008). In the United Kingdom, the magnitude of maximum subsidence is much larger at approximately 90% of the seam’s extracted thickness (National Coal Board, 1965). This marked increase observed in the United Kingdom coalfields is attributed to the lack of strong competent beds in the overburden geology (Hall et al., 1981; Webster et al., 1984; Wagner et al., 1991; Waddington et al., 1998).

The geology of the overburden strata has also been shown to affect the shape of the subsidence profile. Coal measure strata comprised predominantly of massive geological units give rise to a subsidence profile with a smaller angle of draw (McNally et al., 1996). The angle of draw is defined as the angle between the vertical line at the edge of the mine opening and the line connecting the edge of the mine opening to the limit of significant ground displacement (as depicted in Figure 1.4). The angle of draw in Australian coalfields is usually much less than for coalfields in the United Kingdom. Overburdens with predominantly massive strata also generate subsidence profiles with increased maximum tilt and maximum tensile strain. Field measurements of the average subsidence above the edge of the supercritical longwall panels, in many of New South Wales (NSW) coalfields, are less than 12% of the maximum subsidence (Holla, 1985; Holla, 1987; Holla, 1991).

Surface topography and unusual geological features have been reported to be the cause of anomalies in subsidence profiles (Kay et al., 1992; McNally et al., 1996; Holla et al., 2000). Highly variable surface topography, such as cliff faces and gorges, generally experiences significantly more horizontal movements than for flat terrain (McNally et al., 1996; Hebblewhite et al., 2000). Faults or dykes present in the strata can also give rise to anomalies, such as steps in the subsidence profile (McNally et al., 1996). This Thesis will not consider the effects of unusual geological features (such as faults and dykes) or surface topography.

The variables that affect the shape and magnitude of subsidence above single-seam
mining have been primarily identified from correlation of empirical data. It is only through the availability of a large database of empirical data that hypotheses can also be made about the associated geomechanics that give rise to shape and magnitude of subsidence profiles. It has been proposed that the geomechanics driving subsidence can be divided into essentially two independent components: sag subsidence and elastic compression of chain pillars (Mills, 1998). These two independent components are shown schematically in Figure 2.11. Pillar collapse may also contribute to the mechanics of subsidence movement. However, it is uncommon in Australian longwall workings because of the large pillar geometries required to meet acceptable gateroad conditions. Sag subsidence is the subsidence bowl that forms above a single longwall panel that occurs as a result of the overburden strata sagging downwards. Strata compression occurs as a result of the redistribution of overburden load onto the coal pillars that are left in the seam. Varying relative combinations of these two mechanisms have been used to describe the variation among subsidence profiles observed above multiple longwall panels extracted from a single-seam (Mills et al., 2009). Other than this proposal by Mills (1998) there has been very little discussion on the mechanics of the overburden strata that governs the ground surface subsidence deformation. The generally poor understanding of the mechanics of the overburden strata stems from the very little information available on the actual deformation of the sub-surface strata.

![Figure 2.11 – Schematic diagram of the combination of sag and strata compression that govern subsidence (adapted from Mills (1998)).](image-url)
Subsidence above multi-seam panels

It was noted by Haycocks and Zhou (1990) that there have been insufficient studies on the effects of multiple seam mining on surface subsidence. In the few accounts of subsidence observed in multi-seam mining in NSW, Australia, it has been noted that subsidence is generally larger over multi-seam mining than single-seam mining. This effect has been illustrated graphically in a report prepared by Mine Subsidence Engineering Consultants (2007) (see Figure 2.12). The average maximum subsidence observed above single-seam mines is 56% to 66% of the mined seam height in most of the coalfields located in New South Wales. Subsequent mining of a second seam in these coalfields has been observed to cause a further subsidence of approximately 90% of the second seam height. Therefore, the total ground surface displacement is equal to 90% of the second seam height in addition to the subsidence recorded upon extraction of coal from the first seam. Li et al. (2007) presented subsidence case studies from four multi-seam longwall undermining cases (two Australian, one South African, one from the United Kingdom). The maximum subsidence after longwall mining a single-seam ranged from 40 to 80% of the mined seam height, while the maximum subsidence after longwall mining the second seam was always between 95 and 105%.

The results from analysis of a particular mine site suggested that subsidence above multi-seam mines would typically consist of a single-seam subsidence profile with contributions from chain pillar compression and goaf consolidation of the first seam (Gale, 2004). This study also found that the pillar strength, size, positioning and interburden thickness each had an effect on the final subsidence profile. However, given that this report was from a case study, the results would be expected to be very specific to the conditions present at the site.
2.4. Numerical methods

Numerous numerical methods have been developed to suit a wide range of purposes. The most commonly used numerical methods in rock mechanics problems can be broadly grouped into the following three groups: continuum methods, discrete methods and hybrid methods. Examples of continuum methods include finite difference method (e.g., FLAC), finite element method (e.g. ABAQUS) and boundary element method. Examples of discrete methods include discrete element method (e.g., UDEC) and discrete fracture network. A comprehensive review of these numerical methods was prepared by Jing and Hudson (2002) and for a coal mining specific purpose by Larson et al. (2009). Other coal mining specific programs that are used by engineers are based on a theoretical framework (e.g., MUSLIM, LaModel, LaM2D, AMSS) or empirical data (e.g., ALPS).

2.4.1. Numerical modelling of longwall mining

Numerical modelling has only been accessible to longwall mining researchers for about
the last two decades, once the processing power of computers increased sufficiently to allow incorporation of the amount of model detail required and yet still allow a solution to be obtained in a reasonable time. There have been a handful of computer modelling studies of multi-seam mines. However, most of these studies have considered at least one of the seams to be mined using room and pillar method (Singh et al., 2002; Zipf, 2005; Morsy et al., 2006). There is limited published data on numerical analyses of multi-seam mining where both the first and subsequent seams have used the longwall method for coal extraction (Gale, 2004). Numerical modelling of multi-seam longwall mining is more difficult than when modelling the first seam being extracted using the room and pillar method. This is because modelling of a longwall mine requires consideration of the stress-strain behaviour of the caved goaf and fractured overburden.

Morsy and Peng (2002) modelled the longwall goaf material using the finite element package ABAQUS. The study presented predictions of the distribution of vertical stress in the goaf transmitted to the floor of the longwall panel, behind an advancing longwall face. The predictions generally agreed well with field measurements. This study also confirmed that there is progressive transfer of load from chain pillars to the goaf as the latter consolidates. Bard et al. (2003) conducted their analysis using the finite difference package FLAC, employing a strain-softening Mohr-Coulomb model to represent the stress-strain behaviour of the chain pillars. In this model, the goaf material response was initiated once the longwall face had passed any particular location. The conclusion from the study was that the strain-softening model gave a more realistic result than the plastic Mohr-Coulomb model in analysing the response of the yielding pillars to loading. However, it also identified that selecting values for the number of parameters required for the strain-softening model would be the primary limitation in future use. In both of the studies above, non-linear elastic relationships were used to model the behaviour of the longwall goaf.

Many numerical studies have not represented the goaf as a hardening material. In some cases the goaf has been represented as a material with a double yield surface, a bulking factor of 1.5 was adopted, such that the goaf replaces three times the mined seam height (Bigby et al., 2007). Zipf (2005) did not model the goaf bulking behaviour, but represented the longwall goaf material as a softening medium by reducing the cohesion to 10% of the peak value for all elements that had experienced at least 5 millistrains of post-peak axial strain.
Other more sophisticated models have attempted to model the caving process of the overburden (e.g., Singh et al., 2009; Vakili et al., 2009; Tang et al., 2010). One such numerical model of a longwall panel uses coupled rock failure/fluid flow system in FLAC with the Mohr-Coulomb failure criterion implemented to represent the strength of the rock mass (Kelly et al., 1998; Gale, 2001; Gale, 2004; Gale, 2005). This simulation was staged by representing the coal seam removal as a series of 1 m wide excavation increments with the numerical program ‘remembering’ the orientation of fractures as potential ubiquitous planes. However, these sophisticated computer models require detailed geotechnical strata properties and therefore the analysis results only provide site specific solutions.

Subsidence measurements were compared with predictions from a range of independently conducted analysis using various numerical codes in studies by Kay, McNabb et al. (1991) and Coulthard (1999). Most computational methods gave results that did not agree with the field observations, which led to the question of whether it is the numerical method or their inputs and assumptions that causes the differences. Some have argued that it is necessary that a numerical model of a longwall panel needs to accurately represent the blocky nature of goaf formation and possibly the three-dimensional aspect of the goafing mechanisms (Coulthard, 1999). However, the current lack of understanding of how a goaf forms together with the general lack of information on subsurface stratigraphy prior to commencement of a mining project would prevent a discrete method of accurately representing the random nature of three-dimensional goaf formation. The general lack of information on sub-surface stratigraphy can be overcome by representing the sub-surface strata with an equivalent smeared material in the continuum method.

2.5. Constitutive laws

Common to all continuum-based numerical models is the need to represent the constitutive laws of the strata surrounding underground openings. The basic elastic constitutive laws and failure criteria typically used to represent the behaviour of coal measure strata are presented below. The equation used to represent the uniaxial stress-strain behaviour of the caved goaf material is also discussed. More sophisticated constitutive laws are detailed in each Chapter, where appropriate.
2.5.1. Elastic response of coal measure strata

Initially rock masses will respond elastically to additional imposed load and this response will be governed by the elastic properties of the rock mass, primarily its Young’s modulus. Strata with relatively high values of Young’s modulus (e.g., sandstone) are able to support the load with small deformations and thus have smaller zones of influence. Strata with relatively low values of Young’s modulus (e.g., shale) have a high susceptibility to bending and thus are known to transfer loads more extensively.

Coal measure rocks, being sedimentary in origin, usually contain sub-horizontal bedding planes in conjunction with one or more sub-vertical joint sets (Brady et al., 1992). These joint sets in the coal measure stratigraphy often give rise to transverse isotropy. Figure 2.13 shows a schematic representation of a transversely isotropic material. It has been demonstrated that assuming isotropic material behaviour in sedimentary strata can lead to underestimation of the extent of the induced stress distribution (Su et al., 1986; Lightfoot et al., 2010; Seedsman, 2011) as well as poor predictions of subsidence profiles (Wardle et al., 1983; Kay et al., 1991; Coulthard, 1999). In principle, transverse isotropy can be represented in computer models explicitly by including all the discontinuities present in the strata with appropriate contact behaviours. However, this method is time-consuming and computationally taxing, and a thorough field investigation is often required to characterise the location and properties of discontinuities.

If a transversely isotropic elastic model is adopted to represent the stress-strain behaviour of the rock mass, a total of five elastic constants are required: two values of Young’s modulus ($E$), two values of Poisson’s ratio ($\nu$) and an independent value for the shear modulus ($G'$) (Brady et al., 1992). This number may be reduced to just three independent variables by assuming that the $E$ and $\nu$ defining the properties in the plane of isotropy are approximately equal to the $E$ and $\nu$ defining properties in the plane normal to the plane of isotropy. For plane strain analysis, as considered in this Thesis, the compliance matrix for a general transversely isotropic material is given by

$$
\begin{pmatrix}
\varepsilon_{xx} \\
\varepsilon_{zz} \\
\gamma_{xz}
\end{pmatrix} =
\begin{bmatrix}
1 / E & -\nu / E & 0 \\
-\nu / E & 1 / E' & 0 \\
0 & 0 & 1 / G'
\end{bmatrix}
\begin{pmatrix}
\sigma_{xx} \\
\sigma_{zz} \\
\tau_{xz}
\end{pmatrix}
$$

(2.4)
where $E$ and $\nu$ are properties in the plane of isotropy, and $E'$, $\nu'$ and $G'$ are in planes perpendicular to the plane of isotropy.

\[
E_{\text{iso}} = \frac{E}{2(1+\nu)}
\]  

(2.5)

In a transversely isotropic medium, the shear modulus in the planes perpendicular to the plane of isotropy is independent of the values of Poisson’s ratio and Young’s modulus relevant to the plane of isotropy. Thus it is called the independent shear modulus ($G'$).

Independent shear moduli have been measured for many different rock types in laboratory tests, and some of these results have been summarized by Gerrard (1977). Most of the laboratory measurements involved testing intact triaxial rock samples. Generally, published values of the independent shear modulus normalised by the Young’s modulus (i.e., $G'/E$) from this type of laboratory testing are between 0.3 and 1.

The independent shear modulus is likely to be smaller for bedded and jointed coal measure strata when those strata are represented as a single continuum, than for laboratory measurements involving testing intact triaxial rock samples. Back analysis of the heights of failures in the roof at the Donkin Morien tunnel was used by Seedsman (2009; 2011) to propose that the value for Young’s modulus normalised by the independent shear modulus ($E/G'$) should be between 15 and 20. This corresponds to a value of shear modulus normalized by Young’s modulus ($G'/E$) of 0.06 to 0.05.

Transversely isotropic rock mass properties have also been represented by incorporating...
the shear stiffness of the bedding planes into the derivation of the constitutive relations
of the rock mass (e.g., Goodman et al., 1969). If the discontinuities in the rock mass are
equally spaced and parallel to the xy plane, the independent shear modulus is given as
\( G'/E = 1(1+v)+R_s \), where \( R_s = E/SK_s \) represents the shear compliance of the joint set
with respect to the surrounding intact rock and \( S \) is the regular joint spacing. The
relative shear stiffness values \( (K_s) \) obtained from experimental testing of joints primarily
range from 0.1 to 100MPa/mm (Bandis et al., 1983). These stiffness values would
 correspond to a range of independent shear modulus values of about 0.1 to 1 times the
isotropic shear modulus \( (G_{iso}) \).

The mechanical behaviour of the bedding in coal measure strata are incorporated
implicitly and explicitly in numerical models presented in this Thesis. The relative
effects of the implicit and explicit implementation of the bedding on the solution in
question are discussed.

2.5.2. Failure criteria of rock masses

Failures occur when there is no further capacity for the strata to deform elastically.
Failure mechanisms typically observed in coal mines at a local scale can be attributed to
one or more of the following: tensile fracture of intact rock, shear fracture of intact rock,
tensile fracture of bedding (delamination) and shearing of bedding (Gale et al., 1997;
Gale, 2004; Wittles et al., 2007). These are shown in diagrammatical form in Figure
1.2. The high stresses occurring in the vicinity of a longwall face (Figure 2.8) induce
shear stresses that can generate discontinuities ahead of the longwall, which ultimately
influence the caving mechanics (Unrug et al., 1982). Microseismic monitoring at
Gordonstone Colliery in Central Queensland, Australia, indicated that such activity can
occur up to 100 m ahead of the longwall face as a result of compressive shear fracture.
Further, fracturing activity was observed 120 m above the mined seam and 30 m into
the floor (Kelly et al., 1998).

Geotechnical stability and deformation analyses usually require a failure criterion to be
assigned to the materials in question, in this case the rock mass surrounding the cavity.
The use of an appropriate failure criterion to represent the strength of a rock mass is
imperative for obtaining accurate predictions of roof collapse of underground cavities as
well as predictions of subsidence. Engineers have also considered that the rock mass
strength can be approximated with one failure criterion, where essentially the strength of the discontinuities and the intact rock has been smeared into one material. There are several failure criteria available for geotechnical and mining engineers to use when representing the strength of a rock mass, with the Mohr-Coulomb and the Hoek-Brown failure criteria being most commonly used.

The Mohr-Coulomb failure criterion has been widely used in geotechnical engineering stability problems for many decades. In this criterion the shear stress at failure ($\tau$) is governed by the cohesion ($c$) and friction angle ($\varphi$) of the material, as well as the normal stress ($\sigma_n$) acting on the failure plane, as follows

$$\tau = c + \sigma_n \tan \varphi$$  \hspace{1cm} (2.6)

where compression is considered positive. Its popularity and ease of use in many applications has led to the Mohr-Coulomb failure criterion being the primary failure criterion offered by numerical analysis packages designed for geotechnical applications.

Figure 2.14(a) shows the Mohr-Coulomb failure criterion in the shear stress-normal stress plane and also the principal stress plane (Brady et al., 1992). The tensile strength of the material ($\sigma_t$) according to the Mohr-Coulomb criterion is given by $\sigma_t = 2c\cos\varphi/(1+\sin\varphi) = \sigma_c(1-\sin\varphi)/(1+\sin\varphi)$, where $\sigma_c$ is the uniaxial compressive stress of the material defined as $\sigma_c = 2c\cos\varphi/(1-\sin\varphi)$. Equation (2.7) is the Mohr-Coulomb failure criterion in principal stress space:

$$\sigma_1 = N_\varphi \sigma_3 + 2c\sqrt{N_\varphi}$$  \hspace{1cm} (2.7)

where $\sigma_1$ and $\sigma_3$ are the major and minor principal stresses respectively, $N_\varphi = (1+\sin\varphi)/(1-\sin\varphi)$. There have been numerous proposals on methods of implementing a tension cut-off to the Mohr-Coulomb failure criterion (Chen et al., 1982), and only the most common form is considered here. The tension cut-off is applied as shown in Figure 2.14(b) by prescribing a value of maximum tensile stress of the rock mass, denoted by $\sigma_t^\ast$. This method effectively truncates the Mohr-Coulomb failure criterion.
Chapter 2 – Literature review

Figure 2.14 – Schematic representation of: (a) linear form of the Mohr-Coulomb failure criterion (adapted from Brady and Brown (1992)) and (b) prescribed tension cut-off form of the Mohr-Coulomb failure criterion (adapted from Chen (1982) and Clausen and Damkilde (2006)).

An alternative failure criterion for rock masses was developed by Hoek and Brown (1997), which essentially involved the fitting of curves to the results of triaxial strength tests on rock samples. Equation (2.8) is an updated version of the original criterion, which has been refined to consider the effects observed in lower strength materials (Hoek et al., 2002). This criterion assumes that the rock mass is an isotropic material, and it is most conveniently applied to rock masses using the Geological Strength Index (GSI). The assumption of isotropy is usually relevant for either an intact rock or otherwise a heavily jointed rock with randomly oriented discontinuities. However, this criterion is usually not appropriate if there is potential for a preferential failure mechanism to develop in the rock mass, for example as a result of the presence of a dominant set of parallel or sub-parallel discontinuities. The Hoek-Brown criterion can be expressed as:

\[
\sigma'_{1} = \sigma'_{3} + \sigma_{ci} \left( m \frac{\sigma'_{3}}{\sigma_{ci}} + s \right)^{a}
\]  

(2.8)
where $\sigma'_1$ is the maximum effective principal stress, $\sigma'_3$ is the minor effective principal stress, $\sigma_{ci}$ is the uniaxial compressive strength of the intact rock material, $m_b$ is a reduced value of the intact rock constant $m_i$, and $s$ and $a$ are constants for the rock mass which are further defined as:

$$m_b = m_i \exp \left( \frac{GSI - 100}{28 - 14D} \right)$$

$$s = \exp \left( \frac{GSI - 100}{9 - 3D} \right)$$

$$a = \frac{1}{2} + \frac{1}{6} \left( e^{-GSI/15} - e^{-20/3} \right)$$

In the equations above, $D$ is the disturbance factor, as defined in the paper by Hoek et al. (2002). The charts for $GSI$ and $m_i$ in terms of rock types can be obtained from many sources, such as the book by Brady and Brown (1992) or the paper by Marinos et al. (2005).

The Mohr-Coulomb and Hoek-Brown failure criteria both have their place in geotechnical engineering analysis. Calculating equivalent Mohr Coulomb parameters ($c$ and $\phi$) from the Hoek Brown criterion is a highly discussed topic in literature. It is useful to be able to calculate equivalent parameters comparing results from analyses in which each of the two failure criteria have been considered. However, there is no exact method to calculate equivalent Mohr-Coloumb parameters from a Hoek-Brown failure criterion. The source of the problem is an inherent issue that arises when fitting a linear relationship to the non-linear Hoek-Brown criterion.

Hoek et al. (2002) provided a set of curve-fitting equations to calculate equivalent parameters for use in the Mohr-Coulomb failure criterion. This approximation of the equivalent Mohr-Coulomb parameters requires the linear curve to be fitted over a range of stress that might be encountered in the stability problem (Hoek et al., 2002). The term $\sigma'_{\text{3max}}$ denotes the upper limit of confining stress over which the relationship between the Hoek-Brown and the Mohr-Coulomb failure criteria is considered. The magnitude of $\sigma'_{\text{3max}}$ is recommended to be 25% of the unconfined compressive strength of the intact rock ($\sigma'_{ci}$) when simulating a triaxial test (Hoek, 2000). A smaller value of $\sigma'_{\text{3max}}$ is recommended when calculating equivalent parameters of the Hoek-Brown failure criterion for use in the analyses of underground excavations (Hoek et al., 2002).
This is because the strata surrounding an underground excavation are typically exposed to confining stresses that are usually compressive and small, or possibly tensile. This secant approach to obtaining equivalent Mohr Coulomb parameters, proposed by Hoek (2002), usually over-predicts the major principal stress capacity relative to the Hoek-Brown criterion at both low and high confining stresses.

Another proposal is the tangent approach, where an instantaneous relationship is derived between the normal and shear stresses at failure and the corresponding principal stresses. This method was considered by Balmer (1952), and the details are provided in Appendix A. As with the secant approach, the Mohr-Coulomb failure envelope obtained by tangent approximation over-predicts the major principal stress capacity relative to the Hoek-Brown criterion at both low and high confining stresses (Hoek et al., 2002).

The use of numerical methods for stability analysis and prediction of subsidence requires a realistic representation of the strength of the strata present in the overburden and interburden. There is no evidence to support a preference for either the use of the Hoek-Brown or the Mohr-Coulomb failure criterion to represent the strength of coal measure strata when its properties are smeared into a homogeneous material. This Thesis will consider using both the 2002 version of the Hoek-Brown failure criterion and two forms of the Mohr-Coulomb criterion in the prediction of roof collapse and failure surface, which shall be presented in Chapter 5 and 6, respectively. The results are compared to trends observed in subsidence field data.

### 2.5.3. Constitutive law for the caved goaf material

The caved goaf has been noted to respond to load in a hardening manner (Wardle et al., 1983; Smart et al., 1987; Trueman, 1990), which cannot be described using the constitutive laws presented in Section 2.5.1. As described in Section 2.2.2, the caved goaf initially is a pile of caved material that compacts as the overlying strata deflect and apply load to it. The Young’s modulus of caved material is assumed to increase with the compaction of the caved goaf. The properties of the caved goaf have the potential to govern both subsidence and the geomechanics of multi-seam mining interactions.

A field investigation to measure the relationship between stress and strain of the caved goaf has been conducted in a longwall mine in the Bowen Basin, in Queensland, Australia (Wardle et al., 1983). A series of plate loading tests was performed on in situ
caved goaf. The measurements showed a near linear stress-strain behaviour with a magnitude of the Young’s modulus of 21MPa. However, the tests were only conducted to a maximum stress of 1.2MPa, which was only 35% of the estimated overburden stress. Measurements up to a stress of 5MPa have been collected from stone built packs to characterise the behaviour of the caved goaf in the coalfields of the United Kingdom (Trueman, 1990). Packs are pillars built from caved goaf material to support the roof. The results showed a definite trend of strain-hardening. Two stress-strain equations were determined by curve fitting of the field measurements (Smart et al., 1987; Trueman, 1990). It was thought that these equations would probably not be valid if there was strong sandstone present in the roof. Laboratory testing of the caved goaf material also found that the stress-strain relationship exhibited hardening (Pappas et al., 1993). The testing for the study ranged up to a stress of approximately 20MPa.

Stress-strain relationships for granular cohesionless materials have also been proposed (Salamon, 1990; Pappas et al., 1993). These theoretically derived equations have been used to represent the constitutive behaviour of caved goaf in numerical models (Morsy et al., 2002; Badr et al., 2003; Morsy et al., 2006; Morsy et al., 2006). The equation proposed by Salamon (1990) assumes that the secant modulus of caved goaf material is linearly proportional to stress, while the equation proposed by Terzaghi (Pappas et al., 1993) assumes that the tangent modulus is linearly proportional to stress. Both these equations assume one-dimensional deformation.

In general, the lack of measurements of caved goaf material means it is not possible to accurately define the stress-strain relationship of the caved goaf material. Therefore, currently there is no preferred constitutive law that should be used for the caved goaf. The Terzaghi elastic strain-stiffening material model was used in the studies presented in this Thesis, where the tangent Young’s modulus \( E_t \) is defined as:

\[
E_t = E_i + a\sigma \tag{2.9}
\]

In Equation (2.9), \( E_i \) is the initial tangent modulus, \( \sigma \) is the applied uniaxial stress, and parameter \( a \) is a dimensionless constant. The corresponding stress-strain relationship and secant modulus \( (E_s) \) for the caved goaf is given in Equation (2.10) and (2.11), respectively, as detailed by Morsy and Peng (2002):

\[
\sigma = \frac{E_s}{a} (e^a - 1) \tag{2.10}
\]
where $\varepsilon$ is the uniaxial strain. The ranges for parameters $a$ and $E_i$, obtained from laboratory testing of caved goaf made from shales and sandstones, are 10-15 and 5-6MPa, respectively (Pappa et al., 1993). The magnitudes of parameters $a$ and $E_i$, obtained from back analysis in a numerical model, were 355 and 31MPa, respectively (Morsy et al., 2002). These parameters used in the numerical model were selected to ensure that the magnitude of the virgin vertical stress was recovered in the centre of the longwall panel after extraction.

### 2.7 Summary

This Chapter has presented a summary of the available literature on multi-seam longwall coal mining and tools used by mining engineers to assist in prediction of failure and displacements around longwall panels. It is evident that there is some empirical data available on multi-seam interactions as well as subsidence above single-seam panels. However, our ability to predict excessive deformations and failure around multi-seam mining and subsidence above both single-seam and multi-seam longwall panels is still lacking.

Adverse mining conditions when undermining can be avoided if there is a sound understanding of the vertical and horizontal in situ stresses present in the strata around the second seam. Armed with the knowledge of the stress environment in the second seam mining engineers are able to determine areas of excessive deformation and failure. The stress environment in the second seam can be predicted by using the current knowledge of how vertical and horizontal stresses redistribute at the level of the first mined seam after longwall extraction. Chapters 3 and 4 shall present a detailed study of how controlled and fixed variables affect the initial in situ vertical and horizontal stresses, respectively, prior to mining below previously extracted longwall panels.

Currently there is no evidence to support a preference for either the use of the Hoek-Brown or the Mohr-Coulomb failure criterion to represent the strength of coal measure strata when its properties are smeared into a homogeneous material. Chapters 5 and 6 will present the prediction of collapse mechanisms and stability numbers using the 2002 version of the Hoek-Brown failure criterion and two forms of the Mohr-Coulomb
criterion, respectively. The results are compared to typical trends of the critical ratio \((W/H)_{\text{crit}}\) observed in subsidence field data and which criterion predicts better the strength of the coal measure strata are inferred.

Further investigation into the constitutive laws that best represent the deformation characteristics of coal measure strata are conducted by comparing predictions from finite element simulation of single-seam and multi-seam longwall panels. As mentioned in this review of literature, a wide range of constitutive laws has been used to represent the coal measure strata when predicting subsidence above a single-seam longwall panel. The more sophisticated constitutive laws are becoming increasingly popular, even though they can be computationally expensive and may not necessarily lead to improved predictions. There has been no detailed comparative study to identify differences in the predictions obtained using commonly assumed constitutive laws. Chapter 7 investigates where these more sophisticated constitutive laws lead to more accurate subsidence predictions above a single-seam longwall panel. Chapter 8 presents a similar study for multi-seam longwall panels. To validate the findings presented in Chapters 7 and 8, a case study from a multi-seam mine in the Hunter Valley, Australia is used to gain further insight into the geomechanics of the overburden and interburden and this is presented in Chapter 9.
CHAPTER 3. VERTICAL STRESS CHANGES UNDER SUPERCritical LONGWALL PANELS

3.1. Introduction

This Chapter attempts to identify which variables contribute to the redistribution of in situ stress in strata underlying the first mined coal seam, which has been extracted using supercritical longwall panels. This Chapter considers only the vertical normal stress distribution. The other key component is the horizontal normal stress, which will be studied in the next Chapter. Of particular interest is the magnitude and variation of vertical stress along the length of the second coal seam. Knowledge of the vertical and horizontal stress in the second seam can then be used to predict the general areas of potential failure around roadways in the second seam by interpreting the stresses relative to an assumed failure criterion.

The problem considered in this Chapter is shown schematically in Figure 3.1. The results presented are for the vertical stress distribution along a length of the second seam, which is positioned below the first mined seam. In this problem it has been assumed that the first seam has been extracted using a series of parallel supercritical longwall panels. Plane strain conditions are assumed. The geometrical parameters in the problem are the overburden depth \( H \), interburden thickness \( B \) and pillar width \( w \). The effects of transverse isotropy in the strata underlying the first mined seam are considered.

In this analysis a realistic vertical stress distribution at the level of the first mined seam is critical to predicting representative stresses in the second mined seam that would occur in the field. The two theoretical extremes for the solution to the problem being
investigated would involve either the overburden not collapsing or the whole overburden collapsing onto the longwall floor. No collapse of the overburden would require strata in the overburden to bridge all of the overburden load to the adjacent chain pillars. This would induce the highest possible maximum vertical stress into the chain pillar and therefore also into the strata underlying the chain pillar. Complete collapse of the overburden onto the longwall floor would cause vertical stress in the underlying strata to remain geostatic. In reality, the solution is somewhere between these two extremes.

There is no consensus on the best approach for arriving at the vertical stress distribution below the first mined seam. This distribution could in principle be determined using a continuum-based model with suitable constitutive laws to represent the material behaviour, and such an approach is presented in Chapters 7 and 8. In this Chapter, instead of modelling the overburden explicitly, the vertical stress distribution at the level of the first mined seam is applied as a surface load to the strata below the first seam. The magnitude of the surface load to the strata below the first seam was determined using equations derived by Wilson(1980) using analytical methods. As it is thought that the overburden vertical stress is achieved in the centre of the longwall panel (see Section 2.2.3), the width of the analysis was only limited to the chain pillar and enough distance either side for the vertical stress in the first seam goaf to return to overburden stress (as shown in Figure 3.1).

### 3.2. Background

Excessive deformations or failure of the roof, ribs or floor of the underground opening is often referred to as adverse mining conditions. Adverse mining conditions usually occur when the ground control is not adequate. Effective ground control can only be designed if there is correct prediction of both the vertical and horizontal in situ stresses present in the coal measure strata prior to extraction and during extraction of the second seam. The redistribution of in situ stresses caused by longwall mining the first seam need to be appreciated, otherwise adverse mining conditions can arise, which are often referred to as multi-seam interactions.

As mentioned in Section 2.1, the severity of multi-seam interactions has often been linked to the overburden and to the interburden thickness ratio (OB/IB), which shall be referred to as the ratio $H/B$ in this Chapter. In an undermining environment, the
overburden is the strata above the first mined coal seam and the interburden is the strata between the first and second mined seam (Figure 3.1). However, as illustrated in Figure 2.3, it is difficult to draw strong conclusions based solely on the ratio OB/IB as there is a wide scatter in the data and any correlations are unlikely to be entirely reliable.

The report by Ellenberger et al. (2003) assessed the severity of the multi-seam interactions and the influence of the ratio OB/IB did not mention if the study acknowledged the effect of the pillar width used in mining. The width of pillars has been shown to affect significantly the stress magnitudes and distributions in pillars (Su et al., 1986; Chekan et al., 1988). If both wide and yield pillars were used when mining in the cases presented in Figure 2.3, then the different pillar types would have redistributed vertical stresses differently. It may not be possible to find a good correlation for multi-seam mining when different pillar types are used, and this issue may be the cause of the large scatter observed in the data (Figure 2.3).

It has been recognised that the anisotropic response of coal measure strata can potentially affect the stress distribution in the strata after mining. However, this effect is often ignored when undertaking numerical modelling. In many cases such modelling involves treating the overall rock mass as a continuum (also referred to as ‘equivalent material analysis’) (Gerrard et al., 1985). As identified by Lightfoot and Liu (2010), an isotropic elastic medium often does not represent coal measure rocks accurately, because the bedding in coal measure stratigraphy generally inhibits the lateral spread of stress. For the purpose of stability analysis of underground coal extraction, it has been claimed that considering the transverse isotropic behaviour of the rock mass is more significant than considering its nonlinear behaviour (Seedsman, 2011). A study conducted by Su (1991) confirmed that considering the planes of weakness in coal measure rock was more important than consideration of non-linear material behaviour when attempting to predict magnitudes of subsidence above a longwall panel.

The above discussion indicates that there are likely to be many factors controlling the state of stress below an existing longwall, including the ratio IB/OB, the pillar width and anisotropic constitutive behaviour of the rock mass. These aspects will be investigated further in this Chapter.

3.2.1. In situ stresses surrounding previously mined longwalls

Prior to extracting the first seam in an underground profile, the stress field is usually
relatively uniform across the stratum. Generally, the magnitude of vertical stress within a seam is a function of its depth and the unit weight of overburden material. This is commonly referred to as the overburden stress, but it is also refer to as the cover load by Wilson (1983). However, in multi-seam coal mining the in situ stress conditions (prior to mining the second and subsequent seams) are not the same to those which apply to single-seam mining. The method of extraction of coal from the first seam influences how the in situ stresses are redistributed in the strata. Longwall mining generally redistributes in situ stresses to a greater extent than room and pillar mining (Haycocks et al., 1990). The vertical in situ stress is redistributed to form pressure arches giving rise to vertical stress windows (Figure 3.2). These pressure arch formations impose extra vertical load, in excess of the original overburden stress, onto adjacent pillars. This additional load is referred to as the abutment load.

Figure 3.2 - Vertical stress redistribution due to underground mine excavation (adapted from schematic diagram in Chekan et al., 1993).

Mining of any future seams, above or below the seam that is first mined, is usually considered once there is no more coal to be safely removed from the current seam. At the completion of longwall mining, the remnant structures left behind include a series of chain pillars with the collapsed goaf material in between. Analysis of the stresses that are present below a previously extracted seam can be understood by considering a typical plane strain section parallel to the longwall face (Figure 3.1).

A theoretical hypothesis on the vertical stress redistribution after a longwall panel has been mined was first presented by Whittaker (1974) (Figure 2.8). For a single longwall
Chapter 3 – Vertical stress changes under supercritical longwall panels

Panel, this theory shows that abutment stress in the ribs dissipates to the original overburden stress with increasing distance from the rib-edge, and the peak vertical stress induced in the ribs is off-set from the rib-edge. The loading in the goaf area returns to a maximum vertical stress at a certain distance behind the longwall face, after the goaf material has undergone hardening. However, very few field measurements have been taken of the chain pillar stresses or stresses within the goaf, after extraction of longwall panels, as such measurements are difficult and potentially dangerous to obtain. As a consequence this has limited our understanding of the vertical stress redistribution on completion of all longwall mining activity and makes validating empirical and numerical models difficult.

3.2.2. Abutment angle

Whittaker’s (1974) hypothesis indicates that, in general, the in situ stresses in the rock strata do not return to what they were before mining. It is important to be able to calculate the load above the longwall panel that is not carried by the goaf but is transferred to the pillars. As previously indicated, this is referred to as the abutment load and it is typically calculated as the weight of a wedge of material defined by an abutment angle ($\beta$), also referred to as the shear angle. When longwall panels are wider than the overburden depth, the triangles formed by the abutment angle usually reach the surface without intersecting (Figure 3.3). Therefore supercritical longwall panels apply the full potential abutment loads onto the adjacent pillars.

The abutment angle should not be considered as a physical entity, such as the angle of draw, but rather as an entity that helps to easily quantify the overburden load above a goaf carried by adjacent pillars (Mark, 1990; Colwell, 1998). It has been suggested that the abutment angle correlates with the geological strength and bridging capacity of the overlying strata. However, no evidence has been collected to prove this assertion (Colwell, 1998).

![Figure 3.3 – Load redistribution of overburden material after mining of a supercritical longwall panel (adapted from King et al., 1971; Wilson et al., 1972)].
There is limited information on how to determine appropriate abutment angles. The reasons are two-fold; there is limited field data from which to calculate it and there are many interpretations of appropriate values, both of which are discussed below. Some studies have recommended the use of a single abutment angle for all longwall panel designs (Mark, 1990; Colwell, 1998), while others suggest that it is variable and may depend on the overburden depth (Heasley, 2000).

Mark (1990) back-calculated abutment angles from six side abutment stress measurements in the maingate from five US mines. The calculated values of abutment angle varied significantly, ranging from 10.7 to 25.2 degrees. An abutment angle of 21 degrees was recommended to ‘yield appropriately conservative estimates of side abutment loads for longwall pillar design’. It is understood that this value is now pre-defined in all analyses conducted using the longwall chain pillar design program ALPS (Mark et al., 1986; Mark, 1990). The small data set combined with the large range of measured values used to interpret a single value for an abutment angle presents arguably unreliable statistical evidence.

An Australian Coal Association Research Project (Colwell, 1998) conducted a more detailed analysis of abutment angles relevant to Australian coal mines. Field data were collected from several mines from each of the NSW and Qld coalfields and reported abutment angles ranging from 10 to 26 degrees. The study recommended using an abutment angle of 21 degrees for most Australian coalfields, with 2 exceptions: for the Central Colliery $\beta = 26$ degrees and for the Southern coalfields $\beta = 10$ degrees.

There is an alternative means of back-calculating the abutment angle, which involves using the estimated distance from the rib-edge for the original overburden stress to be mobilised in the goaf. A large number of corresponding abutment angles have been proposed from these estimated distances, viz., $0.3H$ to $0.4H$ which corresponds to $\beta$ of 16.7 to 21.7 degrees (Whittaker, 1974; Wilson, 1983); and $0.12H$ which corresponds to $\beta$ of 6.8 degrees (Smart et al., 1987). Trueman (1990) also suggested that the abutment angle should depend on the seam thickness.

It is clear that there is varied information about what the values of the abutment angle should be for longwall panels. As a result, the analyses reported in this Chapter will consider a range of abutment angles in order to assess its affect on the stresses induced in the underlying strata.
3.2.3. Vertical stress distribution in a chain pillar

Most pillars in Australia are now typically designed to ensure serviceability of tailgate roadways (Mark, 1990; Colwell, 1998). This is because it has been observed that roadways often become unserviceable before failure or as the ultimate load is approached in a pillar. This means that pillars are now more likely to exhibit the classical stress distribution of a conventional or wide pillar, as previously discussed.

A study by Mark (1990) compared four theoretical models that describe the vertical stress in chain pillars as a result of the removal of an adjacent longwall panel. The study made comparisons of the predictions of the models with case histories in order to assess their robustness and to make recommendations on appropriate input parameters. The models suggested by Wilson (1983) and Mark (1990) were rated to be the most flexible for any longwall design project, since they can consider a variable number of gateroads and mined seam heights. The Wilson model also considers yielding of the pillar at the rib-edge, and the potential for yield to occur only in the seam (i.e., in the pillar) or also in the roof, seam and floor of the longwall panel.

For the purpose of this Chapter, Wilson’s robust analytical model provides equations that contain enough flexibility to consider all the possible potential variables in the problem of interest. These equations also have the advantage of ensuring that vertical equilibrium is always maintained.

Wilson’s vertical stress distribution in chain pillars

The Wilson method for determining the stresses in pillars was analytically derived from rock mechanics principles and was presented through a succession of papers (Wilson et al., 1972; Carr et al., 1982; Wilson, 1983). Wilson’s derivation of stresses in underground openings in coal seams is based on the fundamentals of cavity expansion theory, which assumes that a cylindrical underground opening is driven into an ideal homogeneous isotropic elasto-plastic medium. Yielding of the elastic material occurs when the peak strength is first mobilised as given by the Mohr-Coulomb failure criterion. Once the material has reached its peak strength, it is assumed that it softens instantaneously to a residual strength. The stress state in the yielded zone has been derived by a number of investigators (e.g., Westergaard, 1940; Terzaghi, 1943), and Wilson chose to use the solution derived by Airey (1977).

The equations, derived from the cavity expansion theory, must be adapted appropriately...
to suit the underground coal mining environment. It is recognised that strata surrounding coal mining roadways are not homogeneous isotropic elasto-plastic materials and a coal seam is a relatively weak stratum, which in general is not able to support high stress near any unconfined boundaries. Wilson made use of the ‘stress balance’ method to estimate the final distribution of stresses in the coal pillars, whereby any additional stress carried by the coal must be equal to the difference between the initial overburden stress and any vertical stress carried by the goaf. In particular, Wilson’s general distribution for the vertical stress in and around a longwall pillar consists of a yielded zone at the rib-edge and an elastic zone within the pillar, with a linear stress rise in the goaf (Figure 3.4).

As noted, the increase in vertical stress in the goaf with distance from the rib-edge is assumed to be linear until it reaches the magnitude of the overburden stress. This assumed pressure rise with distance into the goaf has been queried by Mark (1990), and Salamon (1992). Yavuz (2004) has predicted an approximately similar stress distribution in the goaf using a derivation which considered subsidence measurements combined with goaf formation and hardening.

![Diagram of vertical stress changes](image)

Figure 3.4 – Vertical stress in the vicinity of the rib-edge of a supercritical longwall panel (adapted from Wilson (adapted from 1983)).
Wilson’s derivation acknowledges that the stress distribution would be different in cases where only the seam yields (i.e., the coal pillar) or otherwise if the roof, seam and floor of the pillar all yield. In the case of seam only yielding (SO), it is assumed that the roof and floor strata are much stronger than the coal material and therefore can be considered to be effectively rigid. For the case where yield occurs in the roof, seam and floor (RSF), it is assumed that the roof and floor rocks have a similar strength to the coal pillar. The primary difference between the two cases in terms of the final vertical stress distribution is that the yielded zone for the RSF is usually much larger, sometimes double that for the SO. In this Chapter, the SO case is considered in detail in order to enable the presentation of a detailed review of the many variables that contribute to the stresses likely to be encountered in multi-seam mining. However, some of the differences between the SO and RSF cases have also been reported.

For yielding only in the seam, the final vertical stress distribution in the yielded zone of the pillar is calculated from

$$\sigma_y = k(p + p')\exp\left(\frac{x_F}{M}\right)$$  \hspace{1cm} (3.1)

This stress in the pillar is calculated as a function of the horizontal distance from the rib-edge, denoted by the symbol \(x\) (Figure 3.4). The yielded coal extends a distance \(x_b\) from the rib-edge, which can be calculated from

$$x_b = \frac{M}{F} \ln\left(\frac{q}{p + p'}\right)$$  \hspace{1cm} (3.2)

where

$$F = \left(\frac{k - 1}{k^{1/2}}\right) + \left(\frac{k - 1}{k^{1/2}}\right)^2 \tan^{-1}k^{1/2}$$  \hspace{1cm} (3.3)

And \(k = \frac{1 + \sin\phi}{1 - \sin\phi}\), which is the same as the Rankine passive stress state constant, \(p' = \sigma'_0(k - 1)\), \(p\) is the lateral restraint applied to the rib-edge boundary, \(\sigma'_0\) is the uni-axial (residual) compressive strength of yielded coal, and \(M\) is the extracted seam height (thickness). Wilson assumed exponential stress dissipation with distance into the elastic zone, so that the vertical stress in the elastic zone can be calculated using

$$\sigma_v = (\bar{\sigma} - q)\exp\left(\frac{(x_v - x)}{C}\right) - q$$  \hspace{1cm} (3.4)
where \( \hat{\sigma} = kq + \sigma_0 \) is the maximum stress at the boundary between yielded and elastic material. A constant \( C \) is used to determine how quickly the exponential curve decays, and its value can be calculated by stress balance. For yield in the seam only (SO), the constant \( C \) may be calculated from

\[
C = \frac{0.15H + x_b - Mk/F}{(k-1) + \sigma_0/q} \tag{3.5}
\]

For the case where yield occurs in the roof, seam and floor (RSF), equations for the vertical stress distribution in the yield zone (\( \sigma_y \)), the width of the zone of yielded coal at the rib-edge (\( x_b \)), and the vertical stress in the elastic zone (\( C \)) are given as follows:

\[
\sigma_y = k \left( p + p' \right) \left( \frac{x + M/2}{M/2} \right)^{k-1}, \quad x_b = \frac{M}{2} \left[ \left( \frac{q}{p + p'} \right)^{1/(k-1)} - 1 \right], \quad \text{and} \quad C = \frac{0.15H - M/2}{(k-1) + \sigma_0/q} \tag{3.6}
\]

Several studies have incorporated Wilson’s equations into their work, and have made recommendations on some of the input parameters. In one of Wilson’s original papers (Carr et al., 1982), \( p' \) was set to 0.1 MPa when \( k \) was 3.0 and therefore the corresponding value of \( \varphi = 30 \) degrees (Carr et al., 1982), while in another paper (Wilson, 1983) \( p' \) was set to 0.05 MPa when \( k \) was 4.0 and therefore \( \varphi = 37 \) degrees. It has been suggested by Mark (1990) to use \( k = 3.2 \), which corresponds to \( \varphi = 32 \) degrees, as this value compares best with empirical data.

Wilson (1983) recommended that the minimum width for a conventional pillar should be no less than \( 2(C + x_b) \). This minimum width was determined by approximating the exponential stress dissipation in the elastic zone by a linear stress distribution (as shown by the dashed line in Figure 3.4) corresponding to the same overall vertical load as the exponential stress distribution. The distance over which the linearly varying vertical stress (in excess of the original overburden pressure) dissipates fully in the elastic zone is \( 2C \), where \( C \) is defined in Equations (3.5) or (3.6) for the SO and RSF cases, respectively. Therefore, the minimum pillar width would also need to include yield zones either side of the elastic core, yielding a minimum pillar width of \( 2(C + x_b) \).

Wilson’s method (Wilson, 1983) for estimating the vertical stresses around a longwall panel were initially developed to help with the design of roadway drives and the selection of minimum pillar widths. The proposed methodology has compared favourably, both qualitatively and quantitatively, with field experience (Mark, 1990;
Scovazzo, 2008). The focus in this Chapter is to predict the stresses below previous mined seams to allow for rational planning and the design of ground control in multi-seam mining projects. Wilson’s solution has been adopted for this purpose.

3.3. Problem definition

The aim of this Chapter was to identify the changes to the in situ vertical stresses in the strata underlying a longwall-mined coal seam, assuming the plane strain model shown in Figure 3.1. Only the case where the first seam has been extracted as a supercritical longwall panel has been considered here. Analysis of the problem has been simplified by only considering the areas where the vertical loading differs significantly from the magnitude of the initial overburden stress. This corresponds to the pillar region and some of the goaf, as shown schematically in Figure 3.5.

For the cases studied here the vertical stress changes imposed on the underlying strata due to mining of the first seam, as predicted by Wilson’s equation, were applied to a finite element model. It was assumed that the in situ stress state in the ground is initially isotropic, i.e., at any location the vertical and horizontal stress components are initially equal. It has been assumed that the magnitude of the unit weight of the overlying strata was 0.025 MN/m$^3$, the mined seam height was 3 m, and the lateral restraint applied at the rib-edge was zero. Values of the other variables used in this Chapter are discussed in the following Section. It is also noted that for simplicity all changes in horizontal loading corresponding to the extraction of the longwall panel have been ignored. Because the width of the panel is generally much larger than the thickness of the extracted seam, this latter assumption is considered reasonable as a first approximation.

A parametric study was executed in relation to Wilson’s methodology in order to
understand how a number of variables affect the stress state at various depths below a previously mined longwall. These include the effect of the abutment angle, the overburden depth, the ratio of overburden depth to interburden depth ($H/B$), the friction angle, unconfined compressive strength, the transversely isotropic behaviour of the deformation response of the rock mass and the pillar width.

### 3.3.1. Solution method

The changes to the in situ stresses in coal seams underlying a seam being mined using the longwall technique were analysed using the displacement finite element software ABAQUS. Only the strata beneath the first seam mined were included in the spatial model. A plane strain model with appropriate in situ stresses was meshed (Figure 3.6), and an elastic analysis conducted. The rectangular section of rock analysed was 500 m wide and 200 m high. As with all other simulations conducted as part of this Thesis, the simulations used six-noded reduced integration triangular elements. A large number of elements, often totalling 100,000 or more, were used to achieve a high fidelity solution. Also, the density of elements was varied to accommodate regions where the gradient in the stress field was high, as illustrated in Figure 3.6. Displacements were fixed on the lowermost boundary, and only displacements normal to edges were fixed on the left and right boundaries. Wilson’s equations for the vertical stress changes were applied to the top surface of the finite element model to represent the stresses after extraction of the first mined seam.

Table 3.1 presents the input parameters considered in the model and adopted in Wilson’s equations. A set of parameters was selected for the reference case to allow for easy assessment of the effect each variable has on the stresses in the underlying strata. These reference case values are not intended to be indicative of any specific mine. The values varied in the parametric study were selected as representative of what is typical of Australian longwall coal mines. Representative ranges in values of the pillar width ($w$) (Colwell, 1998), the peak strength of the coal seam at zero confining stress $\sigma_0$ (Hirt, 1991; Seedsman, 2004), and the depth to the top seam for a supercritical longwall panel ($H$) (Peng et al., 1984; Colwell, 1998) were adopted as outlined in Table 3.1. Values of the independent shear modulus ($G'$), ranging from 0.1 to 1 times the isotropic shear modulus ($G_{iso}$) in the plane of the strata, and a Poisson’s ratio ($\nu'$) of 0.25 were considered. Values of the friction angle of the coal have been back calculated from the
suggested values of $k$, as indicated in Section 3.2.3.

![Finite element mesh used for the elastic analysis in ABAQUS for this Chapter.](image)

**Figure 3.6** - Finite element mesh used for the elastic analysis in ABAQUS for this Chapter.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
<th>Reference case</th>
<th>Other values considered</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H$</td>
<td>Depth to top seam</td>
<td>150 m</td>
<td>100, 125, 175, 200 m</td>
</tr>
<tr>
<td>$M$</td>
<td>Mined seam height</td>
<td>3 m</td>
<td></td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Friction angle of coal</td>
<td>32 degrees</td>
<td>30, 34, 36 degrees</td>
</tr>
<tr>
<td>$\sigma_0$</td>
<td>Strength of coal at zero</td>
<td>10MPa</td>
<td>5, 20, 30MPa</td>
</tr>
<tr>
<td></td>
<td>confinement</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_0'$</td>
<td>Strength of yielded coal</td>
<td>0.25MPa</td>
<td></td>
</tr>
<tr>
<td></td>
<td>at zero confinement</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p$</td>
<td>Lateral restraint at the</td>
<td>0MPa</td>
<td></td>
</tr>
<tr>
<td></td>
<td>rib-edge</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>Abutment angle</td>
<td>20 degrees</td>
<td>10, 15, 25, 30 degrees</td>
</tr>
<tr>
<td>$w$</td>
<td>Pillar width</td>
<td>30m</td>
<td>25, 35, 40m</td>
</tr>
<tr>
<td>$G'/G_{iso}$</td>
<td>Independent shear</td>
<td>1</td>
<td>0.5, 0.25, 0.167, 0.125, 0.1</td>
</tr>
<tr>
<td></td>
<td>modulus</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IB</td>
<td>Interburden thickness</td>
<td>$H/B = 6$ (25m)</td>
<td>$H/B = 6, 8, 10, 12$</td>
</tr>
</tbody>
</table>

**Table 3.1** - Variable definition and values used in parametric study.
When exponential stress dissipation is assumed in the elastic zone of a pillar of finite width, the stress balance (vertical equilibrium) is not maintained. For example, when Wilson’s recommended minimum pillar width is adopted, the stress in the elastic zone dissipates to 14% of the peak value, i.e., $(\hat{\sigma} - q)$, at the centre of the pillar, rather than reducing to the original overburden pressure. Therefore, pillars wider than the recommended minimum pillar width were chosen in this Chapter, in an attempt to maintain stress balance as much as possible. For example, for the reference case, corresponding to the parameter values provided in Table 3.1, the stress increase dissipates to less than 2% of the peak value at the opposite elastic/yield boundary of the pillars considered in this Chapter.

As discussed in Section 3.2.2, the abutment angle has been reported as ranging from approximately 10 to 30 degrees. Wilson’s equations originally considered that the abutment angle was a constant of 16.7 degrees, such that the distance to return to overburden stress in the goaf was approximately $0.15H$. In terms of the abutment angle $\beta$, the distance for the vertical stress exerted by the goaf to return to the magnitude of the overburden stress corresponds to $(H\tan\beta)/2$. This expression has been substituted into Wilson’s equation for the constant $C$ (Equation (3.5)) thus providing a revised expression to calculate $C$, as given by,

$$C = \frac{0.5H \tan \beta + x_b - Mk/F}{(k - 1) + \sigma_o/q}$$

(3.7)

### 3.4. Results and discussion

Many design options need to be considered to provide optimal ground control conditions for single-seam mining. For example, consideration of the orientation of the longwall panels with respect to the major and minor horizontal stress directions, the joint spacing and orientation, and the groundwater conditions are important in this regard. In multi-seam longwall mining all of these considerations also apply, but the effects of previous workings in other strata also need to be appreciated. The results from this theoretical study provide information and insight to allow better understanding of the potential effects of previous workings on the stress field in the second seam to be mined.
3.4.1. Reference case

Figure 3.7 shows the final vertical stress at the level of the first-mined seam determined from Wilson’s equations for the reference case of a 30 m wide pillar, located at a depth of 150 m with an abutment angle of 20 degrees and all other variables with values as presented in Table 3.1. The incremental loading applied to the top surface of the finite element model corresponds to the difference between this stress distribution and the initial overburden pressure at seam level. Contours representing the change in vertical stress state in the underlying stratum resulting from this reference case of pillar and goaf loading are presented in Figure 3.8.

Figure 3.8 shows that the changes to the original in situ vertical stresses occur as two lobes under the maximum peak pillar loadings. The major zones of primary influence extend about one pillar width down into the underlying strata. The changes in vertical stress towards the edges of the model are approximately equal to zero. This is to be expected, as the vertical stress transmitted through the goaf, towards the edges of the model, is the same as the initial overburden stress. The predicted stress distribution for the reference case appears to be similar to that imposed by a rigid footing foundation on an elastic half-space (Poulos et al., 1974). The key difference arises because Wilson’s approach considers yielding in the coal seam, and thus the maximum vertical stress does not align with the rib-edge (Figure 3.7). Instead, the peak stresses occur at the boundary separating the yield and elastic zones.
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Figure 3.7 – Vertical stress distribution calculated using Wilson’s equation for yield in SO for the reference case.

Values of the stresses in the rock strata at various depths beneath the first-mined seam are shown in Figure 3.9. The stresses at various depths below the top of the model (i.e., below the level of the first seam) are presented relative to the ratio \( H/B \) (refer to Figure 3.1). Larger magnitudes of ratio \( H/B \) correspond to smaller interburdens for a given overburden height. The maximum value of the ratio \( H/B \) considered here is 12, as...
values greater than this will correspond to very small interburden thickness. Mining under very small interburden is often referred to as ultra-close seam mining, which usually presents unique instability problems (Haycocks et al., 1990; Seedsman, 2003). Since the values of stress change dissipate with depth through the strata, a minimum value of the ratio $H/B$ of 4 has been selected in this Chapter.

Figure 3.9 (a) shows the predicted change in vertical stresses for various values of the ratio $H/B$. These changes in the vertical stress distribution at each depth contain a similar inflection point located under the rib-edge, and differ only in the relative stress magnitudes. For large ratios of $H/B$, higher magnitudes of vertical stresses can be expected under the pillar, with a very sharp decrease in magnitude to less than the original in situ stress elsewhere. The change in vertical stress then diminishes and eventually becomes insignificant, and the lateral extent of significant stress changes is generally contained to a zone within approximately three pillar widths from the vertical axis through the centre of the pillar. Figure 3.9 (b) presents the final vertical stress after mining of the first seam ($\sigma_{vf}$) normalised by the initial vertical stress at the corresponding depth ($\sigma_{vi}$). For a value of the ratio $H/B$ of 12, the normalised vertical stress directly under the centre of the pillar corresponds to approximately 2.3 times the initial in situ vertical stress prior to extraction. It is noted that at the level of the upper mined seam, the maximum peak stress generated in the pillar was approximately 5 times the in situ stress ($\sigma = 22.5 \text{ MPa}$, Figure 3.7), so that for $H/B = 12$, the maximum vertical stress has attenuated by a factor of more than 2.

The reference case considered in this Chapter used Wilson’s equations for situations where yielding occurs only in the seam (SO). Figure 3.10(a) shows a comparison of the vertical stress distribution derived from Wilson’s equations for the reference case (SO) and for the case where similar parameter values have been assumed but where yield occurs in the roof, floor and seam (RSF). For these two cases the differences in vertical stress after the first seam is mined are relatively small, although the width of yielded pillar is almost doubled. Similarly, changing the type of yielding from SO to RSF for the reference case has negligible effects at a depth corresponding to $H/B$ of 6 (Figure 3.10(b)). The approximate doubling of the width of the of yielded zone of pillar coal for the RSF case has not led to any significant differences in the stress changes below the extracted seam when compared to those stress changes predicted for the case of seam only (SO) yielding. Therefore, even though the majority of examples presented in this
Chapter assume SO yielding conditions, the trends discussed should also apply broadly to RSF conditions.

![Diagram](image)

Figure 3.9 – Vertical stress distributions at various depths below the first seam for the reference case: (a) change in stress values, and (b) change in vertical stress normalised by the initial vertical stress at the particular depth.
Figure 3.10 - Vertical stress distributions below the first seam for both SO (reference case) and RSF: (a) vertical stress distribution at first-mined seam level calculated using Wilson’s equation, and (b) vertical stress distribution normalised by the initial vertical stress at the particular depth for H/B=6.
3.4.2. Effect of abutment angle, $\beta$

As discussed previously, the abutment angle is an important variable when estimating the pillar loading and therefore the associated stress below a previously mined longwall. In this Chapter the abutment angle directly affects stress dissipation in the elastic zone of the pillars (Equation (3.4)) and the distance from the rib-edge for the overburden stress to be reached in the goaf.

Figure 3.11(a) presents the vertical stress distribution at the level of the first-mined coal seam for varying values of $\beta$, while still maintaining the values of all other variables as for the reference case (Table 3.1). Larger values of $\beta$ increase the total load carried by the pillar, the stress dissipation parameter $C$, and the distance from the rib-edge required for the vertical stress in the goaf to return to the overburden stress. The value of the parameter $C$ increased to 8.85 m for the abutment angle value of 30 degrees. Therefore, as recommended by Wilson, the minimum pillar width for an abutment angle of 30 degrees is 24.96 m, which is less than the 30 m pillar width adopted in this Chapter.

The changes in vertical stress induced in the underlying strata for different abutment angles, for a value of $H/B$ of 6, are shown in Figure 3.11(b). The distributions of change in vertical stress exhibit the same shape for each abutment angle but they vary in the relative magnitudes in both the horizontal and vertical directions. For a ratio of $H/B = 6$, the maximum vertical stress increases from 1.27 to 2.24 times the in situ vertical stress as the abutment angle increases from 10 to 30 degrees.

Values of the imposed maximum vertical stress, for a range of values of both the ratio $H/B$ and the abutment angle, have been compiled in Figure 3.12. The maximum vertical stresses have been taken from the vertical stress plots for a range of abutment angles, such as those presented in Figure 3.11(b), considering three different ratios $H/B$. These results show that the maximum vertical stress induced in the underlying strata is approximately linearly proportional to the abutment angle for a given ratio $H/B$. The linear relationship between the angle $\beta$ and the maximum normalised vertical stress is not the same for different $H/B$ ratios. For smaller interburdens, i.e., large ratios of $H/B$, a deviation from the true abutment angle generates larger differences in the predicted stress than for deeper interburdens.
Figure 3.11 – For the reference case and a range of abutment angles, (a) the vertical stress distribution at first-mined seam level calculated using Wilson’s equation, and (b) normalised vertical stress distribution below the first seam for $H/B = 6$, shown as change in vertical stress normalised by the initial vertical stress at the particular depth.
Figure 3.12 – For a range of abutment angles: (a) Maximum normalised vertical stress variation under pillar and $H/B$ ratios, and (b) Maximum normalised vertical stress variation for depth below upper seam, shown as depth normalised by pillar width.

Figure 3.11(b) shows the maximum vertical stress dissipation at depths under the centre of the pillar (expressed as a function of the pillar width, $w$). The results for all values of $\beta$ considered show much quicker reduction in values of the maximum vertical stress.
closer to the mined seam, than at greater depths. This highlights the fact that in cases of mining involving smaller interburdens, significantly larger stresses than for thicker interburdens should usually be expected, in particular under pillars with loading corresponding to large values of $\beta$.

The abutment angle has been shown to affect significantly the vertical stress magnitudes in the strata under a pillar as well as the horizontal extent of areas exposed to vertical stresses less than the overburden stress. As the pursuit of multi-seam mining increases, there is likely to be an increasing emphasis placed on the ability to predict appropriate values of abutment angle.

### 3.4.3. Effect of overburden depth and pillar width

Figure 3.13(a) shows the vertical stress distribution at the level of the first-mined coal seam for varying overburden depths, while keeping all the other variables the same as for the reference case (Table 3.1). It may be observed that deeper overburdens cause an increase in the distance from the rib-edge for the vertical stress to return to the overburden stress in the goaf, an increase in the extent of the yield zone ($x_b$), and also an increase in the total load carried by the pillar, as expected. Figure 3.13(a) highlights the point that if the abutment angle is kept constant, then the rate of vertical stress increase in the goaf is also constant.

Figure 3.13(b) shows the effect of overburden depth on the normalised vertical stresses for a ratio of pillar width divided by interburden depth ($w/B$) of 1.2. These results are for a constant value of IB of 25 m, since the pillar width is 30 m. As expected, for a constant interburden thickness, increasing the overburden depth causes the magnitudes of the stress distribution to be magnified. The increase in maximum peak stress is approximately linear, as would be expected.

The distributions of final vertical stresses normalised by the initial in situ vertical stress for a ratio $H/B$ of 6 (Figure 3.13(c)), exhibit almost identical shapes and similar maximum stress magnitudes under the pillar. However, deeper overburdens require larger horizontal distances for the vertical stress to return to overburden stress magnitude, which is reflective of the predicted larger distance to reach overburden stress in the goaf of the first mined seam (Figure 3.13(a)).
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(a) Distance from pillar centre

(b) Distance from pillar centre
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Figure 3.13 – For the reference case and a range of overburden depths: (a) the vertical stress distribution at first-mined seam level calculated using Wilson’s equation, (b) normalised vertical stress distribution below the first seam for $w/IB = 1.2$, (c) normalised vertical stress distribution below the first seam for $H/B = 6$.

Changing the width of the pillar ($w$) causes the peak in vertical stress occurring at the yield-elastic boundary to be separated by a larger trough in the elastic core of the pillar. The total abutment load carried by each pillar is identical. Therefore, when the horizontal distance is normalised by the pillar width, it can be seen that wider pillars carry effectively less load per unit length (Figure 3.14(a)). The effect on the underlying stratum of varying the pillar width, for a value of the ratio $H/B$ of 6, is minimal (Figure 3.14(b)). So for the reference case, irrespective of pillar width, the maximum change in the in situ vertical stress at a value of $H/B$ of 6 is in the order of approximately 0.7 times the initial in situ stress. However, for a smaller interburden, there are variations in the distribution and maximum value of the vertical stress. Figure 3.14(c) shows this effect for a value of $H/B$ of 12. For thin interburdens, wider pillars cause the maximum peaks in the vertical stress to reduce in magnitude and indeed result ultimately in the generation of two peaks.
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(a) Distance from pillar centre

Vertical stress (MPa)

Distance from pillar centre

(b) Distance from pillar centre

Normalised vertical stress ($\sigma_v / \sigma_i$)

Goaf

Pillar

Goaf
The depth of the interburden and the pillar width affects the shape of the vertical stress distribution beneath the first mined seam. Shallower interburdens, or wider pillars, generate 2 lobes of maximum vertical stress in the underlying seam, usually under the rib-edges. Deeper interburdens, or otherwise narrower, conventional pillars, only contain a single maximum peak under the middle of the pillar. This is because deeper interburdens dissipate stress changes to a greater extent and narrower pillars do not contain a large distance between the two maximum stress peaks within the pillar.

### 3.4.4. Effects of transverse isotropy

Figure 3.15(a) shows the vertical stress variation for a range of independent shear modulus values and a ratio $H/B$ of 6, while keeping all the other variables the same as for the reference case. The independent shear modulus ($G'$) has been normalised by the isotropic shear modulus ($G_{iso}$), such that the result for an isotropic material are plotted as $G'/G_{iso}$ of 1. As the ratio of $G'/G_{iso}$ decreases, the degree of material anisotropy increases, as discussed previously. Decreasing the independent shear modulus has the effect of transferring vertical stresses deeper into the strata. By reducing the
independent shear modulus to a value of one-tenth of the value for isotropic conditions, the change in maximum vertical stress increased from 1.75 times the original in situ stress to 2.29 times the original in situ stress. This increase in vertical stress is 74% higher than the isotropic case.

Figure 3.15(a) also shows that if isotropic conditions are assumed, the maximum vertical stress of 1.75 times the original in situ stress occurs locally at the centre of the overlying pillar. However, as the independent shear modulus is reduced, the maximum vertical stress no longer occurs at a single point. In extremely transversely isotropic strata it is predicted that two localized maximum vertical stress peaks would occur which are located approximately under the edge of the yield/elastic boundaries.

To understand further the relative effect of transversely isotropic behaviour, the maximum vertical stresses predicted for each of the ratios of $G'/G_{iso}$ are presented in Figure 3.15(b). Results are presented for several ratios of $H/B$. At any given depth the relationship between the maximum predicted vertical stress and the ratio of the independent shear modulus to the isotropic shear modulus ($G'/G_{iso}$) is not linear. It was not possible to find a common relationship between the maximum vertical stress and the independent shear modulus that could describe each of the curves corresponding to different values of $H/B$. The non-linear relationships shown in Figure 3.15(b) indicate that the vertical stress changes are most pronounced in the more highly anisotropic materials, i.e., low values of the modulus ratio, $G'/G_{iso}$. 
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Figure 3.15 – (a) Normalised vertical stress variation for a range of independent shear modulus values for a ratio $H/B$ of 6.  (b) Variation of the maximum normalised vertical stress with respect to independent shear modulus normalised by the isotropic shear modulus for a range of $H/B$ ratios.

It has been observed that transversely isotropic behaviour in a stratum increases the maximum vertical stress induced at any given depth beneath a mined coal seam, relative to that which would be observed in an isotropic medium. Previous publications have
documented that field measurements often record higher than expected vertical stresses (Lightfoot et al., 2010) as a result of coal seam extraction. In the particular case described by Lightfoot and Liu (2010), the expected vertical stresses were obtained from computer analyses conducted assuming that the coal measure strata could be approximated as an isotropic material. Lightfoot and Liu’s main conclusion was the need for numerical procedures that could model effectively and efficiently laminated strata for underground coal measure stability analyses. The methodology presented in this Chapter has shown how anisotropic effects, arising presumably due to stratification in sedimentary rocks, and possibly sub-vertical jointing, can be considered implicitly by including an independent shear modulus in the stress-strain relationship of the rock strata.

The results presented in this Chapter also show that if an analysis does not consider the effects of jointing in laminated rock strata (either implicitly or explicitly) the analysis is likely to underestimate the vertical stresses in a second seam underlying an extracted supercritical longwall panel. It is imperative when mining to be able to predict the expected vertical stresses in the lower seam. The predicted vertical stresses in conjunction with the expected horizontal stresses will govern any ground control instability issues, for which an effective management plan should be devised.

Transversely isotropic rocks cause more rapid variation of vertical stress with horizontal distance relative to isotropic strata. The consequences for sudden changes in vertical stress could be more rapid changes in rock mass behaviour including its deformation response. The implication for multi-seam mining practice is that changes to mining conditions would occur more rapidly along the lower seam than would be predicted by an analysis assuming isotropic material behaviour. The potential for rapid (or even gradual) changes in rock mass behaviour would need to be acknowledged in the development of an effective risk management plan. The mine site may need to be prepared with resources to implement different ground control measures for sudden (or gradual) changes in stress conditions. The inability to deal with sudden changes in stress conditions could increase the potential for failures to occur with consequent safety implications and reduction in productivity.

Including transverse isotropy in the elastic analyses results in a very strong ‘columnisation’ of the vertical stresses, which is not observed in isotropic elastic analyses. This columnisation can provide explanations for much of the rock mass
behaviour observed when undermining. For example, floor heave and poor rib conditions are sometimes observed when undermining under the goaf/solid boundary or under chain pillars (Oram et al., 1997; Lightfoot et al., 2010), and at greater depth (Dunham et al., 1978; Mark et al., 2007).

A limitation in using the transverse isotropic elastic model with an independent shear modulus to model stratified rock is the limited field measurements that are available to calculate the parameter values and their variability. As an initial estimate, it would be useful to collate field measurements of vertical stress in the lower seam and use them to back analyse the independent shear modulus of the stratum.

3.4.5. Effects of other variables

The friction angle of the coal ($\phi$) and its unconfined compressive strength (UCS) all influence the shape of the vertical stress distribution in the pillar, but only to a minor extent when compared to the effects of the parameters previously discussed. This therefore means that the predicted changes in stress in the strata under the mined seam are not very sensitive to the strength parameters of the coal. As an example, the change in the UCS of the coal seam material alters only the magnitude of the stress change applied within the elastic section of the pillar (Figure 3.16(a)). The resultant stress changes at depth are approximately the same, irrespective of the value of UCS (Figure 3.16(b)). Similar observations were made when varying the friction angle and the residual strength of the failed coal.
Figure 3.16 - The reference case and a range of unconfined compressive strengths (UCS): (a) vertical stress distribution at first-mined seam level calculated using Wilson’s equation, and (b) Normalised vertical stress distribution below the first seam for $H/B = 6$. 
3.5. Discussion

The quantitative results from this theoretical study show that the abutment angle, pillar width, interburden depth and transverse isotropic behaviour of the rock mass are important parameters to consider when estimating the vertical stresses that are induced in an underlying second seam. The results from this analysis need to be considered in conjunction with the findings on horizontal stress redistribution around a series of longwall panels in the first seam, that shall be presented in the next Chapter. With knowledge of the predicted ratio of vertical stress to horizontal stress it will be possible to determine the typical type of ground instability issues that might arise. Studies by Gadde and Peng (2004) found that the maximum roof stability occurs when the horizontal to vertical stress ratio ($K$) is between 0.5 to 1.0. Larger magnitude of $K$ only cause a marginal change in roof stability, however, very high horizontal stresses are unfavourable. If high horizontal stresses can be expected during the mining of the second seam, this stress environment may in fact be advantageous for the stability of the interburden. This idea was first presented by Hill (1994; 1995) for the example of a longwall-mined coal seam over an existing room and pillar-mined coal seam, as shown schematically in Figure 3.17. The interburden would probably be unstable when both principal stresses are tensile.

The finding that the abutment angle played an important role in governing the induced vertical stress changes into the underlying strata highlights that more emphasis should be placed on investigating methods of determining its magnitude during site investigations prior to starting to designing the mine. As identified in Section 3.2.2, it has been recommended that an abutment angle of 21 degrees can be used to yield conservative estimates of side abutment loads (Mark, 1990). This magnitude is even set as the default in the pillar stability analysis program Analysis of Longwall Pillar Stability (ALPS) which feeds into the multi-seam pillar stability Analysis of Multiple Seam Stability (AMSS). Yet, magnitudes of abutment angle have been recorded to vary from 10 to 26 degrees. The finding identified that extreme care should be taken when selecting the magnitude of the abutment angle. If there is uncertainty in the magnitude of the abutment angle, the effects of the largest and smallest possible magnitudes should be assessed and the associated implication on in situ stress environment in the second seam.
The finding that the pillar width and interburden depth play an important role in the vertical stress change in the second seam possibly justifies the pore correlation in the plot of empirical data comparing $H/B$ to severity of multi-seam mining interactions. Approximately half the data points presented in Figure 2.3 have values of the ratio $H/B > 10$. Of these points, almost equal proportions of cases are classed as extreme, moderate and no interaction. This Chapter has shown that for large values of the ratio $H/B$, different pillar widths will affect the magnitude and distribution of vertical stress in the second seam. Therefore, when second seams are mined with a large ratio of $H/B$, the pillar widths in the overlying seam must also be considered when assessing the vertical stress magnitude and therefore the potential for creating adverse conditions for future mining.

There are two significant implications from the finding that vertical stresses induced in the strata underlying a mined longwall seam can vary in magnitude but also very rapidly with horizontal distance. The predicted magnitude of maximum induced vertical stress was as large as 2.5 times the initial in situ vertical stress under the remnant chain pillar
in the first seam (see Figure 3.14). This large vertical stress increase would need to be considered in the ground control design probably by increasing the capacity of the rock bolts or the cables. Large changes in the in situ vertical stress with horizontal distance in the second seam was predicted when a transversely isotropic strata was used to represent the interburden. The predicted vertical stress change was 2.6 times the original in situ stress over a horizontal distance equal to the width of the chain pillar. This is for an independent shear modulus ($G'/G_{iso}$) of 0.167 which was proposed by Seedsman (2009) from a back analysis. Such a sudden change in the vertical stress with horizontal distance would result in adverse mining conditions unless the ground control measures were altered to accommodate the changing in situ stress. This rapid change in mining conditions with horizontal distance when undermining a pillar has been observed in the field (Lightfoot et al., 2010). Significant changes in magnitude of the vertical stress with horizontal distance would pose high risks to the safety of personnel during roadway development. The development rate would need to be slowed down to allow for the possibly complicated ground control design to be implemented.

The results presented in this Chapter imply that the staggered arrangement of longwall panels would in general provide the most stable conditions for gateroads in the second seam. This is because generally it is predicted that high vertical stresses can be expected under remnant chain pillars present in the first seam and either side of the remnant pillar the magnitudes of vertical stresses would be less than the overburden stress. The areas located under the centre of the first seam longwall goaf would expect to have in situ stresses equal to approximately the overburden stress and there would be less vertical stress variation with horizontal distance. This suggestion supports the proposal made by Gale (2004) that the most stable arrangement of gateroads for Australian cases occurs when they are located under the centre of the overlying extracted longwall panel. It is noted that all longwall panels considered in Gale’s study were also supercritical.

3.6. Conclusions

In this Chapter, predictions of the vertical stress in the strata under a series of supercritical longwall panels have been presented. The results show that the magnitude and distribution of the vertical stress are primarily affected by the abutment angle, overburden depth, pillar width and the anisotropic behaviour of the rock mass. For the particular cases considered here, increasing the abutment angle by 20 degrees tripled the
change in vertical stress at a depth of 25 m below the overlying seam. As a practical conclusion, an appropriate value for the abutment angle should be carefully considered early in the design process. Thin interburdens and wide pillars typically generate 2 lobes of maximum vertical stress in the underlying seam, as opposed to a single lobe of maximum vertical stress for thicker overburdens. Decreasing the independent shear modulus of the rock strata increased the magnitude of the vertical stresses observed in the lower seam, such that a shear modulus one tenth of the value of an isotropic shear modulus increased the maximum vertical stress in the lower seam by 74% relative to the isotropic case. Transversely isotropic rock masses also exhibit more rapid variation of vertical stress with horizontal distance than isotropic rocks which lead to more rapid variation.

The implications of these findings is that the ground control will need to be able to accommodate the effect of large vertical stresses under remnant chain pillars and change very rapidly to accommodate the rapid changes in rock mass response as a result of variation of vertical stress with horizontal distance. These vertical stresses and stress changes need to be considered in light of the effects of the in situ horizontal stresses (considered in Chapter 4) to assess stability during longwall mining operations within the second seam.

The findings of this Chapter inform designers of multi-seam mines about which parameters have the most significant influence on the pre-mining vertical stresses induced in seams underlying those already mined. The effect of the identified parameters will have implications for the design of ground support measures for the lower seam pillars and roadways. The combination of the finite element technique and semi-analytical methods used to assess the stress state around coal seams already mined has proved useful for this purpose.
CHAPTER 4. HORIZONTAL STRESSES UNDER SUPERCritical LONGWALL PANELS

4.1. Introduction

The research described in this Chapter identifies which variables are most significant in influencing horizontal stress redistribution in the strata around a series of supercritical longwall panels extracted from the first seam. The predicted horizontal stress needs to be considered together with the vertical stress to design effective ground control for the second mined seam. As mentioned in Section 2.3.3, there is no understanding of the horizontal stresses that are present or induced into the goaf material above an extracted longwall. Also, there is no information on the lateral load capacity of the goaf material.

The problem considered in this Chapter is schematically shown in Figure 4.1. It is assumed that the first seam has been extracted using a series of parallel supercritical longwall panels. Since no information is available on the lateral load capacity of the goaf material, it is assumed that all of the goaf material above each longwall has no stiffness in the horizontal direction. This assumption would predict the largest possible horizontal stress that could be induced into the strata below the first mined seam. The analysis is conducted by assuming that effectively the block of strata above all of the longwalls panels extracted from the first seam does not support any lateral load. The width of this block of strata is defined by an equivalent extracted width ($W_{eq}$) as shown schematically in Figure 4.1. The ratio of the original horizontal to vertical in situ stress components, ($K$) and anisotropy and inhomogeneity typically present in coal measure strata are considered.

![Diagram showing the structure of the strata before multi-seam mining commences.](image)

*Figure 4.1 - Schematic representation of the structure of the strata before multi-seam mining commences.*


4.2. Background

The redistribution of high horizontal stresses present in coal measure strata has been reported as the cause of instabilities in longwall mine roadways, such as compressive roof failures (often referred to as cutter roof or guttering), directionality of roof falls, and maingate failures (Mark et al., 1994). High magnitudes of horizontal stress have been a major concern for ground control coal mining engineers involved in single-seam mining, and yet the origin of the horizontal stress and how it is redistributed in a stratum after mining are poorly understood (Hebblewhite et al., 2000; Tarrant, 2005; Tarrant, 2005). Horizontal stress around an extracted longwall panel

The horizontal stress redistribution discussed in literature is primarily based on redistribution in the horizontal plane (see Section 2.2.3). The horizontal stress redistribution in the vertical plane has only been studied extensively around underground tunnels, where the horizontal stress redistributes both above and below the extracted tunnel (Brady et al., 1992). The final stress distribution around a circular tunnel in an isotropic elastic material is governed by the initial stress ratio ($K$) and the overburden depth. If the initial in situ stress ratio $K$ is constant, the final stresses around an elliptical tunnel are generally higher than those predicted around a circular tunnel for regions of the rock mass adjacent to sections of the tunnel boundary with higher curvature. These general observations of horizontal stress redistribution can be applied to subcritical longwall panels (refer to Figure 4.2). However, the horizontal stress for extracted supercritical longwall panels needs further study and so they are the focus here.

![Figure 4.2 – Conceptualised horizontal stress distribution around a mine opening (adapted from Mucho & Mark 1994).](image)

The primary limitation for understanding how horizontal stresses redistribute in vertical planes for supercritical longwall panels, is most likely to be due to the lack of
knowledge about the goaf behaviour. This lack of knowledge about the goaf arises because difficult and dangerous conditions are confronted when attempting to access or drill into goaf material. Without understanding the material properties or spatial extent of a goaf, it is impossible to accurately predict how much horizontal stress, if any, is transmitted through the goaf material after the coal has been extracted. The most conservative assumption for a shallow supercritical longwall panel is that the goaf extends to the surface and it transmits no lateral stress. This assumption shall be the basis for work presented in this Chapter.

4.2.1. Numerical modelling

As discussed in Chapter 2, numerical modelling provides engineers with a tool to obtain detailed predictions of stresses and deformations around underground excavations. It is used extensively to assess stability when designing coal mines. Numerous numerical modelling packages are now capable of including sophisticated material behaviour, such as yielding of the rock mass strata, strain softening and non-associated plastic flow rules (Morsy et al., 2002; Badr et al., 2003; Esterhuizen et al., 2010). However, most of the studies that have used these models have focused on the vertical stress redistribution and the consequential subsidence at the ground surface. They rarely acknowledge the redistribution of the virgin horizontal stress. The limitation of the analyses that include strain hardening of the longwall goaf by either considering Terzaghi’s linear equation for compressive behaviour of granular fill (Morsy et al., 2002) or Salamon’s hyperbolic equation for compressive behaviour of granular fill (Esterhuizen et al., 2010), is that they only consider strain hardening of the goaf material in the vertical direction. Although sophisticated numerical modelling techniques are available, confidence in stress prediction also depends on the availability of accurate stress measurement data obtained from the field (Lambe, 1973). Availability of accurate field data is the primary limitation on verification and validation of the numerical modelling of coal mining works and the associated goaf material.

A further consideration is that when the longwall panel is not orthogonal to the maximum and minimum horizontal stress directions, stress analyses need to be conducted in three dimensions to obtain an accurate prediction of the final stresses around the roadways (Meyer et al., 1999; Coggan et al., 2012). A plane strain analysis can only be considered if the longwall panel aligns orthogonally with the maximum and
minimum horizontal stress directions, otherwise a 3D model is essential. This Chapter shall only consider plane strain analyses.

The analyses presented in this Chapter have considered all strata as elastic continuum materials in order to obtain information on the horizontal stress redistribution in the vertical plane. A more sophisticated constitutive model was not used as it could mask the fundamental features of the stress redistribution. It is difficult to identify precisely what variable governs the stress redistribution in a more sophisticated computer model. A simpler elastic approach was deemed appropriate, as the focus of the analysis is to understand the relative horizontal stress re-distribution at the scale of a whole longwall panel. Further investigations should be conducted to consider the effect of a yielding floor. The effects of variables predetermined by geological conditions (often referred to as fixed variables), and those selected when designing a mining operation (often referred to as mining variables), were investigated in this Chapter.

### 4.2.2. Geological conditions

In reality, coal measure strata probably do not behave as isotropic media, as they contain bedding planes, layers of rock with different stiffness and interfaces with limited shear strength. As discussed in Section 2.5.1, each of these features will confer non-isotropic behaviour on the rock mass and thus affect how the stresses are distributed in the strata surrounding an extracted seam. The effects of these three features on the redistribution of in situ horizontal stress shall also be assessed in this Chapter, as described in more detail in the following sections.

**Bedding planes**

The bedding planes in coal measure strata generally cause the shearing resistance along the bedding planes to be less than on planes inclined to the bedding. These strata behave anisotropically and can be further defined as transversely isotropic materials. As mentioned in Section 2.5.1, for a transversely isotropic material the shear modulus in planes perpendicular to the plane of isotropy is independent of the values of Poisson’s ratio and Young’s modulus. Thus, it is called the independent shear modulus ($G'$). This Chapter has assessed the relative contribution of the bedding in coal measure strata to the stresses induced in strata underlying a mined seam. The rock strata were modelled as an equivalent continuum elastic material with an independent shear modulus included in the relevant anisotropic elastic constitutive laws.
Layered strata with varying stiffness

Coal measure strata usually consist of a number of layers of sedimentary rocks (e.g., sandstone, mudstones, and siltstones) and each layer may vary in stiffness. The original in situ stress distribution in strata consisting of layers of rock of varying stiffness will be dependent on the specific properties of each of the rock layers. In addition, the layered strata with varying stiffness will also affect the redistribution of in situ horizontal stress as a result of extracting a series of parallel longwall panels.

An equivalent homogeneous orthorhombic elastic material can be used to represent strata with layers of varying stiffness. The advantage in considering an equivalent orthorhombic model in this Chapter is that the stiffness and the original in situ horizontal stress of each individual layer do not need to be explicitly included in the finite element model. Equations to calculate the equivalent elastic moduli have been derived by Salamon (1968) and then extended by Gerrard (1982). These models assume that there is no relative displacement along the interfaces between rock layers. It has been assumed that each individual rock layer is isotropic, such that:

\[ E_{ii} = E_{2i} = E_{3i} = E_i \]  \hspace{1cm} (4.1)

\[ \nu_{12i} = \nu_{21i} = \nu_{13i} = \nu_{31i} = \nu_{23i} = \nu_{32i} = \nu_i \]  \hspace{1cm} (4.2)

where \( E \) represents Young’s modulus, \( \nu \) is Poisson’s ratio, \( i \) is the \( i \)th layer in the layered system, and subscripts 1, 2 and 3 correspond to Cartesian coordinates \( x_1, x_2 \) and \( x_3 \), so that, for example, \( E_{2i} \) represents Young’s modulus of layer \( i \) in the \( x_2 \) coordinate direction. The Poisson ratio \( \nu_{12i} \), for example, corresponds to the ratio of tensile strain in the \( x_2 \) direction to the compressive strain in the \( x_1 \) direction as a result of applying a compressive stress in the \( x_1 \) direction.

When the conditions given in Equations (4.1) and (4.2) are substituted into the original equations derived by Gerrard (1982) to calculate the elastic moduli of an equivalent orthorhombic material the result is the following set of simplified equations:

\[ E_i = E_2 = \frac{\sigma^2 - \zeta^2}{\alpha} \cdot \frac{1}{E_3} = \sum \left( \frac{\nu_i}{E_1} - \frac{\nu_i}{E_2} \right) \chi_i + \sum \left( \frac{\nu_i}{E_2} - \frac{\nu_i}{E_3} \right) \lambda_i, \ \ \ \nu = \chi - \frac{\lambda \zeta}{\alpha} \]  \hspace{1cm} (4.3)
\[ \alpha = \beta = \sum_i \frac{t_i E_i}{1 - v_i^2} \quad \zeta = \sum_i \frac{t_i E_i v_i}{1 - v_i^2} \quad \chi_i = \lambda_i = \sum_i \frac{t_i (v_i^2 + v_i)}{1 - v_i^2} \quad \lambda = \sum \chi_i \quad (4.4) \]

Equations to convert stresses from the equivalent orthorhombic material to those that act in the individual layers are provided in Equation (4.5):

\[ \sigma_{li} = \sigma_1 + C_i \left( E_i A_i + v_i E_i B_i \right) \quad \sigma_{l2} = \sigma_2 + C_i \left( E_i A_i + v_i E_i B_i \right) \quad \sigma_{l3} = \sigma_3 \quad (4.5) \]

where

\[ A_i = \left( \frac{1}{E} - \frac{1}{E_i} \right) \sigma_1 - \left( \frac{v_i}{E} - \frac{v_i}{E_i} \right) \sigma_2 - \left( \frac{v_i}{E} - \frac{v_i}{E_i} \right) \sigma_3 \quad (4.6) \]

\[ B_i = \left( \frac{v_i}{E} - \frac{v_i}{E_i} \right) \sigma_1 - \left( \frac{1}{E} - \frac{1}{E_i} \right) \sigma_2 - \left( \frac{v_i}{E} - \frac{v_i}{E_i} \right) \sigma_3 \quad (4.7) \]

\[ C_i = \frac{1}{1 - v_i^2} \quad (4.8) \]

For the purpose of this Chapter, the orthorhombic material model has been used to assess the redistribution of in situ horizontal stress as a result of extracting a series of parallel longwall panels. Equations (4.3)-(4.4) were used to calculate the parameters required to represent a layered stratum as an equivalent continuum material in a finite element analysis. Equations (4.5)-(4.8) were then used to convert the results obtained from the finite element analysis to prediction the final in situ stress in each rock layer.

**Low shear stress layer**

A layer in a stratum with limited shear strength can also govern how the stresses are redistributed into the surrounding strata after excavation. It is common for coal measure strata to contain a layer (or layers) of material with very low shear strength (Hutton, 2009). Often these low shear strength layers may be located at the boundary between the coal seam and the underlying strata. They may be in the form of a bedding plane, a clay band, or a claystone layer.

The strength of the layer is commonly described using the Mohr Coulomb failure criterion, which is given in Equation (4.9),

\[ \tau = c + \sigma \tan \varphi \quad (4.9) \]

where \( \tau \) is the shear strength of the weaker layer, \( c \) is the cohesive component of
strength, $\sigma_n$ is the normal stress acting on the shearing interface and $\varphi$ the friction angle of the interface. In this Chapter, two forms of limited shear strength layers were considered, viz., purely cohesive ($\varphi = 0$) and purely frictional ($c = 0$).

### 4.3. Problem definition

The aim of this study was to identify those variables that govern horizontal stress redistribution below a series of parallel longwalls, assuming the plane strain model shown in Figure 4.3(a). The goaf and pillars of the series of parallel extracted supercritical longwalls in the first seam (Figure 4.1) have been represented as an equivalent extracted width for the purpose of the analysis. The results from this Chapter can therefore be applied to multi-seam mining operations involving either single or multiple longwall panels extracted from the first-mined seam.

This Chapter only considers the case where the longwalls are aligned with the maximum horizontal stress direction for reasons discussed in the previous Section. Therefore, in a plane strain analysis the horizontal stress acting in the plane of the analysis corresponds to the minimum horizontal stress. In all cases it was assumed that the magnitude of the unit weight of the strata ($\gamma$) is 0.025MN/m$^3$ and Poisson’s ratio ($\nu$) is 0.25 for all analyses. Values of the other variables used in this Chapter are discussed in the following Section.

![Figure 4.3](image)

*Figure 4.3 – Schematic representation of the model used for the elastic analyses for: (a) isotropic and anisotropic strata, and (b) low shear strength layer or interface.*
4.3.1. Solution method

The changes to the in situ horizontal stresses of the strata being mined using the longwall technique were analysed using the displacement finite element software ABAQUS. A plane strain model with appropriate in situ stresses was meshed (shown in Figure 4.4) and an elastic analysis conducted. The mesh and boundary conditions used in the analysis was similar to the one described in Chapter 3. The boundaries of the model were placed at a sufficient distance from the extracted material to ensure minimal effect on the predicted results. Simulations were performed in two stages. First, an initial geostatic stress field was imposed for the entire rectangular region, with no cavity. In the second stage, the cavity of equivalent width $W_{eq}$ was removed using the element removal procedure available in ABAQUS, allowing the initial geostatic stresses to redistribute.

In this Chapter, a set of parameters was selected for the reference case to allow for easy assessment of the effect each individual variable has on the final stresses induced in the strata. The values used in the reference case are not intended to be indicative of any specific mine. The values varied in the parametric study were selected as representative of what is typical of Australian longwall coal mines. The absolute magnitude of the Young’s modulus of the surrounding strata ($E$) used in the analyses has not been presented, as it does not affect the final stress distribution. Of course, this only applies when the value of Young’s modulus is uniform throughout the continuum. The horizontal stress acting in the plane of the analysis was assumed to correspond to the minimum horizontal stress in the ground. For the reference case, the initial stress state in the ground was chosen to be isotropic, i.e., the value of in situ stress ratio $K$, in the analysis plane, was equal to 1. The effect of varying the value of this ratio was also assessed.

![Figure 4.4 – Finite element mesh used for reference case, where $W/H=8$.](image)
Three different models of horizontal stress redistribution were identified and considered in this Chapter. Separate sets of analyses were conducted for each of the three assumed models. Details of the methods of analysis are given below.

**Isotropic elastic strata**

This first model of horizontal stress redistribution assumed that the strata surrounding the extracted longwall panels does not contain a dominant discontinuity that might otherwise govern how the horizontal stress is redistributed. Any discontinuities that might be present in the real strata are considered to be sufficiently rough, allowing the rock strata to be modeled reasonably well as a continuum. The in situ stresses redistribute as if the surrounding stratum behaves as a homogeneous isotropic elastic material.

Table 4.1 presents the input parameters considered in this model. In the reference case it was considered that the goaf supports no horizontal load due to the assumption that the goaf material has no stiffness in the horizontal direction. This condition of zero goaf stiffness would predict the largest horizontal stresses that could be observed in the strata below the extracted longwall panels while multi-seam mining. The goaf with no horizontal stiffness \((E_g = 0)\) was represented by removing the elements corresponding to the goaf material and placing a uniformly distributed vertical load on the longwall floor equivalent to the weight of the goaf material. This model effectively allowed the strata either side of the goaf to relieve horizontal stress. To satisfy horizontal stress equilibrium, all the horizontal stresses originally supported by the overburden strata must redistribute into the underlying strata. Heave of the longwall floor was insubstantial as it was still confined by the original overburden stress. For the reference case there is no question of how much of the horizontal stress is redistributed into the underlying strata, but rather what is its profile with depth.

A minimum ratio of the equivalent extracted width to the overburden depth \((W_{eq}/H)\) of 2.0 was selected in this Chapter. This is in keeping with the observation that, in Australia, a single longwall panel can be supercritical if its ratio of panel width to overburden depth is 1.4 or larger. Typically, single supercritical longwall panels in Australia correspond to ratios of panel width to overburden depth larger than 1.4, as it is generally more economical to extract wide panels. However, in the context of multi-seam mining, the second seam is mined usually only once the mining of the first seam
has been completed. Therefore, the effective value of the ratio $W_{eq}/H$ of a series of parallel longwall goafs would be quite large. For this reason a maximum value of $W_{eq}/H$ of 16 was used.

Table 4.1 – Variable definition and values used for parametric study of elastic strata.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Values used in the reference case</th>
<th>Other variables considered</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{eq}/H$</td>
<td>Ratio of equivalent extracted width to overburden depth</td>
<td>8</td>
<td>2, 4, 16</td>
</tr>
<tr>
<td>$G'/G_{iso}$</td>
<td>Independent shear modulus</td>
<td>1</td>
<td>0.5, 0.25, 0.167, 0.1</td>
</tr>
<tr>
<td>$E_N/E_M$</td>
<td>Layer stiffness</td>
<td>1</td>
<td>0 to 1</td>
</tr>
<tr>
<td>$t_N/t_M$</td>
<td>Relative ratio of layer thickness</td>
<td>1</td>
<td>0 to 1</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Friction angle of discontinuity</td>
<td>45$^\circ$</td>
<td>20, 25, 30, 35, 40$^\circ$</td>
</tr>
</tbody>
</table>

Anisotropic strata

The profile of horizontal stress with depth is likely to be dependent on the material properties of the strata. Two forms of anisotropy were taken into account to represent different aspects of coal measure strata. Firstly, anisotropy was implemented by considering values of the independent shear modulus ($G'/G_{iso}$) ranging from 0.1 to 1. Including an independent shear modulus represents the coal measure strata that possess parallel bedding planes as an equivalent transversely anisotropic continuum material.

Secondly, the stratum was considered to consist of layers with varying stiffness. The layered strata consisted of two alternating layers, which are labelled $M$ and $N$. The stiffer layer was labelled $M$. Each layer was isotropic but its thickness ($t$) and Young’s modulus ($E$) were varied. Both the relative layer thickness ($t_N/(t_N+t_M)$) and the ratio of Young’s moduli ($E_N/E_M$) were varied from 0.1 to 1.0. Equations (4.3)-(4.4) were used to determine parameters for the equivalent homogeneous orthorhombic elastic material which were used in the finite element analysis. The final stress results predicted by the finite element analysis were then converted to predictions of final stress for each layer using Equations (4.5)-(4.8).

Low shear strength layer

The properties of the boundary between a coal seam and the underlying strata can affect the mechanism by which horizontal stress is redistributed after the extraction of
longwall panels. The effects of a discontinuity or a layer with either purely frictional or purely cohesive properties were considered. The layer was assumed to be located at the base of the first extracted seam (Figure 4.3(b)). The material property values considered in this Chapter were selected as being representative of measurements typically recorded in the field (e.g., Gale (2011)) and are provided in Table 4.1. It was still assumed that the goaf material supported no horizontal stress, so that all the horizontal stress initially transmitted through the overburden needed to be redistributed into the sub-strata, beneath the coal seam.

In addition to reviewing the horizontal stress in the strata below the first extracted seam, an analysis was also made of the distances required to transfer the horizontal stress to the lower strata by shear transfer along the low shear strength layer or interface. A hand calculation was used to obtain an initial estimate of the distance along the discontinuity required to transfer all the initial horizontal load in the overburden into the substrata. The total horizontal load that was initially supported by the overburden strata corresponds to \(0.5K\gamma H^2\), where \(K\) is the in situ stress ratio, \(\gamma\) is the unit weight of the strata, and \(H\) the overburden depth. This situation is depicted in Figure 4.3(b).

Discontinuities in geomaterials whose strength can be characterised as being purely frictional in nature can be described by friction coefficients calculated as \(\mu = \tan \phi\). According to Equation (4.9), the maximum shear strength that can be mobilised along a frictional surface is calculated as \(\tau_{\text{max}} = \sigma_v \tan \phi = \gamma H \mu\). Thus the distance required to transfer all the horizontal force in the overburden through shear stress transfer (denoted by \(x\)) is calculated as:

\[
x = \frac{KH}{2\mu}
\]  

(4.10)

If a layer is purely cohesive, then the maximum shear strength it can support is \(\tau_{\text{max}} = c\). Therefore the distance required to transfer all the horizontal load via the transfer of shear stress across the interface (denoted by \(x\)) is calculated as follows:

\[
x = \frac{K\gamma H^2}{2c}
\]  

(4.11)

The ranges of values for the friction angle, cohesion and stress ratio \(K\) considered in this Chapter are presented in Table 4.1. These hand calculations will be compared with the results from numerical analysis conducted assuming the same interface properties.
4.4. Results and discussion

4.4.1. Elastic strata

Figure 4.5(a) shows a contour plot of the change in horizontal stress for the reference case considered in this Chapter. The reference case consisted of the complete removal of the goaf elements and their replacement with an equivalent vertical load applied to the seam floor. The single uniform stratum is isotropic, the ratio of $W_{eq}/H$ is 8, $H$ is 100m and $K$ is 1. The plot axes have been normalised by the overburden depth $H$. The contour plot shows that the horizontal stress increases in the area below the goaf of the first extracted seam. This stress increase is localized to approximately one overburden depth below the first extracted seam. There are areas of horizontal stress relief either side of the equivalent extracted width, which extend laterally for a distance approximately equivalent to 4 overburden depths.

The area of maximum change in horizontal stress typically occurs around the rib-edge and within half of the overburden depth below the seam floor. The magnitude of the maximum change in horizontal stress is in the order of approximately 10 times the original in situ stress. This Chapter is focused on the stress changes that occur in this area of maximum stress change, as this would imply a multi-seam mining case where the interburden would be very thin. Multi-seam mining with a very thin interburden separating the first seam and the second seam is referred to as ultra close mining, which usually presents its own unique instability problems (Haycocks et al., 1990; Seedsman, 2003). The contours shown in Figure 4.5(a) indicate that for larger interburdens the horizontal stress changes remain relatively insensitive to the horizontal position under the longwall panel. Therefore, the results from the centre line can be used as an approximation for much of the stratum under the longwall panel.

Contour plots of the changes in vertical and shear stresses resulting from horizontal stress redistribution around the equivalent extracted width are presented in Figure 4.5(b) and (c), respectively. The magnitudes of the changes in both the vertical and shear stress components are generally smaller than those predicted for the horizontal stress changes. Vertical stress changes are localized to the area near and under the edges of the equivalent extracted width. These predicted vertical stress changes need to be considered in light of the much larger vertical stress changes that occur as a result of the concentration of vertical stresses in longwall chain pillars (Suchowerska et al., 2013).
Vertical-horizontal shear stress changes are also induced in the area around the edge of the extracted panel. This is the area of high gradients in the horizontal stress change.

![Figure 4.5](image1)

**Figure 4.5** – For the reference case considered in this Chapter, contours of the distribution of final change in: (a) horizontal stress, (b) vertical stress and (c) shear stress in MPa, where H is the overburden depth.

The profile of the final induced horizontal stress (not the stress change) along the vertical centreline of the equivalent extracted width for the reference case is presented in Figure 4.6(a) together with the results for a range of initial in situ stress ratios (K). The maximum increase in horizontal stress occurs just at the floor of the first extracted seam for all values of the initial stress ratio. The horizontal stress gradually returns to the original in situ stress value for increasing depth below the first mined seam. This trend can be better appreciated in Figure 4.6(b), where the profile of change in horizontal stress with depth is plotted against the normalised depth. The horizontal stress change is insignificant at large depth below the extracted seam, i.e., typically at a depth equivalent to 10 times the overburden thickness.

Although the initial in situ stress ratio influences the magnitude of the change in horizontal stress predicted in the analysis, the results obtained after normalizing the
stress change by the original in situ horizontal stress exhibit a common curve for all values of the initial in situ stress ratio. The common curve for the reference case, where the ratio of $W_{eq}/H$ is 8, is plotted in Figure 4.7. The predicted results for the final horizontal stress ($\sigma_{hf}$) have been normalized by the original in situ horizontal stress ($\sigma_{hi}$) at the depth of the second mined seam, i.e., $\sigma_{hf}/\sigma_{hi}$. For all values of the initial stress ratio the maximum normalised horizontal stress change is approximately 11% of the original in situ horizontal stress when $W_{eq}/H = 8$. Additional analyses were run to confirm that the same result was obtained for any values of $W_{eq}$ and $H$, provided the ratio $W_{eq}/H$ had a value of 8.
Chapter 4 – Horizontal stress changes under supercritical longwall panels

Figure 4.6 - Horizontal stress along the centreline of the equivalent extracted width for the reference case where $W_{eq}/H=8$ showing: (a) horizontal stress, and (b) change in horizontal stress for different aspect ratios ($K$).

Figure 4.7 – Horizontal stress distributions below the centreline of the first extracted seam normalised by the initial horizontal stress for different ratios of $W_{eq}/H$. 
Equivalent goaf geometry

Figure 4.7 presents the normalised final horizontal stresses along the centreline of the equivalent extracted width for a range of values of the ratio $W_{eq}/H$. For all values of $W_{eq}/H$ considered, the maximum horizontal stress change occurs in the immediate floor of the first-mined seam. The horizontal stress change dissipates back to almost the initial horizontal stress at approximately 2 to 4 overburden depths below the extracted width. Larger ratios of $W_{eq}/H$ exhibit smaller maximum normalised horizontal stress but the changes to horizontal stress occur deeper below the first-mined seam floor.

The profiles of normalised horizontal stress for various ratios of $W_{eq}/H$ for horizontal cross-sections beneath the equivalent extracted width are presented in Figure 4.8. Plots are presented for selected values of the ratio $H/B$, where $B$ is the interburden thickness, as shown in Figure 4.1. In general, the strata below the equivalent extracted width exhibit an increase in horizontal stress, while the strata not underlying the equivalent extracted width exhibit a slight relief in horizontal stress. Increasing the depth below the first seam (i.e., increasing the interburden depth ($B$)) reduces the maximum horizontal stress for any given value of the ratio $W_{eq}/H$.

![Graph showing normalised horizontal stress profiles](image)

Distance from centre of extracted width

- $H/B = 6$
- $H/B = 4$
- $H/B = 2$

(a)
Figure 4.8 - Horizontal stress distribution below the first extracted seam for: (a) $W_eq/H=2$, (b) $W_eq/H=8$, and (c) $W_eq/H=16$. 

*Chapter 4 – Horizontal stress changes under supercritical longwall panels*
For the smallest ratio $W_{eq}/H$ of 2 considered here (Figure 4.8(a)), the final horizontal stresses ranged from 1.2 to 1.4 times the original in situ horizontal stress. There is not a significant difference between the maximum horizontal stress and the horizontal stress at the centerline of the equivalent extracted width. This may be because the two lobes of very high horizontal stress change that are localized to the rib-edge, as shown in Figure 4.5(a), extend to the centre of the longwall panel.

For larger values of $W_{eq}/H$, the horizontal stress under most of the equivalent extracted width is relatively constant with the maximum horizontal stress occurring as peaks under the edges of the equivalent extracted width. The results from analyses with ratios $W_{eq}/H$ of 8 and 16 are presented in Figure 4.8(b) and Figure 4.8(c), respectively. The curves show that under the equivalent extracted width the horizontal stress change is in the range of 1.03 to 1.1 times the original in situ horizontal stress. The maximum horizontal stress that occurs as a peak under the edges of the equivalent extracted width ranges from 1.1 to 1.3 times the original in situ horizontal stress. Overall, there appears to be very minimal variation in horizontal stress values between both plots (Figures 9(b) and (c)) for curves corresponding to the same value of $H/B$.

Multi-seam mining is usually undertaken once mining of the first seam has been completed, i.e., numerous longwall panels in the first seam have been extracted, and therefore the equivalent extracted width is large relative to the overburden thickness. The results presented here indicate that the change in horizontal stress relative to the initial in situ horizontal stress becomes small for large ratios of $W_{eq}/H$. Typically, the horizontal stress change below the extracted seam increases by 11% of the original in situ horizontal stress. Localized values of higher horizontal stress occur under the edges of the equivalent extracted width.

### 4.4.2. Anisotropic strata

Anisotropy arising from the presence of bedding planes in the coal measure strata was the first form of anisotropic rock mass behaviour considered in this Chapter. An equivalent transversely isotropic elastic material was utilized by assigning an independent shear modulus in the constitutive laws. The horizontal stress results obtained for the range of values of independent shear modulus considered are presented in Figure 4.9. The effects of a transverse isotropic elastic stratum relative to an isotropic elastic stratum are twofold: an increase in the maximum horizontal stress
induced in the immediate floor of the first extracted seam; and an increase in the rate of
dissipation of horizontal stress with depth below the mined seam. The latter means that
anisotropic rock strata require less depth below the first mined seam than do isotropic
strata for the horizontal stress to return to the initial in situ condition. For a 10 fold
reduction in independent shear modulus relative to the isotropic shear modulus the
maximum horizontal stress change increased from 11% to 25% for the reference case
(Figure 4.9). For the same 10 fold increase in independent shear modulus, multi-seam
mining at a ratio of $H/B$ of 2 would increase the horizontal stress change from 7% to
14% for the reference case.

The second form of anisotropy considered was that arising from the varying stiffness of
the geological units present in the coal measures. In such cases the rock mass may be
represented as a homogeneous but orthorhombic elastic medium. The strata considered
in this case consisted of 2 repeating layers with thickness $t_N$ and $t_M$ and Young’s moduli
$E_N$ and $E_M$. Figure 5.11 shows the ratio of minimum to maximum shear modulus for the
equivalent orthorhombic elastic material for the complete range of values of $t_N/(t_N+t_M)$
(i.e., the relative thickness) and $E_N/E_M$ considered in this Chapter. The plot is limited to
values of the ratio $t_N/(t_N+t_M)$ from 0.5 to 1.0 as the plot is symmetrical about a vertical
axis corresponding to a relative thickness of 0.5. As shown by the curves plotted in
Figure 5.11, strata composed of layers with equal thickness but a large difference in
their Young’s moduli exhibit larger differences between values of the shear modulus of
the equivalent orthorhombic material. Larger variation between shear modulus values
leads to greater anisotropy of the strata.

Figure 4.11 shows normalised results for the final horizontal stress below the centerline
of the equivalent extracted width for the case of an equivalent orthorhombic elastic
material. These results were obtained from the finite element analyses. As observed for
the first form of anisotropy considered in this Chapter, stronger anisotropy leads to an
increase in the maximum stress observed in the floor of the first extracted seam and
more rapid dissipation of horizontal stress change with depth.
Chapter 4 – Horizontal stress changes under supercritical longwall panels

Figure 4.9 – Horizontal stress normalised by the initial horizontal stress for different ratios of $G'/G_{iso}$ below the centreline of the first extracted seam for the reference case.

Figure 4.10 – Ratio of maximum and minimum shear modulus for the equivalent material calculated using Gerrard’s equations for a two layer orthorhombic material.
Because of space limitations it is not possible to present the final horizontal stress in the individual soft and stiff layers associated with all permutations of the relative thickness and Young’s modulus considered in this Chapter. Therefore, a short discussion involving examples of the effects of each of these parameters is presented. In the first instance, consider the reference case where \( W_{eq}/H = 8 \), and the values 0.1 and 1.0 were selected for the ratios \( E_N/E_M \) and \( t_N/t_M \), respectively, and \( t_N = t_M = 5m \). The ratio \( G_{min}/G_{max} \) for this example is 0.33. Figure 4.12(a) presents the final horizontal stress predictions for both the equivalent orthorhombic material and the alternating soft and stiff layers. The equivalent orthorhombic material induces the maximum horizontal stress change of 15% into the floor of the first mined seam. By considering the relative stiffness of the layers, the plot shows that the softer layers support less horizontal stress than is predicted for the equivalent orthorhombic material and the stiffer layers support more horizontal stress than the equivalent orthorhombic material. Further, the softer layers have larger differences than the stiffer layers between the final stresses in the individual layers compared to the equivalent orthorhombic material. For both the soft and stiff layers the horizontal stresses eventually converge to the original in situ stress with depth.

Figure 4.12(b) presents results for cases where the thickness of the layers is increased to
10 m, with all other parameters the same as for the general example. Since the relative thickness remained the same, the horizontal stresses still plot on the same minimum and maximum horizontal stress curves as shown in Figure 4.12(a). The maximum and minimum horizontal stress for both cases are 1.17 and 1.06 times the original in situ horizontal stress. For this case where the relative thickness is equal to 0.5, the order of the layers can be reversed, and the stresses would still plot on the relevant soft layer and stiff layer curves.

Figure 4.12(c) and (d) present the results for the same value of the ratio $E_N/E_M$ adopted in the general example, but for cases where the ratio $t_N/t_M$ is 0.1 and 10, respectively. The ratio $G_{\text{min}}/G_{\text{max}}$ is 0.60 for both these examples. When the majority of the rock strata consist of stiffer layers with thin interbeds of softer layers, the horizontal stresses in the stiffer layer are very similar to those calculated for the equivalent orthorhombic material (Figure 4.12(c)). The stresses in the softer layers are much smaller than for the equivalent orthorhombic material. For the case presented in Figure 4.11(c) the softer layers have a horizontal stress in the order of a quarter of the equivalent orthorhombic material. When the majority of the rock strata consist of softer layers with thin interbeds of stiffer layers, the horizontal stress for both the soft and the stiff soft layers differs from the equivalent orthorhombic material by a similar amount (Figure 4.12(d)).

Figure 4.12(e) present the results for cases where the ratio $E_N/E_M$ is increased to 0.2 and the ratio $t_N/t_M$ is retained as 1.0, where $t_N=t_M=10$ m. The increase in the ratio $E_N/E_M$ corresponds to a reduction in the difference between the two Young’s moduli. This reduction in the difference between the two moduli results in a reduction in both the equivalent anisotropy of the strata and in the predicted stress differences between the two layers. For this case the ratio $G_{\text{min}}/G_{\text{max}}$ corresponds to 0.56, which reduces the anisotropic effects relative to the results presented for the general example (Figure 4.12(a)). Therefore, there is a reduction in the maximum stress observed at the seam floor from 15% down to 13%. Furthermore, the increase in the ratio $E_N/E_M$ reduces the difference in the range of maximum and minimum stress observed in the soft and stiff layers. The maximum and minimum horizontal stresses for this case are 1.15 and 1.08 times the original horizontal in situ stress.
Chapter 4 – Horizontal stress changes under supercritical longwall panels

(a)

(b)
Normalised horizontal stress \( \left( \frac{\sigma_{hf}}{\sigma_{hi}} \right) \)

- **(c)**
  - Depth below mined seam: 0.0H, 0.5H, 1.0H, 1.5H, 2.0H
  - Equivalent orthorhombic
  - Layered strata
  - Softer layers
  - Stiffer layers

- **(d)**
  - Depth below mined seam: 0.0H, 0.5H, 1.0H, 1.5H, 2.0H
  - Equivalent orthorhombic
  - Layered strata
  - Softer layers
  - Stiffer layers
It is evident from an examination of the results presented in Figures 13(a) to (e) that the horizontal stress profile predicted from the assumption of an equivalent orthorhombic material is a complex function of the relative thicknesses of the alternating layers and their relative stiffness. For cases where the layering is dominated by stiffer and thicker strata, the equivalent orthorhombic prediction lies close to the stress profile predicted for the stiff layers, as might be expected. In contrast, the equivalent orthorhombic profile lies closer to the profile of horizontal stress in the softer layers when the layering is dominated by thinner softer layers.

The example results presented here for the two forms of anisotropy considered in this Chapter show that the horizontal stress change in anisotropic strata varies depending on the degree of anisotropy. The presence of anisotropic strata leads to larger increases in the maximum stress in the floor of the first mined seam and more rapid dissipation of the horizontal stress change with depth. The results presented here also indicate that strata containing individual layers of varying stiffness should not be approximated as a homogeneous isotropic medium, particularly if the thickness of the layers are approximately equal or if there is a large difference in the Young’s moduli of the individual layers.
4.4.3. Low shear strength layer

The effect of the properties of a discontinuity between a coal seam and the underlying strata on the redistribution of horizontal stress after the extraction of longwall panels was also assessed. The predictions of final stress along the vertical centreline of a longwall panel do not differ very much from the results presented for the isotropic elastic material (Figure 4.7). However, there are variations from the results for an isotropic elastic material in the area along the discontinuity. A longer distance was required to transfer the horizontal force to the lower strata by shear transfer along the low shear strength layer or interface. The effects of a discontinuity or a layer with either purely frictional or purely cohesive properties were considered.

An initial estimate of the distance required to transfer shear stress ($x$) through an interface between a coal seam and sub-strata for a purely frictional interface is given by Equation (4.10). The derivation of this equation assumes the goaf supports no lateral (horizontal) load. From Equation (4.10) it is obvious that the distance to transfer all the horizontal force in shear is linearly proportional to the original in situ stress ratio ($K$) and the overburden depth ($H$), and inversely proportional to the friction coefficient ($\mu$). According to Equation (4.10), mines with deeper overburdens or strata with larger in situ stress ratios would exhibit significant shear stresses in the low shear strength layer at greater horizontal distances from the mined area. Therefore, unlike the cases previously investigated the results for a low shear strength layer are no longer independent of the values of $K$ and $H$.

Figure 4.13(a) shows values of the distance ($x$) required to transfer all the horizontal force in shear across the frictional interface as predicted by Equation (4.10). This figure shows that for the values of $K$ considered, the required transfer distance is in the range from 0.2 to 6 times the overburden depth. Figure 4.13(b) presents the results for the distance ($x$) required to transfer all the horizontal force in shear across the frictional interface as predicted by the non-linear finite element analyses. These distances are typically much longer than those predicted by the approximate hand calculations, i.e., in the order of 6 to 14 times the overburden depth. The reason for this large variation can be attributed to the assumed conditions for shear stress development along the interface. For the hand calculations the shear stress was assumed to uniform and equal to the shear strength of the interface for a length sufficient to transfer the entire horizontal force.
from the overburden. However, the numerical analysis predicts that the shear stress along the layer dissipates with distance from the edge of the equivalent extracted width (Figure 4.14). The results presented in Figure 4.13(b) plot the distance predicted by the finite element analysis for the shear stress to dissipate to 1% of the maximum shear stress.

Figure 4.13 – Shear distance for frictional layer obtained from: (a) hand calculations, and (b) finite element analyses.
The distance required to transmit all the horizontal force through shear transfer in a cohesive layer is given in Equation (4.11). This equation indicates that in contrast to a frictional surface, the overburden depth is a governing factor in the distance required to transfer all the horizontal force. In particular, the shearing distance ($x$) is predicted to be quadratically proportional to the overburden depth ($H$), inversely proportional to the maximum shear strength (i.e., cohesion) and linearly proportional to the in situ stress ratio ($K$) and unit weight ($\gamma$).

Figure 4.15 shows the results for shear transfer distances for the purely cohesive interface according to Equation (4.11). The results show that the shear transfer distances are in the order of 5 to 50 times the overburden depth. This is significantly larger than what was predicted for the frictional surface and in practice would correspond to distances in the order of kilometres.

![Shear stress distribution along frictional surface from the edge of the extracted equivalent width](image)

*Figure 4.14 – Shear stress distribution along frictional surface from the edge of the extracted equivalent width.*
A comprehensive set of finite element analyses was not conducted for the cohesive layer at the boundary of the coal seam and the underlying strata, for several reasons. The finite element analyses with a frictional layer predicted shear transfer distances that are much larger than those determined using the approximate hand calculation method. The same phenomenon of dissipating magnitude in shear stress was observed in preliminary finite element analyses conducted assuming a cohesive surface.

4.5. Discussion

As previously discussed, the in situ stress field is a governing factor in predicting instabilities when underground mining (Gadde et al., 2004). The research detailed in this Chapter has considered the effects of a range of geological and mining variables on the horizontal stress redistribution around a series of multiple parallel longwall panels. By assuming that the goaf can sustain no lateral load, this Chapter has identified the maximum possible horizontal stresses to be expected in the substrata beneath an extracted longwall of coal. If, in reality, the longwall goaf is capable of transmitting some lateral load, then at least some of the original horizontal stress in the overburden may remain transmitted through the goaf material. In this case, the stresses in the strata underlying the extracted seam would have a magnitude somewhere between the original in situ horizontal stress and the predicted values of stress presented in this Chapter.
Further studies could be conducted to consider the effect of stiffness of the goaf material with depth above the first mined seam and yielding in the floor of the first extracted seam.

Yielding of the floor of the first extracted seam can occur for a number of reasons, one of which may be that yielding occurs due to the additional horizontal stress induced in the longwall floor from the redistribution of horizontal stress around the extracted longwall panel. Even though this situation was not explicitly considered in this Chapter, an initial approximation of the redistributed stresses can be made using the results presented in this Chapter. Yielding of the floor of the first mined seam can also be approximated as a deeper section that cannot support horizontal stress, i.e., corresponding to an increase in the parameter $H$. Therefore, arguably a reasonable initial approximation of the horizontal stress redistribution could be made by considering a smaller value of the ratio $W_{eq}/H$ for those cases where the floor strata might yield.

4.6. Conclusions

This Chapter has identified which geometric and geological variables govern horizontal stress redistribution below a series of parallel extracted longwall panels. For the most severe condition considered, where the goaf supports no lateral load and the strata behave as an isotropic material, the maximum change in horizontal stress was approximately 11% for cases where $W_{eq}/H=8$. The maximum stress change occurs in the immediate floor of the first mined seam. The percentage change in horizontal stress relative to the original in situ stress was independent of the magnitude of the initial in situ stress ratio ($K$). Larger magnitudes of the ratio $W_{eq}/H$ generally correspond to smaller changes in horizontal stress, but these stress changes extend deeper below the first mined seam.

Taking into account anisotropy of the coal measure strata increased these predicted magnitudes of the horizontal stresses at shallow depths below the first mined seam, when compared to the values predicted for isotropic rock mass conditions. Also, the horizontal stress change dissipates to zero below the first mined seam more rapidly than for the case of isotropic strata. Consideration of a low shear strength layer immediately beneath the mined coal seam did not alter the horizontal stress changes predicted in the substrata associated with multi-seam mining. The key difference was in the shear stress.
distribution, where a longer horizontal distance was required to transfer the horizontal load originally borne by the overburden into the substrata.

The findings show that the horizontal stress changes to the in situ stresses in the rock strata below the mined seam are significantly smaller than those for the vertical stress change. The implications of this are that when determining the in situ stress environment of the second seam, the horizontal stress will not be significantly different to the original in situ stress. The significance of this Chapter is that it informs designers of multi-seam mines about how various parameters influence the pre-mining horizontal stresses induced in seams underlying those already mined. The effects of these identified parameters need to be considered in conjunction with the vertical stresses induced below extracted longwall panels, in order to design effective and efficient ground support measures for the pillars and roadways located in the lower seam.
CHAPTER 5. PREDICTION OF UNDERGROUND CAVITY ROOF COLLAPSE USING THE HOEK-BROWN FAILURE CRITERION

5.1. Introduction

Failures when mining the second seam can be predicted using more than one method. One possible method is to predict the in situ stress environment in the second seam, which was investigated in the previous two Chapters, and consider which areas violate the selected failure criterion (Gadde et al., 2004). In recent times, numerical techniques such as the displacement finite element method and finite element limit analysis have become viable methods for direct assessment of failures mechanisms and corresponding stability numbers. However, in order to be able to effectively predict the roof collapse of underground openings, whether it is above a roadway or within the overburden, it is imperative that a realistic failure criterion is used to represent the surrounding rock mass. There is still much uncertainty as to the appropriate failure criterion to use for coal measure strata. As discussed in Section 2.5.2, the Hoek-Brown failure criterion is often preferred because of its development from laboratory data. This Chapter presents an analysis of roof collapse based on the Hoek-Brown failure criterion. Chapter 6 will consider the Mohr-Coulomb failure criterion.

A single rectangular cavity in a rock mass that is surrounded by a uniform material is analysed, as shown in Figure 5.1. The geometry of the problem is in terms of the cavity width ($W$) and the overburden depth ($H$) to an overlying free surface. Three methods have been used to assess the stability of underground rectangular openings in rock, namely: 1) an analytical upper bound method proposed by Fraldi & Guarracino (2009); 2) the finite element (FE) upper and lower bound (UB-LB) formulations of Lyamin & Sloan (2002; 2002) and Krabbenhoft et al. (2005); and 3) displacement finite element modelling conducted using the program ABAQUS. All three methods were verified by comparing their predictions in a series of proposed stability charts. Some aspects of the results are also compared to the findings obtained from field measurements of subsidence.
5.2. Background

The demand to extract coal in close proximity to previous workings such as multi-seam mining, in parallel with more stringent occupational health and safety regulations, has been driving the need to understand the variables and parameters governing the overburden and interburden collapse of underground cavities. This Chapter considers the roof collapse in the context of multi-seam mining, however, the results can be applied to all forms of underground opening or cavity.

5.2.1. Failure mechanisms for cavities

The collapse of the roof of an underground cavity potentially affects any overlying materials and structures. In order to gain a better understanding of the observed ground movements, the mechanisms by which the material above underground cavities fails need to be understood. Although circular cavities are generally considered to be more stable than rectangular cavities, in the field of underground mining, cavities cross-sections are usually rectangular in shape in the form of roadways or longwall panels. Consequently, rectangular cavities are the focus here.

When the width to height ratio of the underground cavity ($W/H$) is large, such as in longwall panels, the failure surface will extend through the entire overburden depth ($h_f = H$). Conversely, when the width to height ratio is small, such as in tunnels, mining roadways and energy caverns, the failure surface will be contained entirely within the rock cover ($h_f < H$). In the latter case there will be sufficient cover thickness to generate a pressure arch acting through the rock mass, which supports the overlying material, usually leading to reduced subsidence of the overlying free surface.
5.2.2. Previous studies to assess cavity stability

*Empirical Methods*

In the latter half of the 20th century, understanding when cavity roofs might fail was usually assessed using empirical data. Empirical data compiled from field observations were used to develop tunnel support design methods based on rock mass classification systems, viz., the Rock Mass Rating (RMR) (Bieniawski, 1973) and the Q index (Barton et al., 1974). Lang (1994) presented a detailed empirical stability chart relating the Rock Mass Rating (RMR) to cavity span (Figure 5.2). This stability chart was subdivided into three categories of possible outcomes: stable, potentially unstable and unstable. The chart was then further refined by Ouchi et al. (2004) for cavities in lower quality rock masses. In general, the uncertainties in the stability zonings are high because of the large distribution in data values. The significant drawback in using these methods that are based on empirical data is that stability is correlated with rock mass classification coefficients RMR or Q. Both the RMR and Q systems are used directly for the design of tunnel support and do not give detailed information about the inherent rock mass properties. On the other hand, numerical and analytical methods require specification of the detailed rock mass properties as input variables. For these reasons, it is not generally possible to compare or validate results predicted using empirical methods with those obtained from numerical or analytical methods.

![Stability chart for entry-type underground excavations (Lang, 1994).](image)

*Figure 5.2 - Stability chart for entry-type underground excavations (Lang, 1994).*
Chapter 5 – Prediction of underground cavity roof collapse using the Hoek-Brown failure criterion

Analytical methods

Using analytical methods to assess ground stability allows engineers to develop mathematical models to understand and hopefully predict the mechanics of failure. Such methods include the approximate Limit Equilibrium Method (LEM) and the more rigorous Limit Analysis, which is based on plasticity theory. These methods are widely used in practice due to their simplicity and generality, yet they are underpinned by simplifying assumptions. The Limit Theorems of plasticity provide a very powerful means of conducting stability analysis. Limit analysis involves bracketing the theoretical solution for collapse by determining both an upper bound (UB) and a lower bound (LB) for the true collapse load. The true collapse load can be approximated well if the difference between the UB and LB is small.

There have been several papers detailing solutions for underground cavities in soil and rock. For example, Carranza-Torres and Fairhurst (1999) provided analytical results for circular and cylindrical cavities in a Hoek-Brown material based on the previous work of Carranza-Torres (1998), whereas Gesualdo et al. (2001) presented analytical UB results for a rectangular cavity in a Mohr-Coulomb material.

Upper bound limit analysis of underground cavities has been considered previously using both the Mohr Coulomb and Hoek-Brown failure criteria. Lippmann (1971) presented the results of limit analysis for an underground cavity considering an UB mechanism with a Mohr-Coulomb failure criterion. The cavity was assumed to be rectangular. This UB solution was further developed by Fraldi & Guarracino (2009) to include the Hoek-Brown failure criterion. The collapse mechanism of a circular cavity has been studied using UB limit analysis by Fraldi & Guarracino (2010) and more recently by Yang & Huang (2011). For cases in which it is assumed that the rock mass fails according to the Hoek-Brown criterion, application of this method involves fitting Equation (5.1) to the Hoek-Brown criterion by calculating appropriate values for the dimensionless parameters $A$ and $B$, defined as follows:

$$
\tau = \pm A \sigma_{ci} \left( \frac{\sigma_n - \sigma_t}{\sigma_{ci}} \right) B
$$

(5.1)

where $\sigma_t$ is the maximum tensile strength of the rock mass. Exact closed-form expressions for $A$ and $B$ cannot be established for the reference case, but it is noted that Equation (5.1) represents the Mohr Coulomb failure criterion exactly for the special
case where \( A = \tan\varphi, B = 1, \) and \( \sigma_t = c(\tan\varphi)^{-1}. \) Approximate values of \( A \) and \( B \) can be calculated by considering the approximate equivalence of the curve represented by Equation (5.1) with the Hoek-Brown failure criterion using the method described in Appendix A. A more thorough discussion of the regression technique most suited to determining the values of \( A \) and \( B \) is also provided in Appendix B.

**Numerical modelling**

The increase in power and accessibility of computers has enabled numerical modelling to become a popular means of investigating underground stability. There are numerous examples in the literature of the application of the displacement finite element method to underground cavity problems. In the main, these have usually been quite specific instances involving case studies, rather than more general studies of the stability problem.

### 5.3. Problem definition

In order to conduct a general assessment of the stability of rectangular underground cavities a plane strain model was developed in this Chapter, as shown in Figure 5.1. Details of the values of the variables assumed in the parametric study of this problem are presented in Table 5.1. The values of the ratio \( W/H \) were varied from 1 to 8 to encompass both small cavities such as tunnels and mining roadways, and also wide cavities such as longwall panels. The model was designed to represent a cavity in a rock mass that is surrounded by a uniform material. This is an idealised representation of an underground cavity where there is a reasonably uniform overlying material.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Values considered</th>
</tr>
</thead>
<tbody>
<tr>
<td>( GSI )</td>
<td>Material property</td>
<td>10 to 90, inc of 10</td>
</tr>
<tr>
<td>( m_i )</td>
<td>Material property</td>
<td>5 to 30, inc of 5</td>
</tr>
<tr>
<td>( H )</td>
<td>Cover depth</td>
<td>10m to 80m, increments of 10m</td>
</tr>
<tr>
<td>( W )</td>
<td>Width of rectangular cavity</td>
<td>10m to 80m, increments of 10m</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Unit weight of material</td>
<td>Optimised</td>
</tr>
<tr>
<td>( \sigma_{ci} )</td>
<td>Compression strength of intact rock</td>
<td>100MPa</td>
</tr>
</tbody>
</table>

This problem has been established in order to identify the influences of the basic variables such as geometry and rock strength on roof collapse. Therefore the primary
modes of failure considered in this Chapter are shear and tensile failure through the rock mass, which has been characterised by an appropriate value of the Geological Strength Index (GSI). This approach does not include effects such as the influence of ground water and geological structures (e.g., faults, bedding, anisotropy, etc.) on roof stability, although it is generally acknowledged these will influence ground deformations (Gale, 2004). The modelling conducted in this Chapter did not consider shear or tensile failure of bedding planes or pre-existing fractures not accounted for in the GSI classification framework.

The Hoek-Brown failure criterion was used to represent the strength of the rock for the reasons outlined in Section 2.5.2. An associated plastic flow rule was assumed for all calculations, which is consistent with the analytical method used in this Chapter.

Dimensionless stability parameters $N$ and $M$ have been derived as a convenient means of presenting the results in a non-dimensional form. They are defined in Equation (5.2), where $F$ is the Factor of Safety against failure of the roof of the cavity.

$$ N = \frac{\sigma_{ci}}{\gammaWF}, \quad M = \frac{\sigma_{ci}}{\gammaHF} $$ (5.2)

### 5.3.1. Solution methods

Three different methods were used in this Chapter to predict the collapse of an underground cavity. The different predictions were compared in order to provide theoretical validation and implementation of the various solution methods. The first method adopted involves application of numerical UB and LB formulations (Lyamin et al., 2002; Lyamin et al., 2002; Krabbenhoft et al., 2005). The second is an analytical closed form UB analysis (Fraldi et al., 2009). The third is a numerical approach involving the adoption of the conventional displacement finite element method using the commercial software ABAQUS. Further details of these methods are provided as follows.

**Finite element upper and lower bound formulation (FE UB-LB)**

The finite element (FE) upper bound (UB) and lower bound (LB) formulations developed by Lyamin and Sloan (2002; 2002) and Krabbenhoft et al. (2005) have potential for application to underground stability problems. They generally yield more accurate results than limit analysis predictions undertaken by hand calculations. Unlike
analytical derivations, these formulations do not assume a failure mechanism or stress field in advance. Instead, finite elements are used to obtain a kinematically admissible velocity field and a statically admissible stress field in the UB and LB formulations, respectively. These FE formulations involve iterative calculations, which aim to refine the margin between the two bound values. Further, the meshes used in the analyses may include stress and displacement discontinuities, which can therefore allow for abrupt changes in stresses in the lower bound formulation and in the velocities in the upper bound formulation. Despite the obvious power and potential of these techniques, they have until recently seen little application to the problem of underground cavity stability (Wilson et al., 2011; Yamamoto et al., 2011). Readers are referred to the original papers published on the UB and LB formulations for further details (Lyamin et al., 2002; Lyamin et al., 2002; Krabbenhoft et al., 2005; Merifield et al., 2006).

In this Chapter, all the rock mass parameters were assumed to have constant values throughout the overburden. Although not strictly necessary, interfaces were placed at the top and the bottom of the cavity pillar (Figure 5.3). These interfaces were modelled as rough interactions to ensure no planes of weakness were introduced into the model. In the limit analyses, for given cavity geometry \((W, H)\) and mechanical properties of the rock mass \((\sigma_{ci}, GSI, m_i)\), the optimised numerical solutions for the upper and lower bounds have been obtained with respect to the unit weight \((\gamma)\) of the rock mass. The same solution is also obtained by considering realistic unit weights and solving for the maximum cavity width.

The FE UB-LB problem is defined using an input data file that contains information on boundary conditions, materials and initial mesh arrangement. The numerical procedure automatically meshes the problem cross-section based upon the minimum number of elements prescribed in the data file. The formulation can be run with adaptive meshing prescribed, with each adaptive iteration adding extra mesh elements in areas of high variation of stress or velocity, i.e., high stress gradients in the LB approach, and large displacement or velocity gradients in the UB approach. The accuracy of the UB and LB numerical solutions is determined largely by the degree of mesh refinement. The minimum number of elements adopted in each analysis was made large enough to ensure a maximum margin of 10% between the LB and UB predictions of the collapse conditions.
Closed form upper bound analysis (CF-UB)

Upper bound solutions presented by Fraldi & Guarracino (2009) indicate that the critical half-width of an excavation \( L \) and the maximum height of the collapsing rock block \( h \) can be calculated by Equations (5.3) and (5.4) respectively:

\[
L = AB^{-\theta} (1 + B)^\theta \gamma^{-1} \sigma_{si}^{(1-B)} \sigma_i^B
\]  

\[
h = \frac{(1 + B) \sigma_i}{\gamma B}
\]

The critical unit weight that would cause the roof to collapse was calculated by doubling...
the half-width (since $W = 2L$) and equating it to the desired span and then optimising the unit weight ($\gamma$). This upper bound derivation assumes that the cavity is sufficiently deep such that the thickness of the collapsing block would never reach the surface (i.e., $H > h$). In the model considered here, at large cavity spans relative to cover depth, this assumption is not always true. The corresponding span for which the collapsing block height equals the cover depth shall be referred to as the critical width and denoted by $(W/H)_{\text{crit}}$. The critical width $(W/H)_{\text{crit}}$ can be calculated by substituting the cover depth $(H)$ into Equation (5.4) and rearranging for unit weight ($\gamma$) to provide:

$$\gamma = \frac{(1 + B)\sigma_t}{HB} \quad (5.5)$$

$$\left(\frac{W}{H}\right)_{\text{crit}} = \frac{2AB^{-B}(1+B)^{\frac{B}{1-B}}\sigma_{ci}^{(1-B)}\sigma_t^B}{(1+B)\sigma_t} \quad (5.6)$$

Substituting Equation (5.5) and $2L = W$ into Equation (5.3), provides the critical half width for a collapsing block penetrating through the cover depth, as Equation (5.6).

The regression analysis required to calculate the most appropriate values of the parameters $A$ and $B$ was performed using two methods: firstly by the method recommended by Hoek (2000), where 8 equally spaced values of $\sigma'_3$ are adopted in the range $0 < \sigma'_3 < 0.25 \sigma_{ci}$; and secondly using a large number of divisions (i.e., 10,000) over a range of values of $\sigma'_3$ identified to be specific to this problem. The FE formulation, discussed above, was used to identify the principal stresses in the cover depth at failure, in order to assess an appropriate range for $\sigma'_3$ values to use in this regression method. Further details of the second approach are provided in Appendix B.

**Displacement Finite Element Modelling (D-FEM)**

Numerical modelling of the cavity was also conducted using the displacement finite element software ABAQUS. Figure 5.4 shows the plane strain model used in this analysis. The modelling consisted of a two-step process: in the first analysis step the geostatic forces were equilibrated and in the second analysis step a rectangular section of material was removed as a single block excavation and the internal stresses in the rock mass were allowed to come to equilibrium. The Hoek-Brown failure criterion was incorporated into the constitutive model representing the rock mass using the formulation developed by Clausen and Damkilde (2008). The material model was
considered to be linearly elastic – perfectly plastic with the Hoek-Brown failure criterion acting as both the yield criterion and plastic potential.

![Figure 5.4 - Model used in the displacement finite element modelling. A high density of elements was required at the top of the cavity abutments for the analysis to converge.](image)

In this type of analysis failure of the roof of the cavity was defined as the point at which overall equilibrium of the model could just be obtained under the largest material unit weight. The point of instability or collapse was also confirmed by reviewing the shape and magnitude of the displacement curve of the midpoint on the roof of the cavity. The analysis process was automated by using the ABAQUS parametric study function and the built-in meta-command language, python, to cover a sweep of rock material unit weights, in order to locate the point of collapse efficiently. The critical unit weights were then used to calculate the stability number as defined in Equation (5.2). Displacement FE solutions were obtained for all values of \(GSI\) and a value of \(m_i\) of 20, and the predictions were compared with those of the other methods adopted in this Chapter.

### 5.4. Results

#### 5.4.1. Finite element upper and lower bound

The results from the FE limit analysis are an upper and lower bound to the theoretical collapse load. The upper and lower bound values for the stability numbers have been bracketed to within a margin of 10% of each other. The results are presented in terms of the stability factors \(N\) and \(M\) (Equation (5.2)) in Figure 5.5 and Figure 5.6, respectively. Charts of stability factors \(N\) and \(M\) for the values of \(m_i\) considered here were obtained using the UB (solid line) and LB (dashed line) formulations.
Chapter 5 – Prediction of underground cavity roof collapse using the Hoek-Brown failure criterion

Decreasing stability

\[
\left( \frac{W}{H} \right)_{\text{crit}}
\]

Subcritical failure
Supercritical failure

\[
\frac{N}{\sigma_f} = \frac{yF}{c}
\]

\[
W/H
\]

\[
\frac{W}{H}
\]

\[
m_f = 5
\]

\[
m_f = 10
\]

\[
m_f = 15
\]
Figure 5.5 - Charts of stability factor $N$ obtained using FE-UB (solid line) and FE-LB (dotted line).
Chapter 5 – Prediction of underground cavity roof collapse using the Hoek-Brown failure criterion

$m_i = 5$

$m_i = 10$

$m_i = 15$
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Figure 5.6 - Charts of stability factor $M$ obtained using FE-UB (solid line) and FE-LB (dotted line).

$M = \frac{\sigma_{ci}}{\gamma_{HF}}$

$W/H$

$GSI$

$m_i = 20$

$m_i = 25$

$m_i = 30$
There is a common trend between the stability factor $N$ and the ratio $W/H$, which is observed over all the values of $m_i$. For each of the GSI values, the stability factor $N$ is constant at low values of $W/H$, which corresponds with the failure surface remaining within the overburden. At higher values of $W/H$ the values of $N$ increase, corresponding to the failure surface extending through the full overburden. These two types of failure surface are illustrated in Figure 5.7. The changes in slopes in the plots in Figure 5.5 and Figure 5.6 correspond to conditions where the failure surface first extends through the entire cover depth. The grey region in the plots in Figure 5.5 show the range of critical widths $(W/H)_{crit}$, which correspond to the transition zone from subcritical to supercritical, for all the $m_i$ values considered here.

Figure 5.7 – FE-UB predictions of the degree of plastic dissipation showing that the failure surfaces are: (a) subcritical for low ratios $W/H$ and (b) supercritical for high ratios $W/H$. 
5.4.2. Closed form upper bound

The stability numbers calculated using the analytical upper bound solution of Fraldi & Guarracino (2009) (CF-UB) have been calculated for comparison with the numerical bounds solutions presented in Figure 5.5 and Figure 5.6.

An important task in applying this closed-form upper bound solution is that of selecting the most appropriate values of the strength parameters \( A \) and \( B \), defined in Equation (5.1). It will be recalled that they cannot be obtained as closed form expressions in terms of the more conventional Hoek-Brown strength parameters \( (\sigma_{ci}, GSI, m_i) \), which means that approximate techniques must be applied to their evaluation. Two regression methods were used to determine the parameters \( A \) and \( B \), which in turn are necessary to obtain an upper bound to the stability number \( N \). Details of these methods are provided in Appendix B.

Figure 5.8 shows the results for the upper bound on the collapse condition obtained from calculations using the method proposed by Fraldi and Guarracino (2009), for \( m_i \) values of 20. The results show that the stability parameters \( (N \) or \( M) \) increase as \( GSI \) decreases, indicating the factor of safety decreases with decreasing \( GSI \).

The dashed lines in both charts in Figure 5.8 show the limit when the collapsing rigid block extends through the whole cover depth (i.e., when \( H = h \)). This was determined using the method outlined in Section 5.3.1. The maximum achievable span for a given cover depth is observed to occur for a value of \( GSI \) of 20 to 30.

The results corresponding to all other values of \( m_i \) considered in the Chapter can be scaled off the values obtained for \( m_i \) of 20. In particular, the stability factors obtained from Figure 5.8 can be scaled to account for other values of \( m_i \) using the factors presented in Figure 5.9.
Chapter 5 – Prediction of underground cavity roof collapse using the Hoek-Brown failure criterion

Figure 5.8 - Stability charts for $m_i$ of 20, obtained using the CF-UB analysis for stability parameter: (a) $N$ and (b) $M$.
5.4.3. Displacement finite element modelling

Results from the displacement finite element modelling are plotted together with the predicted limits of the UB and LB formulations obtained using the FE method for $m_i = 20$ in Figure 5.10. It may be seen that excellent agreement has been found between the predictions obtained from these different approaches, thus validating the accuracy of the results of these methods.
### 5.4.4. Application example

The charts presented in Figure 5.5 can be used to calculate the Factor of Safety for a proposed cavity. The surrounding rock mass needs to be assessed according to the GSI and the geometry of the proposed cavity and the cover depth also need to be known. As an example, a 40m wide cavity is to be constructed in medium strength sandstone characterised by $\sigma_{ci} = 80$ MPa and a unit weight of 25 kN/m$^3$. The values of GSI and $m_i$ have been quantified as 50 and 20, respectively. The depth to the surface is 30m. Therefore $W/H = 1.34$, and Figure 5.5 indicates a value of $N$ of approximately 50. The Factor of Safety can then be calculated as $F = \sigma_{ci}/\gamma WN = 80,000/25 \times 40 \times 50 = 1.6$.

This information can also be applied to determine the maximum width of cavity required in order to attain a given Factor of Safety. If a minimum Factor of Safety of 2 is required the dimensionless parameter $M$ is calculated: $M = \sigma_{ci}/\gamma HF = 80,000/25 \times 30 \times 2 = 53.33$. Using Figure 5.6, for GSI = 50, the maximum value of $W/H$ is approximately 1.0. Therefore the maximum cavity width to attain a Factor of Safety of 2 would be $1 \times 30 = 30$m.

### 5.5. Discussion

Figure 5.11 shows the values of the stability factor $N$ obtained from the FE-LB and CF-UB methods considered in this Chapter, for a value of $m_i$ of 20. The results of the CF-UB analysis lie between the results of the FE upper and lower bound formulations. The CF-UB analysis appears to overestimate the critical width ratio $(W/H)_{cr}$ There is one notable reason why this has occurred. The derivation of the CF-UB method assumes that the remaining strata left behind, after the roof has collapsed, will be stable. When the depth of the collapsing block ($h$) approaches the full overburden ($H$), this becomes very unlikely in practice. The CF-UB analysis does not include an assessment of the stability of the remaining strata, while the FE UB-LB method does.

Both FE LB-UB and displacement finite element modelling methods were in agreement on the location of the failure surfaces for all values of $W/H$ analysed. As expected, cavities with low $W/H$ ratios exhibited failure surfaces that remained within the overburden, and cavities with high $W/H$ ratios had failure surfaces that extended through the overburden. In the example where $GSI = 50$, $m_i = 20$, $W/H = 2$, both methods showed the maximum height of the caved zone ($h$) to be approximately $1/3 H$. 

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In contrast, for a $GSI = 50$, $m_i = 20$, and $W/H = 8$, the failure surface cuts through the upper free surface at a distance approximately $H$ from the centreline. The significant difference between the two methods is that the displacement finite element modelling results allow additional yielding in the overburden (Figure 5.12).

The results obtained from the FE UB-LB formulation with the Hoek-Brown failure criterion predicted the critical ratio $(W/H)_{crit}$ to range from 1.5 to 8.0 for magnitudes of $m_i$ from 5 to 30. Small magnitudes of $(W/H)_{crit}$ were generally predicted by smaller values of $m_i$ together with higher values of $GSI$. As mentioned in Section 2.3.2, the subsidence recorded in the field has been used to identify the transition from subcritical to supercritical failure of the overburden. In Australia, the critical cavity width in coal measure strata is given approximately by a ratio $(W/H)_{crit}$ of 1.0 to 1.6 (McNally et al., 1996; Mine Subsidence Engineering Consultants, 2007; Mills et al., 2009). It appears that the Hoek-Brown failure criterion predicted much larger magnitude of ratio $(W/H)_{crit}$ than is typically recorded in the field.

![Stability chart with predictions from FE-LB and CF-UB.](image)

Figure 5.11 – Stability chart with predictions from FE-LB and CF-UB.
Chapter 5 – Prediction of underground cavity roof collapse using the Hoek-Brown failure criterion

5.6. Conclusions

In this Chapter, stability charts for rectangular cavities obtained using the Hoek-Brown failure criterion have been presented. Three different methods of analysis yielded results that generally agree well with each other. The limit analysis, with an adaptive mesh feature, bracketed the collapse load to within 10%. The results from the closed form upper-bound analysis and the displacement finite element method lay within the bracket defined by the FE upper and lower bound formulations. This Chapter verified the implementation of the Hoek-Brown failure criterion in the limit analysis and the displacement finite element method. The predicted critical ratios ($W/H_{\text{crit}}$) corresponding to the boundary of subcritical and supercritical failure of the overburden, were larger than those typically observed in subsidence field measurements. This finding suggests that the Hoek-Brown failure criterion is not a good representation of the strength of coal measure strata.

The proposed stability charts should enable designers of underground cavities to predict rapidly the safe widths of underground cavities assuming the surrounding rock mass obeys the Hoek-Brown failure criterion. They also show whether the failure surface will be localized or otherwise will extend through the whole overburden.
CHAPTER 6. PREDICTION OF UNDERGROUND CAVITY ROOF COLLAPSE USING THE MOHR-COULOMB FAILURE CRITERION

6.1. Introduction

In the previous Chapter, the roof collapse of an underground cavity was assessed based on the assumption that the rock mass obeys the Hoek-Brown failure criterion. However, as discussed in Section 2.5.2, considerable uncertainty exists as to whether this criterion adequately characterizes failure of coal measure rock masses. The magnitude of \( (W/H)_{crit} \), when the failure surface transitions from being subcritical to supercritical, obtained from predictions of underground cavity roof collapse using the Hoek-Brown failure criterion appeared to be much larger than typically inferred from field measurements of subsidence. Further investigation of the stability of rectangular underground cavities is therefore required in order to be able to reliably and accurately predict roof collapse and its shape, at least in some coal measure rocks of New South Wales. As discussed in Section 2.5.2, a much simpler alternative to the Hoek-Brown failure criterion is the Mohr-Coulomb failure criterion, which is also commonly used for stability analysis of rock masses.

This Chapter investigates the roof collapse of an underground cavity assuming the rock mass obeys the Mohr-Coulomb failure criterion, with and without a tension cutoff. The same three methods as used in Chapter 5 were again used to assess stability, namely: the finite element upper bound and finite element lower bound formulations (FE UB-LB) derived by Lyamin and Sloan (2002; 2002); the closed form upper bound (CF-UB) derived by Fraldi and Guaraccino (2009); and displacement finite element modelling (D-FEM) using the software ABAQUS. Results are compared to those obtained assuming the Hoek-Brown failure criterion (Chapter 5), and the insights drawn from observed similarities and differences are used to infer which failure criterion is more realistic.

6.2. Background

A discussion on the current methods used to assess underground cavity stability was presented in the previous Chapter. Only the analytical methods that employ the Mohr-Coulomb failure criterion shall be expanded further here. There have been several
derivations of solutions of limit analysis for an underground rectangular cavity that consider an upper bound (UB) mechanism with a Mohr-Coulomb failure criterion (Lippmann, 1971; Gesualdo et al., 2001). The UB solution, developed by Fraldi & Guarracino (2009), assumes that the Hoek-Brown failure criterion for a rectangular cavity, but it can also consider the Mohr-Coulomb failure criterion. The derivation employs Equation (5.1) to represent the Hoek-Brown criterion and requires calculation of appropriate values for the dimensionless parameters $A$ and $B$. Equation (5.1) represents the Mohr-Coulomb failure criterion exactly for the special case where $A = \tan \phi$, $B = 1$, and $\sigma_t = c \cot \phi$. The derivation assumes the linear form of the Mohr-Coulomb failure criterion.

There are no published analytical methods that consider roof stability of underground cavities, assuming plane strain conditions and a Mohr-Coulomb failure criterion with a tension cut-off. For this reason, in this Chapter, finite element methods were used to assess the stability of cavity roofs assuming the Mohr-Coulomb failure criterion with a tension cut-off.

6.3. Problem definition

To assess the stability of rectangular cavities using the Mohr-Coulomb failure criterion a plane strain model was adopted as shown in Figure 6.1. This is the same plane strain model as was used in the previous Chapter with the only modification being the failure criterion used in the analysis. Details of the values of the variables assumed in the parametric study of this problem are presented in Table 6.1. The values of the ratio $W/H$ were varied from 1 to 4 to encompass both small cavities and also wide cavities. The model was designed to represent a cavity in a rock mass that is surrounded by a uniform material. This is an idealised representation of an underground cavity where there is reasonably uniform overlying material.

The Mohr-Coulomb failure criterion was used to represent the strength of the rock mass. It was assumed that the rock mass could be represented as one material, whereby the properties of the intact rock and the properties of any discontinuities present in the rock mass could be smeared into a single material, with properties defined as a single value of $c$ and a single value of $\phi$. The values of friction angle that were considered in this Chapter were selected to be representative of sedimentary strata (Goodman, 1989). The problem was analysed both with and without a tension cut-off to the Mohr-Coulomb
failure criterion. An associated plastic flow rule was assumed for all calculations, which is consistent with the analytical method used in this Chapter.

![Diagram](image)

**Figure 6.1 – Schematic representation of the problem analysed in this Chapter.**

**Table 6.1 - Variables considered in the analysis of cavity roof collapse using the Mohr-Coulomb failure criterion.**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Values considered</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H$</td>
<td>Cover depth</td>
<td>10m to 40m, increments of 10m</td>
</tr>
<tr>
<td>$W$</td>
<td>Width of rectangular cavity</td>
<td>10m to 40m, increments of 10m</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Unit weight of material</td>
<td>Optimised</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Material property</td>
<td>20º to 50º, increments of 10º</td>
</tr>
<tr>
<td>$c$</td>
<td>cohesion of overburden rock mass</td>
<td>1.5MPa</td>
</tr>
<tr>
<td>$\sigma_t*$</td>
<td>tensile cut-off stress</td>
<td>$\sigma_t$ to 3% $\sigma_t$</td>
</tr>
</tbody>
</table>

Dimensionless stability parameters $N$ and $Q$ are used as a convenient means of presenting the results in non-dimensional form. The use of stability parameter $N$ will allow for easy comparison with the results presented in the stability charts obtained using the Hoek-Brown failure criterion (presented in Chapter 5). However, it should be noted, that since the parameter $\sigma_c$ is a function of the friction angle $\varphi$, the plotted results for specific friction angles will be not strictly independent of the stability parameter $N$. For this reason, stability parameter $Q$ is used to show the true trend of varying independent variables $\varphi$ and $c$. The stability parameters $N$ and $Q$ are defined in Equation (6.1), where $F$ is the Factor of Safety against failure of the roof of the cavity;

$$N = \frac{\sigma_c}{\gamma WF}, \quad Q = \frac{c}{\gamma WF}$$

(6.1)
6.3.1. Solution methods

Three different methods were used in this Chapter to predict the collapse of the roof of the underground cavity. The results from the different prediction methods were compared in order to provide theoretical validation of the solution methods. The first method involved application of finite element UB and LB formulations (Lyamin et al., 2002; Lyamin et al., 2002; Krabbenhoff et al., 2005). Both the linear form and the tension cut-off form of the Mohr-Coulomb failure criterion (details provided in Section 2.5.2) were considered in this first method. The second method adopted was the analytical closed form UB analysis proposed by Fraldi and Guarracino (2009). The derivation of the CF-UB considered only the linear form of the Mohr-Coulomb failure criterion. The third was a numerical approach involving the adoption of the displacement finite element method using the software ABAQUS. Further details of these methods are provided as follows.

Finite element upper and lower bound formulation (FE UB-LB)

Solutions for the conditions producing cavity collapse were obtained from numerical techniques developed by Lyamin and Sloan (2002; 2002) and Krabbenhoff, Lyamin et al. (2005). The analysis method used was the same as described in Chapter 5. For any given cavity geometry \((W, H)\) and mechanical properties of the rock mass \((c, \varphi)\) the optimized numerical solutions for the upper and lower bounds have been obtained with respect to the unit weight \((\gamma)\) of the rock mass. The tension cut-off, as the second form of the Mohr-Coulomb failure criterion considered in this Chapter, was implemented in the limit analysis algorithms by prescribing an additional failure surface. This additional failure surface was defined using a cohesion and friction angle, such that \(c = \sigma_t^*\) and \(\varphi = 90^\circ\). The minimum number of elements adopted in each analysis was made large enough to ensure a maximum margin of 10% between the UB and LB predictions of the collapse conditions.

Closed form upper bound analysis (CF-UB)

The UB limit analysis considered in this Chapter was derived for a rectangular cavity in a Hoek-Brown material (Fraldi et al., 2009). The derived UB solution can be used for the Mohr-Coulomb criterion by selecting values of \(A\) and \(B\) such that \(A = \tan \varphi\) and \(B = 1\). These values for parameters \(A\) and \(B\) correspond to the linear form of the Mohr-Coulomb criterion. The limitation of this analysis method is that it only considers
subcritical failure. Equation (5.6) describes the critical span \((W/H)_{\text{crit}}\) for which the collapsing block height \((h_f)\) equals the cover depth \((H)\). The complete derivation of Equation (5.6) was presented in the previous Chapter. The predictions of roof collapse are compared with those of the CF UB-LB formulations adopted in this Chapter.

Displacement finite element modelling (D-FEM)

Numerical modelling of the stability of the cavity was also conducted using the displacement finite element software ABAQUS. The modelling consisted of the same two-step process as used for the Hoek-Brown failure criterion analyses. The Mohr-Coulomb failure criterion with a tension cut-off was incorporated into the constitutive model representing the rock mass using the formulation developed by Clausen and Damkilde (2006).

In this type of analysis, failure of the roof of the cavity was defined as the point at which overall equilibrium of the model could just be obtained under the largest material unit weight. The point of instability or collapse was also confirmed by reviewing the shape and magnitude of the displacement curve of the midpoint on the roof of the cavity. Displacement FE solutions were obtained for a selection of magnitudes of the ratios \(\sigma_t/\sigma_i\) and \(W/H\) and a value of \(\phi\) of 30 degrees. The analyses were conducted for an overburden depth of \(H=150\text{m}\). The predictions are compared with those of the CF UB-LB formulations adopted in this Chapter.

6.4. Results

6.4.1. Linear form of the Mohr-Coulomb failure criterion

Finite element upper and lower bound formulation

The results from the FE limit analysis are an upper and lower bound to the theoretical collapse load. The upper and lower bound values for the stability numbers have been bracketed to within a margin of 10% of each other. The results for the linear form of the Mohr-Coulomb failure criterion are presented in terms of the stability factors \(N\) and \(Q\) in Figure 6.2(a) and (b), respectively. The charts of stability factor were obtained using the UB (solid line) and LB (dotted line) formulations. In Figure 6.2(a) and (b) the curve for \(\phi = 20\) degrees is truncated at both small and large values of the ratio \(W/H\) as the critical predicted failure did not correspond to the roof collapse mechanism, which is schematically defined in Figure 6.1. The observed forms of the predicted failure
mechanisms for $\phi = 20$ degrees are discussed in Appendix C.

![Chart of results](image)

**Figure 6.2** – Charts of the results obtained using the FE-UB (solid line) and FE-LB (dotted line) formulation for: (a) stability factor $N$ and (b) stability factor $Q$.

In general, the larger magnitudes of the friction angle gave larger predicted values of $N$. This prediction indicates that materials with larger friction angles would, in general, be
less stable. This initial observation seems counterintuitive, but this trend arises because the magnitude of $\sigma_c$ is being influenced by the magnitude of $\varphi$. In contrast, the larger magnitudes of the friction angle gave smaller predicted values of $Q$. This result agrees with the findings presented in Chapter 5.

As reported in the previous Chapter, the distinction between supercritical and subcritical failure, assuming the Hoek-Brown failure criterion, was exhibited as two different portions of the curve on the stability charts. The curve where $\varphi = 30$ degrees in the linear Mohr-Coulomb stability chart (Figure 6.2(a) and (b)) appears to consist of three portions. The portion of the curve at low values of the ratio $W/H$, where the curve is approximately horizontal, corresponds to the failure surface remaining within the cover depth. An example of this subcritical failure is depicted in the plot of plastic energy dissipation for $W/H = 0.2$ and $\varphi = 30$ degrees, shown in Figure 6.3(a). Another portion of the curve, at large values of ratio $W/H$, involves the predicted failure surface extending through the cover depth. An example of this supercritical failure is depicted in the plot of plastic energy dissipation for $W/H = 3$ and $\varphi = 30$ degrees, shown in Figure 6.3(b).

![Figure 6.3](image)

*Figure 6.3 – FE-UB predictions of the degree of plastic dissipation showing that the failure surfaces are: (a) subcritical for low value of ratio $W/H$, and (b) supercritical for high value of ratio $W/H$.***

The third portion of the curve for $\varphi = 30$ degrees in the linear Mohr-Coulomb stability chart is located in between the first two portions mentioned previously. This middle portion corresponds to the failure surface transitioning from subcritical to supercritical
failure mechanism, as shown in Figure 6.4. Typical failure surfaces have been marked on the plots of plastic dissipation as dashed lines. There does not appear to be a distinct position along the curve that can be easily marked as the boundary of subcritical and supercritical failure. The grey region in Figure 6.2(b) shows the range corresponding to the transition zone for all the friction angles considered here.

The boundaries between the various portions on the curve occur at different magnitudes of the ratio $W/H$ for each of the curves of varying friction angle $\phi$ (Figure 6.2). The reason for this can be understood by comparing the plastic dissipation for a range of friction angles for $W/H = 3.0$ (Figure 6.5). It can be seen that in these cases of supercritical failure that the angle between the failure surface and vertical ($\theta$) appears to be approximately equal to the friction angle of the rock mass. Therefore, materials with larger friction angles predict a flatter profile for the failure surface. This finding is in agreement with the findings of analyses of the Terzaghi trap-door problem (e.g., Smith, 1998).

Figure 6.4 – FE-UB predictions of the degree of plastic dissipation showing the failure surfaces as dashed lines. There is a progressive transition from subcritical to supercritical failure mechanisms for increasing ratio $W/H$. 

![Figure 6.4](image-url)
Figure 6.5 – FE-UB predictions of the degree of plastic dissipation for ratio W/H of 3.0 and varying friction angle.
Closed form upper bound

The results for the analytical upper bound solution of Fraldi and Guarracino (2009) (CF-UB) for subcritical failure are presented in Figure 6.6(a). The stability parameter \( N \) increases as the friction angle \( (\phi) \) increases. The dashed line shows the limit when the failure surface extends through the entire overburden thickness. This limit was determined using Equation (5.6).

The CF-UB results are in reasonable agreement with those predicted by the FE UB-LB (Figure 6.6(b)). The CF-UB curves for the friction angles 40 and 50 degrees plot within the bounds predicted by the FE UB-LB. As observed in the Hoek-Brown stability chart results, the CF-UB curves do not remain within the UB and LB at larger magnitudes of ratio \( W/H \). This is probably because the CF-UB method assumes that the remaining strata, left behind after the roof has collapsed, will be stable. This is very unlikely as the thickness of the collapsing block \( (h_f) \) approaches the full cover depth \( (H) \). The CF-UB curve for the friction angle of 30 degrees does not plot within the bounds predicted by the FE UB-LB. The stability parameter predicted by the CF-UB is smaller in magnitude than the bracketed bounds. This has probably occurred because the equation assumed for the failure surface in the derivation of the CF-UB is not able to match the most critical shape when \( \phi = 30 \) degrees. Given that the Fraldi derivation was developed for the Hoek-Brown failure criterion, which corresponds to relatively high friction angles in the region of tensile and low confining stresses, then the equation selected to represent the failure surface would necessarily correspond to a smaller inclination of the failure surface from the horizontal. This equation is probably not able to match the larger inclination of the failure surfaces observed in low friction angle materials.
Figure 6.6 – Stability chart for linear Mohr-Coulomb criterion: (a) obtained from the CF-UB analysis, and (b) results from both the CF-UB and FE UB-LB.
Mohr-Coulomb with tension cut-off

Finite element upper and lower bound formulation

Figure 6.7 shows the results from the FE analyses when the tensile cut-off ($\sigma_t^*$) is included in the Mohr-Coulomb failure criterion for friction angles ($\phi$) of 20, 30 and 40 degrees. The results for the stability number $N$ have been plotted against the ratio of the imposed tension cut-off ($\sigma_t^*$) and the tensile capacity predicted by the linear form of the Mohr-Coulomb failure criterion ($\sigma_t$). For each of the values of the ratio $W/H$ the stability parameter $N$ remains relatively constant at large values of the ratio $\sigma_t^*/\sigma_t$. At lower magnitudes of ratio $\sigma_t^*/\sigma_t$, the individual curves corresponding to the different values of ratio $W/H$ combine to plot as a single curve. This curve asymptote to zero as the stability parameter $N$ tends to infinity.

The results presented in Figure 6.7 clearly show that irrespective of the magnitude of other strength parameters, inclusion of a tensile cutoff in the Mohr-Coulomb failure criterion will govern the stability of the cavity and the mechanism of failure of the overburden. In the most extreme case considered, where $\sigma_t^*/\sigma_t = 0.028$, failure in the overburden is located primarily in the area close to the cavity roof (Figure 6.8). The failure surface appears to be relatively flat in nature and separation under tension is probably the primary mode of failure.

The plots of plastic dissipation for $W/H = 3$ and $\phi = 30$ degrees, obtained from the FE limit analyses, provide information about the mechanism by which the overburden fails when the Mohr-Coulomb criterion with a tension cut-off is adopted (Figure 6.8). The failure surface for magnitudes of the ratio $\sigma_t^*/\sigma_t$ greater than 0.5 are all quite similar to the shape obtained when using the Mohr-Coulomb criterion with no tension cut-off. As mentioned previously, the linear form of the Mohr-Coulomb criterion for $W/H=3$ predicts that the failure surface will be relatively linear and inclined to the vertical at an angle approximately equal to the friction angle. Shearing is probably the primary mode of failure along this failure surface. When the magnitude of the ratio $\sigma_t^*/\sigma_t$ decreases below 0.5 the stability number $N$ increases in magnitude and the failure surface changes to be subcritical.
Figure 6.7 - Chart of stability factor $N$ obtained using the FE-UB (solid line) and FE-LB (dotted line) for a range of normalised tensile cut-offs for: (a) $\phi=20^\circ$, (b) $\phi=30^\circ$ and (c) $\phi=40^\circ$. 

$N = \frac{\sigma_c}{\gamma WF}$

$\sigma_t^*/\sigma_t$
Figure 6.8 - FE-UB predictions of the degree of plastic dissipation for a range of normalised tensile cut-offs.

Displacement finite element modelling (D-FEM)

Figure 6.9 presents the results from the displacement finite element analyses conducted in ABAQUS. The results are in good agreement with the predictions obtained using the FE UB-LB. The D-FEM results typically were closer to the LB than the UB.
Chapter 6 – Prediction of underground cavity roof collapse using the Mohr-Coulomb failure criterion

6.5. Discussion

The results obtained from the FE UB-LB formulation with the linear form of the Mohr-Coulomb failure criterion identified that the friction angle governs the shape of the failure surface. This observation has also been observed when analyzing the trapdoor problem (Davis, 1968; Smith, 1998). As a result, larger magnitudes of friction angles led to larger values of the critical ratio \( W/H \)\text{crit}. For the linear form of the Mohr-Coulomb failure criterion with friction angles between 20 and 50 degrees, the predicted critical ratio \( (W/H)_{\text{crit}} \) ranged from 0.5 to 2.5.

Field subsidence measurements have identified the critical cavity width in coal measure strata is given approximately by a ratio \( W/H \) of 1.0 to 1.6 in Australian coalfields (McNally et al., 1996; Mine Subsidence Engineering Consultants, 2007; Mills et al., 2009). This range of critical cavity widths is predicted by a friction angle of approximately 30 degrees in the linear-form of the Mohr-Coulomb criterion. A value of friction angle of approximately 30 degrees has been used in practical examples when representing the coal measure rock mass as a smeared homogeneous material (Coulthard et al., 2008; Seedsman, 2013).

As identified in the previous Chapter, the Hoek-Brown criterion overestimates the
critical cavity width. Typically, the predicted critical ratio \((W/H)_{\text{crit}}\) using the Hoek-Brown failure criterion ranged from 1.5 to 8.0 for \(m_i\) values of 5 to 30. This overestimation probably arose because of the large effective friction angles for tensile or low values of compressive stresses. Although there is no exact method to calculate equivalent Mohr-Coulomb parameters from a Hoek-Brown failure criterion, an approximation can be obtained by using the recommendation made by Hoek (2002). In which case, the equivalent parameters for use in the Mohr-Coulomb failure criterion correspond to friction angles in the order of 35 to 60 degrees for the complete range of GSI values. This discussion about equivalent Mohr-Coulomb parameters has been presented here to highlight that the Hoek-Brown failure criterion corresponds to effectively high friction angles in the range of tensile and very low confining stresses occurring in the strata above underground openings.

The results presented in Chapter 5 and 6 have shown that the predicted stability of an underground opening depends heavily on the failure criterion assumed to represent the surrounding strata, as might be expected. The magnitudes of the stability factors \(N\) for both forms of the Mohr-Coulomb failure criterion are all much lower than for the Hoek-Brown failure criterion. From a practical perspective this implies that the linear Mohr-Coulomb criterion predicts that the underground opening remains stable at much smaller magnitudes of \(\sigma_c\), for given magnitudes of \(\gamma\), \(W\) and \(F\), than predicted using the Hoek-Brown failure criterion. This prediction of apparently greater stability when using the Mohr-Coulomb failure criterion possibly arises because of the much larger tensile capacity associated with the linear form of the Mohr-Coulomb criterion relative to the Hoek-Brown criterion. Inclusion of a tension cut-off in the Mohr-Coulomb failure criterion increases the magnitude of the stability factor \(N\), which predicts a less stable configuration. Further investigation is required to identify which failure criterion predicts roof collapse of an underground excavation in coal measure strata more accurately relative to reality.

The results presented in this Chapter highlight the versatile and robust nature of the FE UB-LB formulation relative to the analytical method and the displacement finite element method. The dependence of the shape of the failure surface on the friction angle might otherwise have been overlooked if only a limit equilibrium method was used as it requires a failure surface to be assumed. The FE UB-LB formulation is flexible enough to find the most critical shape of the failure surface and the solution is
independent of the in situ stress. The FE UB-LB formulation is less computationally intensive and took considerably less time to obtain a solution than the D-FEM method.

6.6. Conclusions

This Chapter has presented the stability charts for rectangular cavities using two forms of the Mohr-Coulomb failure criterion. The FE limit analysis formulations bracketed the collapse load to within 10% for both forms of the Mohr-Coulomb criterion. The two other forms of analysis yielded results that agreed with the limit analysis predictions. The magnitudes of the stability factors $N$ for both forms of the Mohr-Coulomb failure criterion were all much lower than for the Hoek-Brown failure criterion.

For the Mohr-Coulomb failure criterion, the friction angle was shown to govern the shape of the failure surface, and subsequently the critical ratio $(W/H)_{\text{crit}}$. Larger magnitudes of friction angles led to larger values of the critical ratio $(W/H)_{\text{crit}}$. The predictions of the normalized critical cavity width obtained with a friction angle of approximately 30 degrees match field measurements best. The Hoek-Brown criterion overestimates the critical cavity width because in the range of tensile and very low confining stresses it corresponds to the effectively high friction angles. The results obtained by imposing a tension cutoff clearly show that the stability of the cavity and mechanism of failure of the overburden are controlled mainly by the tensile strength.

The FE limit analysis formulation proved to be a robust and efficient means of finding the critical failure surface at which roof collapse shall occur. This is because it does not constrain the solution to an assumed shape of the failure surface, and the solution is independent of the in situ stress conditions.
CHAPTER 7. PREDICTION OF SUBSIDENCE ABOVE A SINGLE-SEAM SUPERCritical LONGWALL PANEL USING FINITE ELEMENT MODELLING

7.1. Introduction

This Chapter assesses the effects of differing assumptions for the constitutive laws, used to represent the coal measure strata and caved goaf, on the accuracy of the predicted subsidence above a single longwall panel when compared with field measurements. The trends typically observed in field measurements of subsidence above single-seam mining, as discussed in Section 2.3.2, are used to infer the possible material behaviour of coal measure strata. In particular, various constitutive laws are assessed on the basis of whether they can predict (1) a realistic subsidence profile and (2) that the vertical stress along the longwall floor returns to the overburden stress in the centre of the longwall panel. In some cases, the predicted vertical stress along the longwall floor of the first seam is compared to the analytically derived vertical stress distribution presented in Section 3.2.3, which in conjunction with field observations is regarded as being close to the true distribution. The constitutive laws used to represent the deformation of the subsurface strata need to accurately predict the vertical stress distribution at the height of the first seam to load appropriately the interburden for accurate prediction of subsidence above multi-seam longwall panels.

A single supercritical longwall panel, as schematically shown in Figure 7.1, is considered. Plane strain conditions are assumed. The overburden above the longwall panel is considered as three mechanically different materials: a purely isotropic linear-elastic material, an elastoplastic material and a jointed material. A linear failure criterion is used for the elastoplastic and jointed material with $\varphi = 30$ degrees, as this was shown to provide the best agreement with field measurements in Chapter 5 and 6.

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**Figure 7.1 – Schematic diagram of problem analysed in this Chapter**
Chapter 7 – Prediction of subsidence above a single-seam supercritical longwall using finite element modelling

7.2. Background

In general, there is poor agreement of predictions of subsidence using numerical methods with measurements made in the field, irrespective of which numerical modelling method is used (Coulthard et al., 1988; Kay et al., 1991; Mohammad et al., 1998; Esterhuizen et al., 2010). Currently, the primary limitation in obtaining accurate predictions of subsidence using numerical modelling methods has been identified to be the lack of constitutive laws that realistically represent coal measure strata. More sophisticated constitutive laws are becoming increasingly popular amongst numerical modellers even though they are numerically expensive, generally take a long period of time to run, and there is no evidence that they lead to more accurate subsidence predictions. Numerical models that incorporate continuum mechanics can represent a bedded rock mass by determining properties of an equivalent material, whereby the mechanical properties of the intact rock, the bedding planes and the jointing are smeared and represented by a homogeneous material. Overall, there appear to be two schools of thinking on the degree of complexity of constitutive laws used for the smeared material: simple or very sophisticated.

The former method, of representing the rock mass as a smeared material with a simple mechanical behaviour, was initially the only form available to numerical modellers. An isotropic linear-elastic material, as the most basic form of constitutive law, has long been known to predict shallower subsidence profiles than those which are observed in the field. An overburden represented as an isotropic linear-elastic material overestimates the subsidence at the panel edges (Coulthard et al., 1988; Kay et al., 1991). Predictions based on transversely-isotropic elastic properties match the monitored subsidence curves significantly better (Coulthard et al., 1988; Su, 1991).

Representing the rock mass through sophisticated constitutive laws has been able to obtain the best agreement between numerical modelling predictions and field measurements (Coulthard et al., 1988; Lloyd et al., 1997). The sophisticated constitutive behaviours have consisted of a combination of numerically expensive relationships, e.g., elastoplastic, strain softening, ubiquitous joint elements, coupled rock failure and fluid flow systems (Lloyd et al., 1997; Gale, 2004; Esterhuizen et al., 2010; Vakili et al., 2010). The limitation of most reports and papers describing numerical models that used sophisticated constitutive laws is that it is often unclear
what the relative contributions of each of the components of the material response and their properties are to the overall mechanics of the subsurface strata deformations and magnitude of the predicted subsidence profile.

The predictions of subsidence using both simple and sophisticated constitutive laws are considered in order to identify to what extent numerically expensive constitutive laws contribute to the accuracy of prediction of subsidence. This Chapter only considers supercritical longwall panels and the subsidence observed in coalfields of New South Wales (NSW), Australia.

7.2.1. Numerical modelling of longwall panels

Numerical models, as very powerful tools to calculate the distribution of stress and strain in a given problem, have the potential to assist in accurately predicting subsidence. There have been reasons proposed as to why numerical models often inaccurately predict subsidence (Holla et al., 1990). The primary being that there is a poor understanding of the mechanical behaviour of coal measure strata material (Gadde et al., 2007). This stems from the scarce measurements obtained from the field, which was highlighted in Section 2.3. However, there are ample subsidence measurements, which effectively are a second order effect of the displacements that have occurred under the ground surface. A summary of the trends typically observed in field measurements of subsidence above single-seam mining was provided in Section 2.3.2. Field measurement of subsidence can be used to back-analyse and infer the possible material behaviour of coal measure strata.

The effect of representing the overburden as an isotropic elastic, transversely-anisotropic or elastic-perfectly plastic materials on the accuracy of subsidence predictions was studied more than a decade ago (Fitzpatrick et al., 1986; Coulthard et al., 1988; Kay et al., 1991; Su, 1991). The transversely isotropic material gave the best agreement when compared with field measurements. However, all of these studies considered a model where the extracted seam was left as a void and allowed for the overburden to sag the full extraction height. For this reason, most of the models would not have predicted a return to overburden stress along the longwall floor because the overburden would not have sagged enough for the roof to come in contact with the floor. In more recent times, significantly more numerically expensive constitutive laws (e.g., an elastic-perfectly plastic, an elastic-strain softening and a ubiquitous joint
material) are being used for coal measure strata when modelling longwall panels. These more constitutive laws shall be defined further below.

Even though there have been numerous subsidence studies conducted for a range of constitutive laws, there has been no single study conducted or aggregation of results from previous studies that provides a comprehensive assessment the effect of complex constitutive laws on the accuracy of predicted subsidence. In addition, most studies that predict subsidence using numerical models typically only report on the subsidence results calculated and the constitutive laws used. These studies rarely provide information on the stresses and strains predicted in the sub-surface strata by the numerical model. This makes it very difficult to assess if the model would have achieved a reasonable stress distribution within the sub-surface strata and along the longwall floor. As mentioned previously, the latter information will be important when predicting subsidence above multi-seam longwall panels. Both of these gaps in our knowledge shall be the focus in the results presented in this Chapter.

*Elastic-perfectly plastic model*

An elastoplastic material constitutes one which will behave elastically until it reaches yield, after which it will behave plastically. Yield is defined by a failure criterion, which is often the Mohr-Coulomb failure criterion. Strain that occurs while the material is still elastic is recoverable on unloading, while the plastic strain that occurs after the material has yielded is not recoverable.

For an elastic-perfectly plastic material, the yield criterion or yield surface does not change with plastic straining. At yield, a perfectly plastic material is assumed to flow at constant stress with no deformation limit. A schematic representation of the relationship of stress to strain for an elastic-perfectly plastic material is given in Figure 7.2(a). Studies that have predicted subsidence above longwall workings using an elastic-perfectly plastic overburden found that a transversely isotropic material gave more accurate results when compared to field observations (Su, 1991), or otherwise that strain-softening was required in order for the predictions to match field measurements of subsidence (Coulthard et al., 2008; Vakili et al., 2010).
Elastoplastic strain-softening model

For an elastoplastic strain softening material, the yield surface contracts with plastic straining (Yu, 2010). Therefore, in the elastic phase the material behaves in the same manner as an elastic-perfectly plastic material. It differs in the plastic phase, where plastic straining reduces the strength of the material (Figure 7.2(b)). The term softening is often used as the yielded material is no longer able to support the stress that initially caused it to yield and so the stress needs to be redistributed elsewhere. The reduction of strength can be applied to any of the strength parameters (i.e., $\phi$, $c$) and be expressed as a function of the plastic strain ($\varepsilon_p$).

Implementation of strain-softening into numerical models has been reported to give rise to issues of numerical instability and sensitivity to mesh size (Pietruszczak et al., 1981). If the rate of softening is greater than the stiffness of the elastic material a numerically stable solution cannot be found. The strain energy in this extremely brittle material cannot dissipate quickly enough to the surrounding strata. Also, a finite element mesh with a small mesh size can cause localization of the plastic flow. Both of these issues can cause no stable solution to be found by the numerical method.

Strain softening has been used in numerical models for rock mechanics problems to represent the brittle nature of the failure of rock (Pietruszczak et al., 1980). Once a rock has failed it is no longer able to support the same load that originally caused it to fail. The inclusion of a strain-softening overburden in numerical models has been found to yield the best agreement of predicted subsidence with field measurements (Mohammad et al., 1998; Coulthard et al., 2008; Esterhuizen et al., 2010; Vakili et al., 2010). However, most of these studies do not provide details on how the softening was
implemented, which makes it difficult if not impossible to validate the findings of these studies. In the model developed by Mohammad et al. (1998) the friction angle was reduced from 35 degrees to 21 degrees over a strain of 0.01, the dilation angle was reduced from 10 degrees to 5 degrees over a strain of 0.005, and the cohesion was reduced from 800kPa to 100kPa over a strain of 0.05. It is unclear what form of strain was used. In this Chapter, strain softening will be implemented by reducing only the cohesion to zero as a function of plastic deviatoric strain ($\varepsilon_{pd}$).

**Ubiquitous joint model**

Some numerical modelling software packages include an in-built material model to represent a material with a high density of parallel joint surfaces. A typical example of such a material is sedimentary rock. The constitutive model allows for the stiffness of the material normal to the joint plane to reduce to zero when the stress normal to the joint becomes tensile. The constitutive model also allows for sliding along the joint system according to either an associated or non-associated flow rule. The ubiquitous joint material has been used for the overburden in numerical models to predict subsidence (Coulthard et al., 1988; Coulthard et al., 2008; Esterhuizen et al., 2010).

It should be noted that the implementation of the ubiquitous joint material is often not the same in each commercially available software package. For example, the FLAC implementation assumes that the failure on the weak plane is defined by the Mohr-Coulomb failure criterion with a tension cut-off. The shear flow rule is non-associated and tension flow rule is associated. In ABAQUS the Drucker-Prager failure criterion is used, with the flow rule specified by the user.

### 7.3. Problem definition

The aim of this Chapter was to assess the effect of material properties used to represent the overburden and the caved goaf of a longwall panel on the accuracy of the predicted final vertical subsidence of the ground surface when compared with field measurements. The overburden material to the longwall panel is considered as three mechanically different materials: an isotropic linear-elastic material, an elastoplastic material and a jointed material.

The predicted vertical subsidence is assessed with regard to three parameters which have been found to be common to subsidence profiles measured in NSW coalfields.
Firstly, that the magnitude of maximum subsidence is able to predict approximately 60% of the extracted seam height ($T$) (Mine Subsidence Engineering Consultants, 2007; Mills et al., 2009). Secondly, the ratio of the subsidence above the edge of the longwall panel ($S_{\text{edge}}$) to the maximum subsidence above the centre of the longwall panel ($S_{\text{max}}$) is within the range of 5 to 15% (Holla, 1985; Holla, 1987; Coulthard et al., 1988; Holla, 1991). Thirdly, that the magnitude of the vertical stress exerted onto the longwall floor in the central region of the panel after extraction of the coal is equal to the original in situ vertical stress.

The numerical model consists of a cross-section parallel to the longwall face and assumes plane-strain conditions. The initial pre-mining geometry of the model has an overburden depth ($H$) of 150m, the width of the longwall panel ($W$) of 300m, and the height of extraction ($T$) of 3m (Figure 7.3(a)). Two different options are considered to represent the post mining strata: Cavity Model and Goaf Model. The Cavity Model assumes that the cavity generated by the extraction of the coal seam can be left as a void, as shown in Figure 7.3(b). Although leaving the void created by the extracted longwall panel as a cavity may not be realistic, this model provides a lot of detailed information on how the overburden behaves, which might otherwise be masked. The Goaf Model assumes that a strain-stiffening material can be used to represent the behaviour of the caved goaf. The caved goaf represents the roof of the longwall panel that has collapsed onto the longwall floor and bulked in volume so as to fill the void left by the extracted coal. The geometry of the Goaf Model is shown in Figure 7.3(c).

A bulking-controlled goaf (as described in Section 2.5.3) is considered here. The bulking factor is assumed to be equal to a magnitude of 1.2 for reasons outlined in Section 2.3.1, such that the height of the caved goaf is equal to 6 times the extracted height above the longwall floor (i.e., 18m). The strain-stiffening constitutive law proposed by Terzaghi is used to represent the behaviour of the caved goaf. Three degrees of stiffening were used for Terzaghi strain-stiffening caved material and they have been labelled as Soft, Average and Stiff. The magnitude of the parameters for the three degrees of stiffness is defined in Table 7.1. The parameters from the Stiff and the Soft caved goaf materials correspond to those determined from a numerical model (Morsy et al., 2002) and laboratory testing (Pappas et al., 1993), and were both considered since it is not possible to independently assess which of the results for parameters $a$ and $E_i$ is more appropriate.
Chapter 7 – Prediction of subsidence above a single-seam supercritical longwall using finite element modelling

7.3.1. Solution methods

Modelling of extracted seam using the finite element method

Initial numerical models of subsidence considered that the extracted seam could be represented as a void, with the overburden allowed to sag the full extracted seam height (Fitzpatrick et al., 1986; Coulthard et al., 1988; Su, 1991). Predicting subsidence using
this type of model will not allow the vertical stress along the longwall floor to return to the overburden stress, unless the overburden is very soft. However, the models where the extracted seam is represented as a void still provide vital information about how the constitutive laws used to represent the overburden behave. Modelling the extraction of the seam as a void effectively provides a prediction of the maximum possible subsidence for the specific properties specified to the overburden. When modelled as a void, the roof and floor of the cavity may come into contact. For modelling performed in this Thesis, self-contact was defined, and frictionless contact was assumed.

To achieve a subsidence prediction closer to reality, it is hypothesised that a caved goaf needs to be included into the model. What is known about the potential geometry of the caved goaf was discussed in Section 2.3.1, and the constitutive law for the caved goaf was discussed in Section 2.5.3. Implementation a strain-stiffening goaf material into a finite element numerical model was considered to be possible using two different implementation methods. The first implementation involves applying initially geostatic stresses to the problem and in a second step in which the elements that are defined by the properties of the coal are replaced with elements that are defined by properties of the strain-stiffening goaf. This is difficult in a finite element analysis because the instantaneous change in stiffness causes numerical instabilities. Furthermore, contact between the caved goaf elements and the surrounding strata leads to instabilities. The instantaneous loss of equilibrium can be avoided in several ways, one of which is to introduce equal but opposite traction forces on the nodes on the boundary of the void of the extracted seam. However, this implementation method would be computationally taxing and additional coding would need to be written for the ABAQUS analysis in the cases where sequential steps are required in the analysis.

The second implementation involves including the caved goaf in all the mined longwall panels in the initial setup. This second implementation means is unable to model the sequential aspect of the mining process. In the single step of the analysis the gravity forces are incrementally ramped up to the required magnitude. This second method would yield different results from the first method only if the materials used for the overburden and the interburden were strongly dependent on the stress path. It was confirmed that the two different implementations gave similar results in cases when the overburden and interburden were not defined as an elastoplastic material. This second implementation was used in the analyses presented in Chapters 7 to 9 because it was
less computationally taxing than the first implementation method.

Constitutive laws

For each form of constitutive law, the results from the Cavity Model are presented and, where appropriate, also for the Goaf Model. For both models, the strata below the longwall panel are assumed to be elastic with a Young’s modulus of 10GPa. The bottom of the finite element mesh is fixed and normal displacements only are fixed on left and right boundaries. Table 7.2 specifies all the parameters used in the reference case. For the reference case, the coal stiffness is assumed to be equal to that of the overburden (i.e., $E_o = E_c$). The initial stress state was assumed to have a ratio of horizontal to vertical stress of 1.5. This will be significant in non-linear analyses.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
<th>Values used in reference case</th>
<th>Values considered</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_o$</td>
<td>Young’s modulus of overburden</td>
<td>10GPa</td>
<td>5, 1GPa</td>
</tr>
<tr>
<td>$E_c$</td>
<td>Young’s modulus of coal</td>
<td>10GPa</td>
<td>2, 1, 0.5GPa</td>
</tr>
<tr>
<td>$c$</td>
<td>Cohesion of overburden</td>
<td>-</td>
<td>1500, 2000, 2500kPa</td>
</tr>
<tr>
<td>$G'/G_{iso}$</td>
<td>Norm. independent shear modulus</td>
<td>1.0</td>
<td>0.334, 0.167, 0.1</td>
</tr>
<tr>
<td>$D$</td>
<td>Spacing of bedding planes</td>
<td>No bedding</td>
<td>30, 15, 7.5m</td>
</tr>
</tbody>
</table>

The primary difference between the Goaf Model and the Cavity Model is the inclusion of the Terzaghi strain-stiffening caved material. The constitutive laws for the caved goaf were implemented in ABAQUS through a Fortran script, which is included in Appendix D. The script adopted Equation (2.11) to update the Young’s modulus of the material as a function of the total vertical strain. The vertical strain was used in this instance as the constitutive law was designed for 1D conditions. Further study should be conducted to assess the effect of using other forms of strain to govern the stiffness of the strain-stiffening material. The script did not allow the newly calculated Young’s modulus to exceed the original Young’s modulus of the overburden strata.

Isotropic linear-elastic overburden

A parametric study of the effects of overburden stiffness ($E_o$) and coal stiffness ($E_c$) was conducted using the magnitudes presented in Table 7.2. Both the Cavity Model and the
Goaf Model were considered. Results are presented for the subsidence, sub-surface strata displacement and stress redistribution.

**Elastoplastic overburden**

An elastic-perfectly plastic and an elastoplastic strain-softening material are considered. The Mohr-Coulomb criterion was used to define failure for reasons identified in Chapter 5 and 6. The results presented in Chapter 6 provide information about the failure mechanism and shape of the failure surface when the Mohr-Coulomb failure criterion is used. The failure surface that best predicted the ratio \((W/H)_{\text{crit}}\) that is typically recorded in the field occurred when \(\varphi = 30\) degrees, so this magnitudes was used here. The focus in this Section is to identify the effects of using the Mohr-Coulomb failure criterion for the overburden material on the predicted subsidence and the sub-surface geomechanics. The magnitude of cohesion was selected to be low enough to ensure the overburden would fail. The upper and lower bound of the stability number \(N\) for ratio \(W/H = 2\), \(W = 300\)m, \(\gamma = 25 kN/m^3\) and \(\varphi = 30\) degrees for the linear Mohr-Coulomb failure criterion is 1.2 and 1.25 (Figure 6.2). This corresponds to a magnitude of cohesion between 2598kPa and 2706kPa that would cause failure. For the elastic-perfectly plastic analysis only the magnitudes of \(c\) of 2000kPa and 1500kPa were considered. Although these magnitudes may seem larger than the typical measured strength of discontinuities or smaller than the typical measured strength of intact coal measure rocks, it is necessary to remember that this magnitude of cohesion is representative of the smeared coal measure strata, including all defects. Although this magnitude selected may not always reflect a realistic value for the smeared coal measure strata, the analyses here were conducted primarily to obtain an indication of the overall behaviour of an elastic-perfectly plastic material. Two magnitudes of stiffness were used (\(E_o = 1\)GPa and \(E_o = 10\)GPa).

The elastoplastic strain-softening overburden also used the Mohr-Coulomb failure criterion, with \(\varphi = 30\) degrees and the magnitudes of the cohesion were selected small enough to ensure the overburden would undergo yielding and subsequent softening. Three magnitude of \(c\) were considered, as detailed in Table 7.2. The rate of softening was defined using the plastic deviatoric strain \((\varepsilon_{pd,\text{max}})\), by which point the cohesion would have been reduced to zero. For this Chapter the ratio of \(c\) to \(\varepsilon_{pd,\text{max}}\) was kept constant at 10,000kPa. Stable numerical solutions could not be found for the elastoplastic strain-softening overburden in the Cavity Model. Therefore, only results
Bedded overburden

Bedding planes typically found in coal measure strata were represented by three different forms of constitutive laws: a transversely isotropic elastic material, an isotropic elastic material with smooth interfaces at the horizontal strata boundaries, and a ubiquitous joint material. The bedding planes were assumed to lie horizontally in the overburden material. The transversely isotropic elastic material was implemented by specifying a value of the independent shear modulus ($G'$). The magnitudes of the normalised independent shear modulus ($G'/G_{iso}$) considered here are presented in Table 7.2. The normalised independent shear modulus has been calculated as described in Section 2.5.1.

Incorporation of smooth interfaces into the isotropic elastic overburden would allow the overburden to shear and slide in the horizontal plane more easily than for an isotropic elastic overburden. The smooth interfaces are positioned at a spacing of $D$ through the whole depth of the overburden. Although in reality the properties of the bedding planes are not perfectly smooth, this Chapter considers the effect of the most conservative of conditions.

The ubiquitous joint material obeys pre-defined constitutive laws included in the ABAQUS software. This material model assumes that the material can fail by either slipping along a set of parallel surfaces or as a bulk material, as discussed earlier in this Chapter. The ubiquitous joints are defined by a joint friction angle $\varphi = 30$ degrees, joint cohesion $c = 0$ kPa, an associated flow rule and are orientated horizontally.

7.4. Results

The results are presented according to the three forms of constitutive laws used to represent the overburden, with results for the Cavity Model and Goaf Model presented separately. Table 7.3 and Table 7.4 (located on pages 193 and 194, respectively) provide a summary of the maximum subsidence above the centre of the longwall ($S_{max}$) and the ratio of the subsidence above the edge of the longwall panel to the maximum subsidence above the centre of the longwall panel ($S_{edge}/S_{max}$) for all types of overburden materials used in the Cavity Model and the Goaf Model, respectively.
7.4.1. Isotropic linear-elastic overburden

Cavity Model

Figure 7.4(a) shows the normalized predicted subsidence caused by the extraction of the single supercritical longwall panel in the Cavity Model (Figure 7.3b). The subsidence profile is symmetrical in shape about the vertical axis. The maximum subsidence for the elastic overburden when $E_o = 10\text{GPa}$ is approximately $9\%$ of the extracted seam height ($T$). The magnitude of ratio $S_{\text{edge}}/S_{\text{max}}$ is $47\%$. This magnitude of ratio $S_{\text{edge}}/S_{\text{max}}$ is much larger than what is typically recorded in the field. An isotropic linear-elastic overburden predicts a subsidence profile shallower and wider than what is normally observed in the coalfields of NSW. This result has been reported previously by many authors (Fitzpatrick et al., 1986; Coulthard et al., 1988).

Figure 7.4(b) shows the principal stress distribution after the extraction of the longwall panel. The initial stress state was such that the ratio of horizontal to vertical stress ($K$) was $1.5$. The principal stress plot shows that the overburden above the first extracted longwall behaves similar to a simply supported beam, whereby the roof above the centre of the longwall and ground surface above the edges of the longwall are in tension. The ground surface above the centre of the longwall panel and the strata directly above the rib-edges are in compression. There is a band of strata under compression that is shaped as an arch, which is sometimes referred to as bridging. Bridging actively redistributes all the vertical load of the unsupported overburden to the remaining coal pillars on either side of the longwall. This induces very high vertical stresses into the adjacent coal pillars. This vertical stress increase in the coal pillars is significantly greater than those calculated from analytical calculations or from stress measurements recorded in the field. Analytical calculations and field measurements indicate that the stress on the longwall floor in the centre of the longwall panel returns to the overburden stress.
As expected, smaller magnitudes of Young’s modulus for the overburden ($E_o$) provides predictions of a deeper subsidence profile (Figure 7.5(a)). For $E_o = 1$ GPa the maximum subsidence increases to 75% of $T$. The ratio $S_{edge}/S_{max}$ remains constant at 47% for all magnitudes of $E_o$ considered. Therefore, a softer elastic overburden increases the maximum predicted subsidence but it does not change the overall shape of the subsidence profile to be closer to what is observed in the field.
The subsidence profiles for the varying magnitudes of $E_o$ are normalized by the Young’s modulus and presented in Figure 7.5(b). This figure shows that where the roof of the longwall does not touch the floor the curves plot as one, i.e., for $E_o = 10$GPa and $E_o = 5$GPa. The overburden has sagged to its full elastic capacity. However, for the softest overburden considered (i.e., $E_o = 1$GPa) the roof and floor of the cavity displace enough to make contact in the centre of the longwall panel. The non-linear aspect to this analysis, where the roof and floor interact, limits the degree to which the overburden sags. It should be noted, that even though the normalized vertical subsidence is 75% of $T$ in the case of $E_o = 1$GPa, the roof and floor of the longwall come into contact because of compression of the coal seam from the abutment loads and some floor heave. When the overburden is soft enough to allow the roof and floor of the longwall to come into contact, vertical stresses are able to transmit from the overburden to the strata in the longwall floor. The final stress distribution in the deformed strata is shown in Figure 7.6. This plot shows that even though there is contact between the roof and the floor, the elastic overburden still bridges most of the load of the overburden onto the adjacent pillars.

Reducing the Young’s modulus of the coal seam ($E_c$) slightly increases the magnitude of the subsidence profile above the longwall panel (Figure 7.7). In these analyses the stiffness of the overburden strata remains at 10GPa. This increase in subsidence as a result of the softer coal seam is negligible relative to the subsidence observed in the field. As observed when varying the stiffness of the overburden strata, reducing the magnitude of $E_o$ does not change the shape of the subsidence profile significantly. The ratio $S_{edge}/S_{max}$ increases by 2% when the $E_o$ is reduced to 1GPa (Table 7.3).
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Figure 7.5 – Plots for elastic overburden material with varying Young’s modulus of the strata ($E_o$) of: (a) vertical subsidence normalised by extracted seam height, and (b) vertical subsidence normalised by the Young’s modulus of the overburden material and extracted seam height.

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**Figure 7.5** – Plots for elastic overburden material with varying Young’s modulus of the strata ($E_o$) of: (a) vertical subsidence normalised by extracted seam height, and (b) vertical subsidence normalised by the Young’s modulus of the overburden material and extracted seam height.
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Figure 7.6 – Plot of the principal stress rosettes plotted on the Cavity Model in the final deformed state, where $E_o=1\text{GPa}$.

Figure 7.7 – Plot of vertical subsidence by extracted seam height for elastic overburden material with varying Young’s modulus of the coal ($E_c$) normalised.

**Goaf Model**

In the Goaf Model, the parameters for the Stiff Goaf and Soft Goaf stiffness (Table 7.1) were used for the Terzaghi strain-stiffening caved goaf material and two magnitudes of $E_o$ were used for the overburden strata (i.e., 1GPa and 10GPa). The Terzaghi strain-stiffening material extended to a height of 6 times $T$ above the floor of the longwall panel (Figure 7.3(e)).
Figure 7.8 shows the subsidence profiles obtained from the four material combinations. The shape of the subsidence profile is relatively similar for both cases with the Stiff Goaf. The maximum subsidence is approximately 4% of $T$. The similar shape of the subsidence profile suggests that the magnitude of the subsidence profile is governed by the stiffness of the caved goaf material. The sub-surface vertical displacement in the caved goaf and overburden is presented in Figure 7.9(a) and (b). Both plots show that the Stiff Goaf compresses to a maximum of approximately 0.1m at the top of the goaf. The secant modulus rose to a maximum of 287MPa and 239MPa for an overburden stiffness of 1GPa and 10GPa, respectively. This maximum secant modulus was reached at the top in the centre of the caved goaf material.

For the cases with the Soft Goaf, the shape of the subsidence profile is different for the two magnitudes of $E_o$ (Figure 7.8). The maximum subsidence is approximately 8% and 50% of $T$ for the $E_o = 10$GPa and $E_o = 1$GPa, respectively. Both the subsidence profiles are significantly larger in magnitude than was predicted for the Stiff Goaf. Plots of the vertical displacements in the caved goaf and overburden show that the variation in magnitude of surface subsidence is governed by the stiffness of the overburden material (Figure 7.9(c) and (d)). A soft overburden with a relatively soft caved goaf material allows for the maximum subsidence to be achieved, while the sagging limit of a stiff overburden governs the compression of the caved goaf and the overall subsidence. The secant modulus rose to a maximum of 12MPa and 6MPa for an overburden stiffness of 1GPa and 10GPa respectively. The ratio $S_{edge}/S_{max}$ for all four goaf cases is presented in Table 7.4 and they all fall in the range of approximately 50-60%. This result continues the trend observed of all elastic overburden, that an isotropic elastic overburden overestimates the relative subsidence above the edge of the longwall panel relative to the maximum subsidence.

Figure 7.10(a) shows the vertical stress at the height of longwall floor for all four elastic cases of the Goaf Model. The vertical stress in the caved goaf returns to the original in situ stress only when $E_o = 1$GPa and the parameters of the Stiff Goaf were used in the analysis. The vertical stress in the centre of the panel was closer to the original in situ stress in the cases with the Stiff Goaf. The maximum vertical stress attained in the Soft Goaf and $E_o = 10$GPa is 610kPa which is 16% of the original overburden stress. This highlights the fact that the displacement of the stiffer overburden is governed by its sagging limit and it does not transfer a lot of vertical stress through to the caved goaf.
Wilson’s equation for the vertical stress on the longwall floor has been plotted in Figure 7.10(b) for three magnitudes of abutment angle ($\beta$). The plotted Wilson curves are for the case of yield in seam only, extraction height $M = 3m$, Rankine passive stress state constant $k = 3$, and unit weight of rock $\gamma = 25kN/m^3$. The combination of the Stiff Goaf and $E_o = 1GPa$ best matches Wilson’s equation for $\beta = 10^\circ$. The significantly higher maximum stress at the rib-edge predicted by the numerical model arises because the coal seam is elastic, while Wilson’s equations consider the finite strength of the coal and the subsequent yield of the coal seam.

![Graph showing normalised surface vertical displacement from the Goaf Model with varying Young’s modulus of the elastic overburden ($E_o$) and Goaf material.](image-url)

**Figure 7.8 – Results of normalised surface vertical displacement from the Goaf Model with varying Young’s modulus of the elastic overburden ($E_o$) and Goaf material.**
Figure 7.9 – Plots of vertical displacements in meters for the Goaf Model with: (a) Stiff goaf and $E_o=10$ GPa, (b) Stiff goaf and $E_o=1$ GPa, (c) Soft goaf and $E_o=10$ GPa and (d) Soft goaf and $E_o=1$ GPa.
Figure 7.10 – Results from the Goaf Model with varying Young’s modulus of the elastic overburden ($E_o$) and Terzaghi strain-stiffening material of: (a) normalised vertical stress at the height of the longwall floor and (b) normalised vertical stress at the height of the longwall floor together with Wilson’s vertical stress distribution for a range of abutment angles ($\beta$) in the yield in seam only case.
Figure 7.11 – Plots of principal stress rosettes on the deformed strata for: (a) Stiff goaf and $E_o=10\text{GPa}$, (b) Stiff goaf $E_o=1\text{GPa}$, (c) Soft goaf and $E_o=10\text{GPa}$, and (d) Soft goaf and $E_o=1\text{GPa}$. 
In those cases where the overburden is elastic, the stress transmitted onto the longwall floor appears to be governed by the relationship of relative stiffness of the overburden to the caved goaf material. This relationship is better appreciated by assessing the principal stress plots for all four cases of the Goaf Model (Figure 7.11). If the overburden is relatively stiff and the goaf is relatively soft, the overburden effectively bridges its entire vertical load to the surrounding strata (as shown in Figure 7.11(c)). If the overburden and goaf are either both relatively stiff or both relatively soft, the goaf attracts some vertical stress but it does not return to the original overburden stress level (Figure 7.11 (a) and (d)). If the goaf is relatively stiff and the overburden relatively soft, the vertical stress in the goaf returns to the original overburden stress along the longwall floor (Figure 7.11 (b)). These results are as expected of an elastic analysis where generally stiff elastic materials attract stress.

The results for an isotropic linear-elastic overburden from the Cavity Model show that, in general, a softer overburden will predict the largest maximum subsidence. However, the isotropic linear-elastic overburden is not capable of predicting the shape of a subsidence profile typically observed in the field. If the results from the four cases of the Goaf Model were assessed purely on predicted maximum subsidence, the Soft Goaf with $E_o = 1$ GPa would be the most appropriate solution, since it was closest to empirical observations (i.e., 60% of $T$). If the numerical model was assessed purely on the stress distribution on the longwall floor returning to the overburden stress then the case with the Stiff Goaf and $E_o=10$ GPa would be the best model. This is because the Stiff Goaf attracts vertical stress. The optimum solution, where both single-seam subsidence and the vertical stress along the longwall floor are predicted accurately, would require a numerical model to consist of a goaf and overburden with an appropriate balance of relative stiffness, which will be case dependent.

### 7.4.2. Elastoplastic overburden

**Cavity Model**

Figure 7.12(a) presents the predicted subsidence for the Cavity Model for two magnitudes of $E_o$ when the overburden material is defined as an elastic-perfectly plastic material. Failure in the overburden is defined by the Mohr-Coulomb failure criterion with $c = 2000$ kPa and $\varphi = 30$ degrees. The analyses were conducted for a ratio of horizontal in situ stress to vertical in situ stress ($K$) of 1.5. Unlike the elastic
overburden, the shape of the elastic-perfectly plastic subsidence profile is different for the two magnitudes of $E_o$. The normalized maximum vertical subsidence is 96% and 82% of $T$ for $E_o = 10\text{GPa}$ and $E_o = 1\text{GPa}$, respectively. The magnitude of ratio $S_{\text{edge}}/S_{\text{max}}$ is 5% and 45% for $E_o = 10\text{GPa}$ and $E_o = 1\text{GPa}$, respectively. In an elastic analysis this would be counter intuitive, but this is no longer the case when plasticity governs the deformations of the overburden. The significantly different shapes of the two subsidence profiles arise from the softer overburden sagging more prior to the onset of failure than the stiffer overburden. This can be better appreciated when comparing the subsidence for the elastic-perfectly plastic overburden to the isotropic linear-elastic overburden results (Figure 7.12(b)). The softer overburden underwent much less plastic straining before the longwall roof touched the longwall floor and further vertical displacements ceased, than for the stiffer overburden.

Distributions of the vertical displacement and maximum in-plane plastic strain for the elastic-perfectly plastic overburden are presented in Figure 7.13(a) and (b), respectively. The areas that have undergone plastic straining, by definition, define the areas that have yielded. It is evident to see that there is a mass downward movement of a trapezium-shaped block of the overburden. This failure mechanism has previously been described as Terzaghi’s trap door problem. The effect of the magnitude of the strength parameters of the Mohr-Coulomb failure criterion on the stability and shape of the failure surface have been presented in Chapter 6. The overall shape of the subsidence curves (Figure 7.12(a)) appear to be primarily governed by the elastic properties of the overburden outside the area of failure, and by the plastic flow rule along the failure surface for the trapezium-shaped block of overburden. For this reason, the subsidence bowl is still relatively wide for the case where $E_o = 1\text{GPa}$. 
Figure 7.12 – Results of the normalised surface subsidence from the Cavity Model and: (a) an elastic-perfectly plastic overburden with varying Young’s modulus, and (b) elastic-perfectly plastic (E-PP) and elastic (E) overburden.
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Figure 7.13 – Results from the Cavity Model for the elastic-perfectly plastic Mohr-Coulomb overburden of: (a) vertical displacement in metres, and (b) plastic strain.

The stress distribution at the longwall floor for both values of overburden stiffness considered is presented in Figure 7.14(a). Both these stress distributions do not return to overburden stress at the centre of the longwall panel and only vary slightly in the maximum vertical stress achieved. The principal stress plot for the elastic-perfectly plastic analysis (Figure 7.14(b)) shows that, even though the roof comes into contact with the floor of the longwall, the overburden bridges a lot of the vertical stress to the remaining coal pillars. This occurs because a yielded elastic-perfectly plastic material is still able to support stresses that caused it initially to fail. Even after yielding the overburden will continue to bridge the same vertical load to the surrounding strata that it was transmitting at the onset of failure. Therefore, the principal stress plot for an elastic-perfectly plastic medium is similar to the principal stress plot for an isotropic linear-elastic overburden (Figure 7.4(b)).

It should be noted that the solution for the elastic-perfectly plastic overburden was very sensitive to the magnitude of cohesion \( c \) and the mesh size. When a magnitude of cohesion was selected much smaller than 2000kPa, the solution became numerically unstable because a plastic surface developed from the rib-edges to the ground surface generating a free block within the overburden.
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Figure 7.14 – Results from the Cavity Model for the elastic-perfectly plastic Mohr-Coulomb overburden of: (a) normalised vertical stress at the height of the longwall floor, and (b) principal stress rosettes.

Goaf Model

Figure 7.15(a) and (b) show the subsidence profiles for the Goaf Model for the elastic-perfectly plastic overburden with $c = 2000\text{kPa}$ and $c = 1500\text{kPa}$, respectively. In both figures $E_0 = 10\text{GPa}$ and predictions for three forms of caved goaf stiffness are represented (Table 7.1). When $c$ is changed the shape of the subsidence profile remains similar for each form of caved goaf stiffness. The Stiff Goaf has supported the overburden such that the overburden has not yielded. The overburden above the Average Goaf and Soft Goaf have both yielded as described in the above Section on...
elastic-perfectly plastic overburden in the Cavity Model (Figure 7.13). Larger maximum subsidence was recorded for \( c = 1500\text{kPa} \) than for \( c = 2000\text{kPa} \) because onset of failure started at lower stresses for smaller magnitude of \( c \). The solution for smaller magnitudes of cohesion was again dependent on the mesh size. A solution for \( c = 1500\text{kPa} \) was not found when a very fine mesh was used. The results for the \( E_o = 1\text{GPa} \) and \( c = 2000\text{kPa} \) were also analysed and the results were similar to those obtained for the elastic overburden, as the overburden did not yield.

Figure 7.16 shows the vertical stress distribution on the longwall floor for \( c = 2000\text{kPa} \). The results for \( c = 1500\text{kPa} \) were almost identical to those for \( c = 2000\text{kPa} \). The trend, as observed for the elastic overburden, was that the Stiff Goaf gave rise to the largest magnitude of vertical stress within the caved goaf. However, the maximum vertical stress has not returned to the magnitude of the original overburden stress. This is not an unexpected result, as it was observed that the elastic-perfectly plastic overburden will continue to bridge the same vertical load to the surrounding strata that it was bridging at the onset of failure.

Figure 7.17 presents the subsidence profile for the Goaf Model with the elastoplastic strain-softening overburden for a range of cohesive strengths. The solution was noted to vary for other magnitudes of horizontal in situ stress to vertical in situ stress (\( K \)). Only the results for the Average Goaf are presented here as the Stiff Goaf did not allow for the overburden to yield and it was not possible to find a stable solution for the Soft Goaf. Smaller magnitudes of cohesive strength of the elastoplastic strain-softening overburden increase the predicted maximum subsidence. The strain-softening material also changes the general shape of the subsidence profile. The ratio \( S_{edge}/S_{max} \) reduces from 48% for an elastic overburden with an Average Goaf to 19% for the elastoplastic strain-softening overburden with \( c = 1500\text{kPa} \) and \( \varepsilon_{pd\_max} = 0.15 \) (Table 7.4).

The vertical stress at the elevation of the longwall floor in the elastoplastic strain-softening overburden is presented in Figure 7.18. Unlike an isotropic linear-elastic overburden or an elastic-perfectly plastic overburden, the elastoplastic strain softening material’s response to a reduction in overburden strength increases both the subsidence and the maximum stress induced into the caved goaf. This relationship arises because of the very essence of the strain-softening material, where the initial stresses that induced failure need to then redistribute as a result of a reduction in strength with plastic straining.
Figure 7.15 – Plots of surface vertical displacement normalised by extracted seam height from the Goaf Model for the elastic-perfectly plastic overburden and varying goaf stiffness for: (a) $c=2000\text{kPa}$ and (b) $c=1500\text{kPa}$. 

\[ \text{Stiff goaf,} \quad \text{Average goaf,} \quad \text{Soft goaf} \]
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Figure 7.16 – Plots of normalised vertical stress at the height of the longwall floor from the Goaf Model for the elastic-perfectly plastic overburden with $c=2000\text{kPa}$ and varying goaf stiffness.

Figure 7.17 – Plots of surface vertical displacement normalised by extracted seam height from the Goaf Model for the average goaf and elastoplastic strain-softening overburden with $E_o=10\text{GPa}$ and varying maximum cohesion.
7.4.3. Bedded overburden

Cavity Model

Figure 7.19 shows the subsidence profiles from the Cavity Model for a range of magnitudes of normalized independent shear modulus \( \frac{G'}{G_{iso}} \) and \( E_o = 10 \text{GPa} \). Greater anisotropy (i.e., smaller magnitudes of \( \frac{G'}{G_{iso}} \)) gives rise to deeper subsidence profiles, while the magnitudes of subsidence above the edge of the longwall panel remain relatively constant. The increasing maximum subsidence and relatively constant subsidence above the edge of the longwall leads to a reduction in ratio \( S_{edge}/S_{max} \) for increased anisotropy (Table 7.3). The transversely isotropic elastic overburden gives rise to a subsidence profile shape closer to what is typically recorded in the field. This finding has also been previously noted by other authors (Wardle et al., 1983; Coulthard et al., 1988; Kay et al., 1991).

Although using a transversely isotropic elastic material increases the surface displacement, it does not affect the predicted stress redistribution in the overburden. For all the magnitudes of \( \frac{G'}{G_{iso}} \) the stress distribution in the overburden is similar to what is predicted for an isotropic linear-elastic overburden (Figure 7.4(b)). Therefore,
the use of transverse-anisotropy as the overburden material may lead to prediction of
more accurate subsidence profiles for single-seam subsidence. However, its use would
not be suitable for multi-seam mining as vertical stress is unlikely to transfer through
the goaf and onto the interburden.

Figure 7.20 shows the subsidence profiles from the Cavity Model where smooth
interfaces are included in the elastic overburden, with \( E_o = 10 \text{GPa} \). The interfaces are
separated by a distance \( D \) starting at the top surface of the model. Including smooth
interfaces in the overburden increases the maximum subsidence and also changes the
shape of the subsidence profile. Smooth interfaces spaced at intervals of 15m or less
allows the cavity roof to touch the floor of the longwall panel and the subsidence to
reach 100% of \( T \). The subsidence at the panel edge (\( S_{\text{edge}}/S_{\text{max}} \)) reduces from 47% for
the elastic overburden down to 1% for the elastic overburden with smooth interfaces
spaced at 7.5m intervals (Table 7.3).

Figure 7.21 shows a plot of the principal stresses for the elastic overburden with smooth
interfaces. As expected, each layer of the overburden behaves as described for the case
of a homogeneous elastic overburden: tension is present in the bottom of each layer
above the centre of the longwall and in the top of each layer above the edges of the
longwall; compression is present in the top of each layer above the centre of the
longwall panel and in the bottom of each layer directly above the rib-edges. Therefore,
although adding interfaces explicitly in the overburden increases the maximum
subsidence predicted, the overburden still bridges a lot of the load of the overburden
into the surrounding strata. This is evident when reviewing the vertical stress on the
longwall floor for the elastic overburden with smooth interfaces (Figure 7.22).
Reducing the layer thickness increases the amount the overburden sags and allows for
the roof and floor of the longwall to come into contact. For the overburden with
interfaces spaced every 30m, because the roof and floor of the longwall do not come
into contact the vertical stress along the longwall cavity floor is zero. The roof and
floor of the cavity come into contact when 15m and 7.5m thick elastic layers are used
for the overburden. The area of roof and floor in contact is smaller for the 15m thick
layers than for the 7.5m layers, and consequently the stress on the longwall floor is
more concentrated. Therefore, the maximum vertical stress on the longwall floor is
2.6\( \sigma_{vi} \) and 1.8 \( \sigma_{vi} \) for the 15m and 7.5m thick layers, respectively.

The results from the elastic overburden with smooth interfaces have shown that the
layer thickness needs to be thin to achieve a desired flexibility in the overburden to be able to predict the desired magnitude of maximum subsidence above the longwall panel. In the Cavity model the predicted maximum subsidence corresponds to the full extraction height. This material type also predicts a significantly reduced magnitude of the ratio of subsidence above the rib-edges to the maximum subsidence ($S_{rib}/S_{max}$), which is the primary problem with the predictions made by using an isotropic elastic or transversely isotropic overburden. An overburden composed of thin layers also leads to less overburden load being transferred to the surrounding strata and more onto the longwall floor. The typical subsidence curves observed in the field could be predicted by a numerical model if closely spaced horizontal interfaces and a strain-stiffening goaf are used.

![Figure 7.19](image-url)

*Figure 7.19 – Plots of surface vertical displacement normalised by extracted seam height for the Cavity Model with an elastic overburden material and varying transverse-anisotropy ($G'/G_{iso}$).*
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Figure 7.20 – Plots of surface vertical displacement normalised by extracted seam height for the Cavity Model with an elastic overburden material and smooth interfaces separated by spacing $D$.

Figure 7.21 – Plot of principal stress rosettes on the deformed strata for the Cavity Model with an elastic overburden material and smooth interfaces separated by spacing 30m.
Figure 7.22 – Plots of normalised vertical stress at the height of the longwall floor from the Cavity Model with an elastic overburden material and smooth interfaces separated by spacing $D$.

**Goaf Model**

The bridging nature of the transversely isotropic overburden and elastic overburden with widely spaced interfaces means that their implementation into the Goaf Model would yield similar results to those obtained for an elastic overburden. Only the elastic overburden with interfaces spaced every 7.5m and the ubiquitous joint overburden were considered in the Goaf Model. It was not possible to find a stable solution for the ubiquitous joint material in the Cavity Model.

The results presented in Figure 7.23 are for the Goaf model with an elastic overburden with smooth interfaces spaced every 7.5m. Three degrees of stiffness are considered for the strain-stiffening material, where the strain-stiffening parameters are provided in Table 7.1. The stiffness of the strain-stiffening goaf material appears to govern the maximum subsidence that is predicted when the overburden is an elastic material with smooth interfaces spaced every 7.5m. The Soft Goaf predicts the maximum subsidence to reach almost the full extracted seam height. The Average Goaf and Stiff Goaf predict a maximum subsidence of 27% and 6% of $T$, respectively. The selection of appropriate magnitudes of strain stiffening goaf parameters would need to be further investigated, possibly through back calculation, to achieve a prediction of maximum subsidence.
equal to the magnitudes typically recorded in the field.

Figure 7.24 presents the vertical stress along the longwall floor for the three degrees of goaf stiffness and an elastic overburden with smooth interfaces spaced every 7.5m. All three degrees of goaf stiffness predict a return to overburden stress in the middle of the longwall panel. The stiff goaf predicts the widest width of the longwall floor to return to the overburden stress.

Figure 7.25 shows the subsidence profiles from the Goaf Model where overburden is defined by the ubiquitous joint material with $E_o = 10\text{GPa}$ and $E_o = 1\text{GPa}$. Numerical instability prevented a solution for the Soft Goaf to be found. All the overburdens yielded by slipping along the horizontal ubiquitous joints except for the case with a Stiff Goaf and $E_o = 1\text{GPa}$. The Stiff Goaf provided support for the overburden so that it did not deflect and yield. The effect of the overburden slipping along the ubiquitous joints on the subsidence profile is quite evident. There is a significant increase in the maximum subsidence.

There are several similarities in the results obtained from the ubiquitous joint material to those obtained from the elastic-perfectly plastic overburden. The ubiquitous joint material predicts a mass downward movement of a trapezium-shaped block of the overburden similar in shape to that which forms in the elastic-perfectly plastic overburden. However, the ubiquitous joint material does not bridge as much stress to the surrounding strata and allows it to transfer into the goaf. Figure 7.26 shows the vertical stress distribution on the longwall floor for all four cases considered of the ubiquitous joint material in the Goaf Model. The Stiff Goaf and $E_o = 1\text{GPa}$ predict a return to overburden stress, while the other three cases all predict a maximum vertical stress of at least 80% of overburden stress.
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Figure 7.23 – Plots of surface vertical displacement normalised by extracted seam height for the Goaf Model with an elastic overburden material and smooth interfaces spaced every 7.5m.

Figure 7.24 – Plots of normalised vertical stress at the height of the longwall floor from the Goaf Model with an elastic overburden material and smooth interfaces spaced every 7.5m.
Figure 7.25 – Plots of surface vertical displacement normalised by extracted seam height for the Goaf Model with a ubiquitous jointed overburden material.

Figure 7.26 – Plots of normalised vertical stress at the height of the longwall floor from the Goaf Model with a ubiquitous jointed material overburden.
Table 7.3 - Normalised maximum subsidence and edge subsidence from Cavity Model parametric study.

<table>
<thead>
<tr>
<th>Overburden</th>
<th>Variable</th>
<th>Magnitude</th>
<th>$S_{\text{max}}/T$</th>
<th>$S_{\text{edge}}/S_{\text{max}}$</th>
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<tbody>
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<td><strong>Elastic</strong></td>
<td>$E_o$</td>
<td>10GPa</td>
<td>9%</td>
<td>47%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5GPa</td>
<td>17%</td>
<td>47%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1GPa</td>
<td>75%</td>
<td>48%</td>
</tr>
<tr>
<td>$E_c$</td>
<td></td>
<td>2GPa</td>
<td>9%</td>
<td>48%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1GPa</td>
<td>10%</td>
<td>49%</td>
</tr>
<tr>
<td>$G'/G_{iso}$</td>
<td></td>
<td>0.334</td>
<td>15%</td>
<td>34%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.167</td>
<td>24%</td>
<td>26%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.1</td>
<td>35%</td>
<td>21%</td>
</tr>
<tr>
<td>$D$</td>
<td></td>
<td>30m</td>
<td>54%</td>
<td>20%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>15m</td>
<td>100%</td>
<td>11%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7.5m</td>
<td>100%</td>
<td>0.5%</td>
</tr>
<tr>
<td><strong>Elastic-Perfectly plastic</strong></td>
<td>$E_o$</td>
<td>10GPa</td>
<td>96%</td>
<td>5%</td>
</tr>
<tr>
<td>(c=2000kPa, $\phi=30^\circ$)</td>
<td></td>
<td>1GPa</td>
<td>82%</td>
<td>45%</td>
</tr>
</tbody>
</table>
### Table 7.4 – Normalised maximum subsidence and edge subsidence from Goaf Model parametric study.

<table>
<thead>
<tr>
<th>Overburden</th>
<th>Variable</th>
<th>Goaf</th>
<th>$S_{max}/T$</th>
<th>$S_{edge}/S_{max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic</td>
<td>$E_o=10\text{GPa}$</td>
<td>Stiff</td>
<td>4%</td>
<td>50%</td>
</tr>
<tr>
<td></td>
<td>$E_o=1\text{GPa}$</td>
<td>Stiff</td>
<td>4%</td>
<td>48%</td>
</tr>
<tr>
<td></td>
<td>$E_o=10\text{GPa}$</td>
<td>Soft</td>
<td>8%</td>
<td>61%</td>
</tr>
<tr>
<td></td>
<td>$E_o=1\text{GPa}$</td>
<td>Soft</td>
<td>32%</td>
<td>50%</td>
</tr>
<tr>
<td>Elastic-perfectly plastic</td>
<td>$c=2000\text{kPa}$</td>
<td>Stiff</td>
<td>4%</td>
<td>50%</td>
</tr>
<tr>
<td>$(E_o=10\text{GPa})$</td>
<td>$c=2000\text{kPa}$</td>
<td>Ave.</td>
<td>10%</td>
<td>34%</td>
</tr>
<tr>
<td></td>
<td>$c=2000\text{kPa}$</td>
<td>Soft</td>
<td>32%</td>
<td>12%</td>
</tr>
<tr>
<td></td>
<td>$c=1500\text{kPa}$</td>
<td>Stiff</td>
<td>4%</td>
<td>50%</td>
</tr>
<tr>
<td></td>
<td>$c=1500\text{kPa}$</td>
<td>Ave.</td>
<td>13%</td>
<td>23%</td>
</tr>
<tr>
<td></td>
<td>$c=1500\text{kPa}$</td>
<td>Soft</td>
<td>46%</td>
<td>7%</td>
</tr>
<tr>
<td>Elastic-plastic strain softening</td>
<td>$c=2500\text{kPa}$</td>
<td>Ave.</td>
<td>8%</td>
<td>42%</td>
</tr>
<tr>
<td>$(E_o=10\text{GPa})$</td>
<td>$c=2000\text{kPa}$</td>
<td>Ave.</td>
<td>11%</td>
<td>29%</td>
</tr>
<tr>
<td></td>
<td>$c=1500\text{kPa}$</td>
<td>Ave.</td>
<td>14%</td>
<td>19%</td>
</tr>
<tr>
<td>Ubiquitous joint material</td>
<td>$E_o=10\text{GPa}$</td>
<td>Stiff</td>
<td>5%</td>
<td>20%</td>
</tr>
<tr>
<td></td>
<td>$E_o=1\text{GPa}$</td>
<td>Stiff</td>
<td>5%</td>
<td>53%</td>
</tr>
<tr>
<td></td>
<td>$E_o=10\text{GPa}$</td>
<td>Ave.</td>
<td>20%</td>
<td>4%</td>
</tr>
<tr>
<td></td>
<td>$E_o=1\text{GPa}$</td>
<td>Ave.</td>
<td>21%</td>
<td>28%</td>
</tr>
</tbody>
</table>

### 7.5. Discussion

The material models used for the overburden in this Chapter have yielded a wide range of subsidence shapes and distributions of vertical stress on the longwall floor. Considering only the subsidence results, some general observations can be made about the advantages and limitations of certain material models. As has been previously documented, in general, isotropic elastic overburdens predict wider and shallower subsidence profiles than that which is recorded in the field (Fitzpatrick et al., 1986; Coulthard et al., 1988). Using an elastic strain-stiffening material to represent the caved goaf can control the sag of the overburden and ultimately the magnitude of the maximum subsidence. Both the elastic-perfectly plastic and the ubiquitous joint
material give rise to a sudden increase in magnitude of subsidence that occurs above the yielded zone. The transversely isotropic material and the elastic material with smooth interfaces show the best agreement with subsidence measurements made in the field. Therefore, if a numerical model need only to predict accurately the shape and magnitude of a single-seam subsidence profile, the findings from this Chapter would imply that a transversely isotropic elastic or an elastic material with interfaces would probably be the best possible materials to use for the overburden. Both of these material models are not very taxing numerically.

The Goaf Model successfully allowed for some of the load of the overburden to be transferred onto the longwall floor. In general, less stress was induced into the caved goaf material when there was a larger difference between the relative stiffness of the caved goaf material and the relative stiffness of the overburden material. In general, if the overburden was allowed to yield, (i.e., in the cases where the overburden was elastic-perfectly plastic, elastoplastic strain softening or a ubiquitous joint material) there was more stress induced into the goaf than for equivalent caved goaf stiffness and non-yielding overburden material. The exception was the isotropic elastic overburden with closely spaced smooth interfaces. If a numerical model needs only to predict accurately the magnitude of vertical stress on the longwall floor, the findings from this Chapter would imply that a relatively stiff elastic strain-stiffening caved goaf material with a relatively soft overburden would be the best possible material combination. However, the effect of the yielding nature of the coal seam, longwall roof and longwall floor would need to be further investigated.

If the primary goal of a numerical model is to accurately predict multi-seam subsidence, the material models used for the overburden would need to ensure an appropriate subsidence bowl shape is obtained as well as allowing for the load of the overburden to be transferred through the first seam goaf and onto the interburden. Given the findings presented in this Chapter, this could possibly be achieved by using a strain-stiffening caved goaf material in the first seam with a strain-softening overburden, a ubiquitous overburden or with an elastic overburden with closely spaced smooth interfaces.

7.6. Conclusions

The findings presented in this Chapter have shown that an elastic overburden with closely spaced smooth interfaces together with a strain-stiffening goaf has the best
potential to predict the subsidence profiles observed in the field and also predict the return of vertical stress in the goaf along the longwall floor to the original overburden stress. The two material models that included softening after the onset of yield (i.e., elastoplastic strain-softening and the ubiquitous joint material) would also be able to achieve a predicted subsidence close to the field measurements, however, these computations are significantly more numerically taxing and the solution can be plagued with numerical instability. The transversely isotropic material and the elastic-perfectly plastic material would possibly be able to predict the shape of the subsidence profile reasonably well. However, these materials were not able to predict the return of overburden stress along the longwall floor because the overburden load is bridged to the surrounding strata. An isotropic linear-elastic overburden is not able to match either the shape of the subsidence profile or the vertical stress distribution along the longwall floor.
CHAPTER 8. PREDICTION OF SUBSIDENCE ABOVE A MULTI-SEAM SUPERCRITICAL LONGWALL PANEL USING FINITE ELEMENT MODELLING

8.1. Introduction

In this Chapter the predicted shape and magnitude of the incremental subsidence profiles above several multi-seam longwall panel arrangements is compared to the trends of subsidence profiles recorded in the field. The problem investigated here considers the effect of the location of the longwall panel in the second seam relative to first-seam longwall panels, as schematically shown in Figure 8.1. It is assumed that the first seam has been extracted as a series of parallel supercritical longwall panels. The constitutive laws used to represent the coal measure strata are transversely isotropic elastic material, isotropic elastic material with equally spaced smooth interfaces, and a ubiquitous joint material. While the extracted longwall was represented as either a void or a region of strain-stiffening material (goaf) in Chapter 7, the analyses presented in this Chapter assume for simplicity that the longwall extracted from the second seam can be represented as a void. This assumption is discussed further at the end of the Chapter.

![Figure 8.1 – Schematic diagram of the problem analysed in this Chapter, where $S_{max}$ is the incremental subsidence caused by the extraction of a longwall panel in the second seam.](image)

8.2. Background

Initially, it was thought that subsidence above multi-seam supercritical longwall panel extractions could be determined by superimposing the subsidence profiles predicted when single-seam mining (Whittaker et al., 1989; Holt, 2001). However, the incremental subsidence recorded above multi-seam longwall panels typically has not matched the subsidence profiles recorded above single-seam longwall panels in either magnitude or shape. Incremental subsidence refers to the additional vertical
displacements at the ground surface that occur as a result of extraction of a single longwall panel. It is unclear why the incremental subsidence above a multi-seam supercritical longwall panel differs from one above a single-seam longwall panel. The limited availability of recorded subsidence profiles above multi-seam longwall mines means that comprehensive empirical databases are not yet available. Further, generally there is a low level of detail about the underground excavation geometry and the properties of the subsurface strata.

Recorded measurements of maximum incremental subsidence above multi-seam supercritical longwall are typically larger than those recorded above single-seam mining. The additional subsidence measured above a longwall extraction in the second seam is typically in the order of 80% of the extracted seam height of the second seam \((T_2)\) (Schumann, 1987; Dynl, 1991; Sheorey et al., 2000; te Kook et al., 2008). This increased maximum subsidence above multi-seam longwall panels has been recorded in many coalfields around the world (Dynl, 1991; Sheorey et al., 2000; Li et al., 2007; Mine Subsidence Engineering Consultants, 2007; te Kook et al., 2008). An exception is multi-seam mining in the coalfields of the United Kingdom, where typical magnitudes of maximum subsidence above the second seam longwall extraction are similar to those recorded above longwalls in the first seam. These findings of increased subsidence upon extraction of longwalls in the second seam have typically been from coalfields where the geology of overburden strata is relatively competent and subsidence in the first seam is usually in the order of 50-65% of the extracted seam thickness of the first seam \((T_1)\).

There appear to be discrepancies in the reported relative effects of multi-seam mining on the general shape of the subsidence profile. It has been noted that the shape of an incremental subsidence profile above a multi-seam longwall panel is wider and flatter than for a single-seam incremental profile (Mine Subsidence Engineering Consultants, 2007). This has been deduced from field data collected from Australian coalfields and applies when comparing longwalls with similar values of the ratio \(W/H\), where \(W\) is the width of the longwall panel and \(H\) is the overburden depth. The incremental subsidence profiles above second seam longwall extractions from two mines in China have been reported to be narrower and steeper than typically recorded for single-seam profiles (Ma et al., 1984). The concave and convex curvatures of the multi-seam subsidence profiles were described as being more pronounced. Although these different trends from
Australia and China could be attributed to different geological conditions in the
different countries, there is a lack of information on which to draw such a conclusion.
In addition, the technical note by Ma and Zhu (1984) did not specify the width or depth
of the longwalls extracted in each seam. It would be appropriate to compare only the
shape of normalized incremental subsidence profiles if the ratio $W/H$ were the same.
This seems unlikely for the described Chinese mines unless the width of the longwalls
in the second seam were wider than those in the first seam and the longwall in the
second seam was made wide enough to achieve the same ratio $W/H$ as for the longwalls
in the first seam. The effects being described by Ma and Zhu (1984) could possibly be
the relative changes to the subsidence profiles observed for supercritical longwall panels
and critical longwall panels.

Magnitudes of maximum subsidence recorded above extracted longwall panels in the
first and second seam have been compared and the results are summarized in Figure 8.1.

Table 8.1 – Summary of the geometry and maximum subsidence ($S_{max}$) for multi-seam mining cases when
longwall mining beneath previously extracted longwall panels (from Li et al., 2010).

<table>
<thead>
<tr>
<th>Location</th>
<th>Seam</th>
<th>Panel</th>
<th>$H$ (m)</th>
<th>$W/H$</th>
<th>IB (m)</th>
<th>$S_{max}$</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sigma Colliery</td>
<td>No. 3</td>
<td>LW4</td>
<td>133</td>
<td>1.59</td>
<td>13</td>
<td>40% of $T_1$</td>
<td>Parallel stacked arrangement</td>
</tr>
<tr>
<td></td>
<td>No. 2b</td>
<td>LW4A</td>
<td>148</td>
<td>1.26</td>
<td>13</td>
<td>96% of $T_2$</td>
<td></td>
</tr>
<tr>
<td>Newstan Colliery</td>
<td>Great Northern</td>
<td>LW6</td>
<td>60</td>
<td>2.58</td>
<td>15</td>
<td>60% of $T_1$</td>
<td>Parallel stacked arrangement</td>
</tr>
<tr>
<td></td>
<td>Fassifern</td>
<td>LW8</td>
<td>75</td>
<td>2.80</td>
<td>15</td>
<td>95% of $T_2$</td>
<td></td>
</tr>
<tr>
<td>Liddell Colliery</td>
<td>Upper Liddell</td>
<td>LW1</td>
<td>160</td>
<td>1.13</td>
<td>40</td>
<td>65% of $T_1$</td>
<td>Perpendicular arrangement</td>
</tr>
<tr>
<td></td>
<td>Lower Liddell</td>
<td>LW3</td>
<td>200</td>
<td>0.90</td>
<td></td>
<td>105% of $T_2$</td>
<td></td>
</tr>
<tr>
<td>North Wambo Mine</td>
<td>Whybrow</td>
<td>LW13</td>
<td>260</td>
<td>0.81</td>
<td>85</td>
<td>-</td>
<td>Diagonal arrangement</td>
</tr>
<tr>
<td>North Wambo Mine</td>
<td>Wambo</td>
<td>LW1</td>
<td>345</td>
<td>0.75</td>
<td></td>
<td>98% of $T_2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Whybrow</td>
<td>LW10b</td>
<td>95</td>
<td>2.21</td>
<td>65</td>
<td>-</td>
<td>Diagonal arrangement</td>
</tr>
<tr>
<td></td>
<td>Wambo</td>
<td>LW1</td>
<td>160</td>
<td>1.63</td>
<td></td>
<td>98% of $T_2$</td>
<td></td>
</tr>
</tbody>
</table>
8.2.1. Case studies of multi-seam longwall subsidence profiles

The subsidence profiles recorded above multi-seam longwall panels are influenced by the thickness and geology of the interburden and the relative orientation of the longwalls in each seam (Li et al., 2007; Mine Subsidence Engineering Consultants, 2007), in addition to the parameters that influence subsidence above single-seam mining. The simplest arrangement of longwalls in multiple seams is where the longwalls are parallel and the longwalls lie directly over each other. This arrangement is also referred to as a stacked arrangement. Sigma Colliery in South Africa used stacked longwall panels with an interburden depth of 15m (Schumann, 1987; Li et al., 2010). The longwalls in the second seam were marginally narrower than those in the first seam. The shape of the subsidence profiles above both seams is similar and the maximum subsidence occurred above the centre of the longwall panels in each seam.

The Newstan Colliery in NSW also used a stacked arrangement of longwalls in multiple seams with an interburden depth of 15m. The width of the longwall located in the lower seam was 1.4 times the width of the longwall located in the first seam. The recorded subsidence profile shows that the maximum subsidence above the first seam extraction ($S_{max1}$) did not occur in the same location as the maximum incremental subsidence above the second seam ($S_{max2}$) (Mine Subsidence Engineering Consultants, 2012). The maximum incremental subsidence above the first extracted seam ($S_{max1}$) occurred along the centerline of the first seam longwall panel. However, the maximum incremental subsidence above the second extracted seam ($S_{max2}$) occurred above the edges of the first seam longwall panel, where low values of subsidence were observed after extraction of the first seam. There is a reduction in the magnitude of the incremental subsidence recorded above the second seam panel where there is no overlying goaf in the first seam. This result suggests that some component of additional vertical movement is occurring in the subsurface strata where goaf material is present, while mining the second seam.

The longwall in the second seam at the Liddell Colliery was positioned approximately perpendicular to the orientation of the previously extracted longwall in the overlying seam. The interburden was 40m thick. The incremental subsidence after extracting the second seam showed that the location of the maximum incremental subsidence did not occur near the centerline of the first seam longwall panel, which is where the maximum
subsidence for the first seam occurred (Mine Subsidence Engineering Consultants, 2012). This result was not evident when assessing the total subsidence profiles (Li et al., 2010). The maximum subsidence for the second seam occurred above the edges of the first single-seam mined panel.

When longwall panels located in multiple seams are extracted at an orientation diagonal to each other, similar behaviours are observed as for the perpendicular and the stacked arrangements. However, the diagonal orientation of an undermining longwall leads to continually varying degrees of subsidence as the longwall panels undermine either the compacted goaf, uncompacted goaf or chain pillars present in the first seam. Both North Wambo Colliery and Blakefield South Mine subsidence profiles show that subsidence is in the order of 80% of $T_2$ when the centre of the second-seam longwall panel undermined the compacted goaf in the centre of the first-seam longwall panel (Mine Subsidence Engineering Consultants, 2012). The magnitude of the subsidence often is less when the second seam longwall panel undermines a chain pillar in the first seam. Conversely, the magnitude of the subsidence often is larger when a second seam longwall panel undermines the edges of the longwall goaf in the first seam, which presumably consists of uncompacted caved goaf material.

### 8.2.2. Mechanics of sub-surface strata deformations

The unique shape and magnitudes of subsidence profiles recorded above multi-seam longwall panels suggest the mechanics driving sub-surface strata deformations around multi-seam longwall panels are more complex than around single-seam longwall panels. Subsidence above single-seam mining has been attributed to sag subsidence and pillar compression (Mills et al., 2009), which was discussed in the previous Chapter. There have been several proposals of the geomechanics of strata that lead to the additional subsidence observed above multi-seam supercritical longwall panels (Ma et al., 1984; Mine Subsidence Engineering Consultants, 2007). These proposals can be divided broadly into two groups: mechanisms that are associated with the strata around the longwalls extracted in the second seam; or mechanisms that are associated with the deformed strata around the longwalls extracted in the first seam.

The former proposal would occur if the caved goaf in the second seam and the interburden deformed differently to the caving and deformations of the overburden upon extraction of the longwalls in the first seam. The caved goaf in the second seam may
not bulk as much as the first seam caved goaf if the interburden is not very thick (Ma et al., 1984). Other reasons could stem from the fact that the overburden has been fractured and it does not have a high bending resistance. This may reduce the amount of bed separation in the interburden. This might especially be the case if the interburden consisted of stiff and strong rocks. Another possibility is that the second seam caved goaf may compact more than the first seam caved goaf given that it is at a greater depth and the fractured overburden does not have a high bending resistance.

The mechanism associated with the deformed strata of the longwalls extracted in the first seam are primarily concerned with the idea that the goaf of the first seam remobilises and undergoes additional compaction (Bai et al., 1995; Gale, 2004; Mine Subsidence Engineering Consultants, 2007). This mechanism of remobilisation and additional compaction of the first seam caved goaf suggests that the additional vertical displacements arise through reconfiguration of the fragments of rock present in the caved goaf and deformed overburden. This idea has been likened to the behaviour of generally loose blocks of stone. The movements of loose non-cohesive blocks is more dependent on stochastic movement processes than on the properties of the individual blocks (Kratzsch, 1983; Mine Subsidence Engineering Consultants, 2007). Remobilisation and additional compaction of the first seam caved goaf is supported by field measurements that show that a larger percentage of the extracted height is recorded as subsidence above undermining than overmining (Li et al., 2007).

8.2.3. Numerical modelling of multi-seam longwall panels

To be able to accurately predict the subsidence above multi-seam longwall panels, the constitutive laws used in numerical models need to appropriately resemble the mechanical behaviour of coal measure strata. The strength and deformation characteristics of both the undisturbed state and the caved form of the strata need to be appropriately represented. Although many laboratory tests have been undertaken to characterise the constitutive laws of coal measure rocks, there is still no robust constitutive laws to use for these rock masses. However, field measurements such as the subsidence recorded above the multi-seam longwall can be used to back calculate the mechanical behaviour of the subsurface strata. As discussed in Chapter 2, there are many numerical methods available to assist in predicting subsidence, such as finite element modelling (FEM), the finite difference method (FDM), the boundary element
method (BEM), and the discrete element method (DEM). Each of these methods has its advantages and limitations in terms of how a numerical solution is sought, as well as the availability and accurate implementation of constitutive laws. This Chapter shall consider three constitutive laws and the FEM to try to predict the subsidence above multi-seam supercritical longwall panels.

Additional compaction of the first seam caved material upon extraction of the second seam would only be possible in a continuum method, such as the FEM, if there is additional stress applied to the caved goaf material. However, additional compaction is deemed to be unlikely if the magnitude of the vertical stress in the centre of the first seam longwall panel has already returned to the magnitude of the original overburden stress. The edges of the caved material near the rib-edges may undergo additional compaction if the constitutive laws defining the overburden strata allow the overburden to deform and apply load onto the uncompacted caved goaf. It is understood that the FEM would not be able to simulate well, if at all, the displacements that might occur due to rock fragments in the first seam reconfiguring into a more compact arrangement. Further investigation should be conducted to see if other numerical methods, such as the discrete element method (DEM), would be able to represent the reconfiguration of the first seam caved goaf as a result of extracting a second seam longwall, and if this would lead to the appropriate shape of the subsidence profile recorded above multi-seam longwall panels.

The failure mechanism involving shearing through the whole thickness of the interburden after the hydraulic shields have moved forward would typically occur if the interburden consists of strong rocks. The strong rocks would not allow for the immediate roof of the second seam longwall to undergo much caving or bulking. The FEM would be able to represent this type of failure mechanism. Given that there is little caving or bulking of the immediate roof of the second seam it would be reasonable to leave the extracted longwall panel in the second seam as a void in the analysis.

**Constitutive laws**

Numerical models of multi-seam mining have been conducted where the coal measure strata were represented as isotropic linear-elastic materials (te Kook et al., 2008). The results showed that the highest magnitude of subsidence from all the various incremental subsidence profiles for each seam was upon extraction of the first seam.
Subsequent longwall extractions in other seams predict less incremental subsidence than was predicted above the first seam. This occurs because the extraction of the first seam causes the strata above and below the first seam to be destressed. When extracting an underlying or overlying seam the surrounding strata do not have a lot of stress to redistribute and as such do not strain much.

Many approaches have been used to accommodate the mechanics of stratified rock when represented as a continuum (e.g., Salamon, 1968; Gerrard, 1982; Salganik et al., 1990). Some approaches have represented the layered rock mass as an ‘equivalent’ homogeneous orthorhombic elastic material. This equivalent material allows for the mechanics of the stratified rock to be represented implicitly, which can save time in constructing the numerical model as well as reducing numerical instabilities due to difficulties in convergence of the contact formulations. This Chapter shall represent the mechanics of stratified rock in the numerical models using both implicit and explicit methods. Two materials shall be used that represent stratified rock mechanics implicitly. The first is a transversely isotropic elastic material where an independent shear modulus ($G'$) is used in the formulation of the elastic material. The second is a ubiquitous joint material that is defined by a reduction of the stiffness of the material normal to any joint plane when the stress normal to the joint becomes tensile. The material that involves the stratifications being represented explicitly shall consist of layers of elastic strata with frictionless interfaces defining the contact between the layers.

The constitutive laws for the caved goaf should normally reflect strain-stiffening behaviour (Wardle et al., 1983; Smart et al., 1987; Trueman, 1990; Pappas et al., 1993). This Chapter shall use the Terzaghi strain-stiffening equation, which was presented in Section 2.5.3. Previous numerical modelling and laboratory studies have shown that the magnitudes of the parameter $E_i$ can vary from 5MPa to 30MPa and magnitudes of the parameter $a$ can vary from 15 to 350. Stress-strain curves for uniaxial compression are presented in Figure 8.2 for a relatively soft and a relatively stiff caved goaf, as used in Chapter 7. For the soft caved goaf $E_i = 5$MPa and $a = 15$ and for the stiff caved goaf $E_i = 30$MPa and $a = 350$. Yet the numerical modelling (Morsy et al., 2002) and laboratory study (Pappas et al., 1993) did not consider the upper limit of the stress-strain relationship prescribed by the proposed magnitudes of $E_i$ and $a$. 
Chapter 8 – Prediction of subsidence above a multi-seam supercritical longwall panel using finite element modelling

When selecting values for the parameters in the Terzaghi strain-stiffening material, the upper limit of the magnitude of strain can be associated (at least approximately) with the displacement required for the goaf material to return to a state of zero air voids. In principle, a state of zero air voids would occur when the particles in the bulked material rearrange and compact such that no voids are left in the material. Considering one-dimensional deformation, the vertical strain required for the caved goaf material to return to a state of zero air voids would correspond to a magnitude of $b-1$, where $b$ is the bulking factor and defined as the void ratio plus 1. Therefore the upper limit of the stress strain curve would correspond to a vertical stress given by

$$\sigma = \frac{E_i}{a} (e^{ac} - 1) = \frac{E_i}{a} (e^{a(b-1)} - 1)$$

(8.1)

Once the strain-stiffening goaf reaches a state of zero air voids, the behaviour of the reconstituted goaf material can be assumed to return back to its original elastic behaviour with Young’s modulus defined as $E_o$. On this basis, it can be assumed that the Young’s modulus of the strain-stiffening elastic material is equal to $E_o$ when $c = b-1$. Therefore, by differentiation of Equation (8.1) the tangent modulus of the strain-stiffening material for a given strain can be determined, as given by

$$\frac{d\sigma}{dc} = E_i e^{ac}$$

(8.2)

When the gradient of the stress strain curve of the strain stiffening material is equal to...
\( E_o \) and \( \varepsilon = b - 1 \), then the parameters for \( a \) and \( E_i \) become mutually dependent, as given in Equation (8.3)

\[
a = \frac{1}{b-1} \ln \left( \frac{E_o}{E_i} \right)
\]

(8.3)

### 8.3. Problem definition

The aim of this Chapter is to investigate the subsidence predicted for multi-seam longwall panels using the displacement finite element method. The constitutive laws used for the overburden and interburden strata correspond to a transversely-isotropic elastic material, an isotropic elastic material with equally spaced smooth horizontal interfaces, and a ubiquitous joint material. These constitutive laws have been identified to predict either (a) subsidence profiles above single-seam supercritical longwall panels similar to those recorded in the field or (b) returned the vertical stress in the caved goaf to the overburden stress level (see Chapter 7).

Two models with different longwall arrangements in multiple seams are considered here: a stacked and a staggered arrangement. The panels in one seam are parallel to those in the other seam. The initial geometry of both the stacked and staggered model arrangements is shown in Figure 8.3(a). This initial geometry consists of an overburden above the first seam \( (H) \) of 150m and an interburden between the first and second seam \( (B) \) of 40m. For both models the width of each longwall panel \( (W) \) is 300m and the height of extraction in the first seam \( (T_1) \) and the second seam \( (T_2) \) is 3m. The Young’s Modulus for all of the coal measure strata was kept constant at \( E = E_o = E_c = 10 \text{GPa} \). In the stacked arrangement, the longwall panel extracted in the first seam lies directly above the longwall panel extracted in the second seam (Figure 8.3(b)). In the staggered arrangement, two longwall panels are extracted in the first seam and one longwall panel extracted in the second seam. The centreline of the longwall in the second seam aligns with the centre of the chain pillar in the overlying seam (Figure 8.3(c)).

The models assume that the caving of the overburden is governed by a bulking-controlled goaf. Thus, the caved goaf created upon the extraction of a longwall panel in the first seam is represented as a strain-stiffening material. The bulking factor \( (b) \) of the caved goaf material was assumed to be equal to 1.2. \( E_i = 5 \text{MPa} \) is assumed for the strain-stiffening caved goaf and with \( b = 1.2 \) and \( E_o = 10 \text{GPa} \) gives \( a = 38 \), according to Equation (8.3). The stress strain curve for this material is shown in Figure 8.2.
The interburden has been assumed to comprise strong rocks. Thus, upon extraction of the longwall in the second seam, the mechanics of failure of the interburden was assumed to be shearing through all of the interburden strata with relatively little caving and bulking of the immediate roof of the second seam. For this reason, the cavity generated by the extraction of the longwall panel in the second seam was left as a void in the finite element models. Although there is probably some caved goaf that falls onto the longwall floor upon extraction of the second seam, it is assumed to be significantly smaller in height and volume than the goaf in the first seam. The difference between modelling the second seam as a void or as a caved goaf material of small height is thought to have a negligible effect on the predicted stress distribution and displacements in the surrounding strata.

![Figure 8.3](image-url)

Figure 8.3 – Schematic drawing of geometry and material properties of: (a) initial conditions of both stacked and staggered arrangement, (b) final conditions for the stacked arrangement, and (c) final conditions for the staggered arrangement.
8.3.1. Solution methods

For the two multi-seam longwall panel arrangements considered in this Chapter, three materials were used for the overburden strata: a transversely isotropic elastic material, an isotropic elastic material with equally spaced smooth interfaces, and a ubiquitous joint material. As a matter of convenience, the isotropic elastic material with smooth horizontal interfaces is referred to here as the bedded material. Although it has been proposed that the independent shear modulus can be represented by $G'/G_{iso} = 0.167$ for coal measure strata (see Section 2.5.1) (Seedsman, 2009; Seedsman, 2011), initial analyses using transversely isotropic elastic material with this value yielded subsidence results where the overburden had not sagged enough to match field measurements. Therefore, a magnitude of ratio $G'/G_{iso}$ of 0.05 was used for the transversely isotropic material, as this allowed the overburden to sag more and to have a similar relative stiffness as the bedded material. It seemed unreasonable to reduce the independent shear modulus to less than 5% of $G_{iso}$ because such a small magnitude would probably be physically unrealistic for coal measure strata. For the bedded material, the interfaces were defined as perfectly smooth such that $\phi = 0$ degrees and $c = 0$ kPa. The smooth horizontal interfaces were equally vertically spaced, every 5m. Detailed information about the implementation of these three materials was provided in Chapter 7.

The interburden has been modelled as a transversely isotropic material and as an isotropic elastic material with smooth interfaces. Although the ubiquitous joint material was also used to represent the interburden, the analysis was plagued with numerical instabilities due to lack of convergence of the contact algorithms and the material response.

Simulations were performed in two steps, in a manner similar to that described in Section 7.3.1. In the first step, geostatic stresses were imposed with the goaf material present in the first seam and the second seam fully intact. In the second step, elements corresponding to the second-seam longwall were removed using the built-in ABAQUS subroutine.

8.4. Results

The results of predicted subsidence are presented as incremental subsidence normalized by the thickness of the extracted seam $x$ ($T_x$). The extracted seam thickness for the first...
seam \((T_1)\) and the second seam \((T_2)\) were 3m. The incremental subsidence is defined as the additional subsidence caused by the extraction of the mentioned longwall panel. The stress and displacement results for extraction of the longwall panels in the first seam were approximately the same, irrespective of whether the longwall floor was defined as a transversely isotropic or a bedded material.

### 8.4.1. Stacked arrangement

Figure 8.4 shows the predicted incremental subsidence after the extraction of each longwall in the stacked arrangement, when a transversely isotropic elastic material was used to represent the overburden. The magnitude of the independent shear modulus was given by \(G'/G_{iso} = 0.05\). The maximum subsidence above the longwall panel in the first seam was 38% of \(T_1\). The curves in Figure 8.4 show that the predicted incremental subsidence after extraction of the longwall panel in the second seam is similar when the interburden was defined as a transversely isotropic material and a bedded material. The maximum incremental subsidence for both forms of interburden was 29% of \(T_2\), which is less than the subsidence observed above the longwall panel in the first seam.

![Figure 8.4](image.png)

*Figure 8.4 – Plot of incremental subsidence profiles from the stacked arrangement after extraction of the longwall in the first seam and second seam with a transversely isotropic overburden.*

Assessment of the vertical displacements in the subsurface strata indicates that the
transversely isotropic elastic material gives the overburden a limited sagging capacity, which hinders accurate prediction of the subsidence above the subsequent second seam extraction. Figure 8.5(a) shows a contour plot of the vertical displacements after extraction of the longwall panel in the first seam below the transversely isotropic overburden. Figure 8.5(b) and (c) show the contour plots of the incremental vertical displacement after extraction of the longwall panel in the second seam below the transversely isotropic interburden and the bedded interburden, respectively. For the case of the transversely isotropic overburden and interburden, the roof and floor of the longwall panel in the second seam do not come into contact. The stiffness of the transversely isotropic material prevents the overburden and interburden from sagging enough for this to occur. In the case of the transversely isotropic overburden and the bedded interburden, the roof and floor of the second seam longwall come into contact but the interburden separates from the overburden. This predicted mechanism of the overburden bridging over the span of the longwall would probably not occur in reality because the fractured overburden would not have the bridging capacity to do so. This bridging mechanism in the overburden is only possible in the model because the goaf and the overburden are represented as elastic continuum materials.

The results presented in the previous Chapter showed that a transversely isotropic elastic overburden predicts a subsidence profile above a supercritical longwall panel in the first seam close to what is typically recorded in the field. In a multi-seam mining application the transversely isotropic elastic material limits the sagging capacity of the overburden as well as creating an artificial bridging of the overburden over the interburden. Both of these features reduce the magnitude of the predicted incremental subsidence above a second seam longwall panel.
Figure 8.5 – Plots of incremental vertical displacement in metres from the stacked arrangement for the transversely isotropic elastic overburden after: (a) extraction of the longwall panel in the first seam, (b) extraction of the longwall panel in the second seam with a transversely isotropic interburden and (c) extraction of the longwall panel in the second seam with a bedded interburden.

Figure 8.6 shows the predicted incremental subsidence after the extraction of each longwall panel in the stacked arrangement when the bedded material was used to represent the overburden. The maximum subsidence above the longwall panel in the first seam was 47% of $T_1$. The predicted subsidence profile with the bedded overburden is narrower and deeper than for the transversely isotropic overburden, as was noted in the results of Chapter 7. The stiffness of the caved goaf material controls the magnitude of maximum subsidence above the first seam. The predicted incremental subsidence after the extraction of the second seam longwall panel is similar when the interburden
was defined as the transversely isotropic material and the bedded material, as shown in Figure 8.6. The maximum incremental subsidence for the transversely isotropic interburden was 101% of $T_2$ and the maximum incremental subsidence for the bedded interburden was 103% of $T_2$. The incremental subsidence profiles for the second seam are primarily contained within the width of the longwall panel. The incremental subsidence for the second seam is larger than the extracted thickness of the second seam because the strain-stiffening caved goaf has undergone additional compaction. This has occurred because the vertical stress in the caved goaf material of the first seam increases upon extraction of the second seam.

Contour plots of incremental vertical displacements in the subsurface strata of the model with a bedded overburden are shown in Figure 8.7. After extraction of the first seam the bedded overburden sags more than the transversely isotropic overburden, which allows the strain-stiffening goaf to become more compressed (Figure 8.7(a)). Similar contour plots of incremental vertical displacements after extraction of the second seam under a transversely isotropic elastic interburden (Figure 8.7(b)) and a bedded interburden (Figure 8.7(c)) show that the mechanisms of deformation are relatively similar for both types of material. The flexibility of the bedded overburden allows for any displacements experienced by the interburden to be reflected in the subsidence predicted at the ground surface.

![Figure 8.6](image)

*Figure 8.6 – Plot of incremental subsidence profiles from the stacked arrangement after extraction of the longwall in the first seam and the second seam with a bedded overburden.*
Figure 8.7 – Plots of total vertical displacement in metres from the stacked arrangement for the bedded overburden after: (a) extraction of the longwall panel in the first seam, (b) extraction of the longwall panel in the second seam with a transversely isotropic interburden and (c) extraction of the longwall panel in the second seam with a bedded interburden.

Figure 8.8 shows the predicted incremental subsidence after the extraction of each longwall panel in the stacked arrangement when the ubiquitous joint material was used for the overburden. The maximum subsidence above the longwall panel in the first seam was 44% of $T_1$. Figure 8.8 shows the predicted incremental subsidence profile curves after extraction of the second seam longwall panel when the interburden was defined as a transversely isotropic material and a bedded material. The maximum incremental subsidence for the transversely isotropic interburden was 84% of $T_2$ and
under the bedded interburden it was 91% of $T_2$. The ubiquitous joint overburden predicts the narrowest subsidence profile of all three material types considered in this Chapter, for both the first seam and the subsequent second seam extraction.

The subsidence profile above the extraction of the first seam longwall has a sudden step because the ubiquitous joint material predicts a mass downward movement of a trapezium-shaped block of the overburden, as shown in Figure 8.9(a). The contour plot of the total vertical displacement after the extraction of the second seam under a transversely isotropic elastic interburden (Figure 8.9(b)) and a bedded interburden (Figure 8.9(c)) show that the mechanisms of deformation are relatively similar to each other. The extraction of the second seam longwall allows the interburden to sag and allows for additional vertical displacements in the overburden, but only within the limits of the trapezium shaped-block. The sagging interburden and vertical displacements contained within the trapezium shaped block in the overburden give rise to the acute peak in the predicted incremental subsidence profile.

Figure 8.8 – Plot of incremental subsidence profiles from the stacked arrangement after extraction of the longwall in the first seam and second seam with a ubiquitous overburden.
Contour plots of vertical stress within the subsurface strata show that only the bedded overburden allows the stress in the caved goaf to return to the same magnitude as the initial overburden stress after the extraction of the longwall in the first seam (Figure 8.10(c)). The results presented in the previous Chapter indicate that the bedded strata bridged overburden load to the surrounding strata unless the layer thickness was made small enough to allow for the overburden to sag onto the longwall floor. As is the case
here, the layer thickness is small enough to allow for the overburden to sag and for the overburden to apply load onto the caved goaf material and compress the strain-stiffening caved goaf. The caved goaf is then able to support a vertical stress equal to the original overburden stress. The vertical stress in the caved goaf almost reaches the original overburden stress when the overburden consists of the transversely isotropic elastic material (Figure 8.10(a)) and the ubiquitous joint material (Figure 8.10(e)). The maximum vertical stress is achieved in the centre of the longwall panel when the overburden material is transversely isotropic elastic or bedded material. On the other hand, the ubiquitous joint material achieves the maximum vertical stress almost along the whole width of the longwall panel. Of the three overburden material types considered here, the vertical stress distribution predicted by the bedded overburden is closest to the analytically derived vertical stress distributions along a longwall floor (e.g., Wilson, 1983) discussed in detail in Chapter 3.

Varied results were obtained for the vertical stress in the subsurface strata after the extraction of the second seam (Figure 8.10(b), (d) and (e)). Only the results for the bedded interburden are presented here. Given that the bedded interburden separated from the transversely isotropic overburden, it is expected that there is a significant reduction of vertical stress in the caved goaf of the first seam and the interburden strata (Figure 8.10(b)). A bedded overburden and interburden give rise to similar vertical stress distribution in both the first seam and the second seam, whereby the original overburden stress is achieved in the centre of the longwall panels with a reduction to zero vertical stress at the rib-edges of the longwall panels (Figure 8.10(d)). The vertical stress distribution in the ubiquitous joint overburden and bedded interburden is complicated and for this reason is probably unreliable and requires further investigation and validation with field measurements (Figure 8.10(e)).
Chapter 8 – Prediction of subsidence above a multi-seam supercritical longwall panel using finite element modelling
8.4.2. Staggered arrangement

Figure 8.11 shows the predicted incremental subsidence after extraction of the longwall panels in each seam for the staggered arrangement with a transversely isotropic elastic overburden. The maximum subsidence above both first seam longwalls was 23% of $T_1$. This magnitude is less than the 38% of $T_1$ predicted for the single panel in the first seam of the stacked arrangement. This is because the strata over the chain pillar are hogging and, together with the elastic material used to represent the overburden, limit the amount the strata over the longwall can sag. The subsidence above the chain pillar is 9% of $T_2$ and forms as a result of the superposition of the sag subsidence over each individual longwall in the first seam. The maximum incremental subsidence above the second seam longwall for the transversely isotropic interburden was 67% of $T_2$ and for the bedded interburden was 78% of $T_2$. 

Figure 8.10 – Plots of vertical stress in kPa for the stacked arrangement with bedded interburden with: (a) transversely isotropic overburden after extraction of the first seam longwall (b) transversely isotropic overburden after extraction of the second seam longwall (c) bedded overburden after extraction of the first seam longwall (d) bedded overburden after extraction of the second seam longwall (e) ubiquitous joint overburden after extraction of the second seam longwall and (f) ubiquitous joint overburden after extraction of the second seam longwall.
Contour plots of the total vertical displacement, for the staggered arrangement with a transversely isotropic elastic overburden, show that the displacements above the two longwalls in the first seam are essentially mirror images of each other, as would be expected of an elastic material (Figure 8.12(a)). Upon extraction of the second seam longwall, similar mechanics are seen in the staggered arrangement as were noted for the stacked arrangement. In the case of the transversely isotropic elastic overburden and interburden, the roof and floor of the longwall do not come into contact (Figure 8.12(b)). The bedded interburden has detached from the transversely isotropic elastic overburden (Figure 8.12(c)). In both the staggered and the stacked arrangements, the predicted subsidence profiles are primarily governed by the sagging capacity of the transversely isotropic elastic material used to represent the overburden strata.

Figure 8.11 – Plot of incremental subsidence profiles from the staggered arrangement after extraction of longwalls in the first seam and second seam with a transversely isotropic overburden.
Chapter 8 – Prediction of subsidence above a multi-seam supercritical longwall panel using finite element modelling

Figure 8.12 – Plots of incremental vertical displacement in metres from the staggered arrangement for the transversely isotropic elastic overburden after: (a) extraction of the first seam longwalls, (b) extraction of the second seam longwall with a transversely isotropic interburden and (c) extraction of the second seam longwall with a bedded interburden.

Figure 8.13 shows the predicted incremental subsidence after extraction of the longwall panels in both seams from the staggered arrangement with a bedded overburden. The maximum subsidence for the first seam longwalls was 49% of $T_1$. The subsidence profile above the first seam longwalls appears to consist of the superposition of two incremental subsidence profiles predicted using a bedded overburden. Unlike the transversely isotropic elastic material, the superposition of the subsidence above each longwall panel does not cause the ground surface above the chain pillar to subside. This
is because the sagging of the bedded overburden strata is limited primarily to the width of each individual longwall panel. The maximum incremental subsidence induced by extracting the longwall in the second seam is 100% of $T_2$ for both the transversely isotropic and bedded interburdens.

Contour plots of the vertical displacement for the staggered arrangement with a bedded overburden (Figure 8.14) show very similar mechanics as was noted for the stacked arrangement. The displacement of the bedded overburden after extraction of the longwall panels in the first seam causes a sagging of the overlying strata (Figure 8.14(a)). Significant displacements only occur in the strata directly above the extracted longwall panel, and do not extend past the rib-edges. This response is also observed in the second seam with some minor lateral spreading of vertical displacements through the strain-stiffening goaf material in the first seam (Figure 8.14(b) and (c)).

![Contour plot of vertical displacement](image-url)
Figure 8.14 – Plots of incremental vertical displacement for the staggered arrangement with bedded overburden after: (a) extraction of the first seam longwalls, (b) extraction of the second seam longwall with a transversely isotropic interburden and (c) extraction of the second seam longwall with a bedded interburden.

Figure 8.15 shows the predicted incremental subsidence profile after extraction of the longwall panels in both seams from the staggered arrangement with a ubiquitous joint overburden. The maximum subsidence predicted above the first seam longwalls is 44% of $T_1$. The ground surface above the chain pillars has subsided by 3% of $T_1$. The maximum subsidence above the second seam longwalls is 90% of $T_2$ and 94% of $T_2$ for the transversely isotropic elastic and the bedded materials, respectively.

Contour plots of the vertical displacements for the staggered arrangement with a
ubiquitous joint overburden are shown in Figure 8.16. Trapezium-shaped blocks define the vertical displacements in the overburden above each of the longwall panels extracted in the first seam, when the overburden is defined by the ubiquitous joint material (Figure 8.16(a)). The location of the longwalls in the first seam does not change the vertical displacements predicted in the interburden and the overburden upon extraction of the second seam longwall (Figure 8.16(b) and (c)). Upon extraction of the second seam the interburden sags and a new trapezium shaped block forms in the overburden directly above the longwall in the second seam.

Figure 8.16 – Plot of incremental subsidence profiles from the staggered arrangement after extraction of the longwalls in the first seam and the second seam with a ubiquitous overburden.

Figure 8.15 – Plot of incremental subsidence profiles from the staggered arrangement after extraction of the longwalls in the first seam and the second seam with a ubiquitous overburden.
Figure 8.16 – Plots of incremental vertical displacement for the ubiquitous joint overburden after: (a) extraction of the first seam longwalls, (b) extraction of the second seam longwall with a transversely isotropic interburden and (c) extraction of the second seam longwall with a bedded interburden.

As a point of reference, adding the goaf into the second seam, for either the stacked or the staggered arrangement, reduced the magnitude of the incremental subsidence profile above the second seam extraction. The results for the staggered case are given in Figure 8.17. This is as expected given this response was also observed in the single-seam subsidence results presented in Chapter 7.
8.5. Discussion

The best correlation of the magnitude of maximum subsidence above the second seam to what is typically recorded in the field was achieved when the overburden was represented as the bedded material or the ubiquitous joint material and the interburden represented as a bedded material. In this instance, the magnitude of subsidence was approximately equal to the second seam extracted height. The bedded material would be the preferred material to use in further analyses because the ubiquitous joint material did not predict a realistic vertical stress distribution along the longwall floor and often encountered numerical instability problems during implementation.

The results from the stacked and the staggered arrangements showed that for a given material, used to represent the overburden or the interburden, similar mechanisms of strata deformation and stress distribution were observed. This led to similar shapes of subsidence profiles above the second seam longwall panels for the stacked and the staggered arrangements, when the overburden was represented as the bedded material or the ubiquitous joint material. For these material types the solution is independent of the location of the longwalls in the first seam. This result occurred because the interburden
and overburden displaced by an amount equivalent to the full extracted height of the second seam in both instances.

For the transversely isotropic elastic overburden, the subsidence profile above the second seam longwall panel was not similar for stacked and staggered arrangements. In general, the transversely isotropic elastic material used here was found to limit the magnitude of the vertical displacements achieved in the strata. Subsequently, the incremental subsidence above the second seam longwall panel in the stacked arrangement was limited and less than the incremental subsidence above the first seam longwall panel. As mentioned earlier, this is to be expected of an elastic material because the extraction of the first seam causes the strata above and below the first seam to be de-stressed. When extracting an underlying or overlying seam the surrounding strata no longer have significant stress to redistribute and as such do not strain much (te Kook et al., 2008). However, in the case of the staggered arrangement the maximum incremental subsidence above the second seam longwall is larger than for the first seam longwalls. This increase in subsidence occurs because the extraction of the longwall panels in the first seam causes some of the load of the overburden to be applied to the chain pillars. Upon extraction of the second seam longwall the additional load carried by the central chain pillar in the first seam is transmitted to the interburden. The load distribution in the subsurface strata of the staggered arrangement of longwalls can be simplified to effectively a point load being applied to the mid-span of a beam, as schematically shown in Figure 8.18. This load distribution gives rise to the additional deflection of the interburden and the subsequent increased incremental subsidence upon extraction of the second seam longwall panel.

The magnitudes of $E_i$ and $a$ used in this Chapter appeared to have governed the
subsidence predicted above the longwall panel in the first seam when the overburden was defined as the bedded material or the ubiquitous joint material. The maximum subsidence for these cases was less than the 60% of extracted seam thickness typically observed in the field. This can be modified in future studies by changing the magnitude of $E_i$ and $a$ in keeping with the derived equation given in Equation (8.3). It should be noted, that the stress achieved in the strain-stiffening material in the analyses was much less than is required to return the goaf to the original overburden stiffness ($E_o$). For the values of strain-stiffening parameters used in this Chapter, of $E_i = 5\, \text{MPa}$ and $a = 38$, the stress required to reach a state of zero air void is $263\, \text{MPa}$. The maximum stress achieved in the strain-stiffening caved goaf material was in the order of only $4\, \text{MPa}$.

8.6. Conclusions

This Chapter has investigated the subsidence profiles predicted using the finite element method and three forms of constitutive laws to represent the stratified coal measure strata. When the overburden and interburden were represented by a bedded material, the subsidence and vertical stress profiles that best matched typical field measurements were predicted. This was the case for both the stacked and staggered arrangement of longwall panels. When a transversely isotropic elastic material with $G'/G_{\text{iso}} = 0.05$ was used to represent the overburden or the interburden it limited the magnitude of the vertical displacements achieved in the strata. The main limitations of using the ubiquitous joint material for the overburden strata are that it is computationally taxing, it often leads to numerical instabilities and it does not predict a realistic vertical stress along the longwall floor.

An interesting aspect of the findings presented in this Chapter was that when the interburden was represented as a transversely isotropic material there was significantly more incremental subsidence above the second seam longwall panel for the staggered arrangement than for the stacked arrangement. This was because the chain pillar in the first seam was effectively behaving as a concentrated load at the midspan of the interburden. As expected of the continuum method, none of the constitutive laws showed indications of increased additional subsidence above the edges of the goaf in the first seam longwall panels due to additional compaction in the caved goaf material.
CHAPTER 9. CASE STUDY: SUBSIDENCE ABOVE SINGLE-SEAM AND MULTI-SEAM LONGWALL MINING IN THE HUNTER VALLEY

9.1. Introduction

The Hunter Valley coalfield has been identified to have the potential for several multi-seam mining projects (Holt, 2001; Howarth et al., 2009). In order for any mining approvals to be granted for such ventures, environmental impact statements need to be submitted with predictions of subsidence that will result from the proposed mining. Therefore, it is necessary to be able to accurately predict subsidence above both single-seam and multi-seam longwall panels in this and other coalfields.

This case study shall use the subsidence measured above extracted longwall panels in the first seam and the second seam at what is currently known as the Blakefield South Colliery, in the Hunter Valley. All the longwalls in each seam were supercritical in width. Longwalls 1 to 6 were extracted from the Lower Whybrow coal seam where the average overburden depth is approximately 150m. Longwalls BSLW1 and BSLW2 are being extracted from the underlying Blakefield coal seam, below an average interburden thickness of approximately 80m.

The numerical models developed in the previous two Chapters shall be used to attempt to match the subsidence profiles that were recorded at the case study location. The constitutive laws used to represent the coal measure strata for the prediction of subsidence above the first seam longwall panels are for (1) an isotropic elastic material, (2) a transversely isotropic elastic material, (3) an isotropic elastic material with equally spaced smooth interfaces, (4) a ubiquitous joint material and (5) an elastoplastic strain-softening material. For prediction of subsidence above the second seam longwall panel only the isotropic elastic material with equally spaced smooth interfaces was used to represent the coal measure strata. This Chapter shall provide insight into the advantages and limitations of using the displacement-finite element method (DFEM) to back analyse the subsidence above single-seam and multi-seam supercritical longwall panels for the Blakefield South Colliery.
9.2. Background information

The Blakefield South Underground Coal mine is part of the Bulga Complex, which is located approximately 5km north of the town Broke in the Upper Hunter Valley of New South Wales. Underground coal mining began at the site in 1994 by Oakbridge Pty Ltd at what was known then as the South Bulga Colliery. Thirteen longwall panels were extracted from the Lower Whybrow coal seam (Flowers et al., 2012) totalling approximately 30 million tonnes of removed coal. Current mining operations are being conducted by GlencoreXstrata plc. The longwalls that are being extracted from the Blakefield coal seam undermine the previously extracted longwall workings in the Lower Whybrow coal seam. Long term plans for the site include mining the Glen Munro and Woodlands Hill coal seams, which both underlie the Blakefield seam (Gale, 2004).

9.2.1. Geology

The Bulga Complex is part of the Hunter coalfields, which lie west of the Newcastle coalfields and east of the Western coalfields. The Hunter coalfields, together with the Newcastle coalfield, Western coalfields and Southern coalfields are part of the Sydney Basin, as schematically shown in Figure 9.1. The Sydney Basin contains sediments from the early Permian to Triassic. Quaternary alluvium overlies the earlier units in erosional valleys and along coastal plains.

The stratigraphy of the Hunter coalfields is presented in Table 9.1. The Lower Whybrow and Blakefield seams lie within the Jerry Plains Subgroup of the Wittingham Coal Measures. A more detailed table of the stratigraphy of the Wittingham Coal Measures is presented in Table 9.2. The Wittingham Coal measures are mainly comprised of frequently bedded sandstones and siltstones. There are also occasional isolated thin beds of conglomerate and tuff, which are generally less than 10 metres in thickness.

Within the case study mine site, the surface topography is generally flat to undulating. Both the Whybrow and Blakefield seam generally dip from the north down to the south. The depth to the Whybrow seam varies from 40m at the northern end of the site to 350m at the southern end of the site. The overburden consists of moderate strength strata near the surface (UCS = 30-40MPa) and at greater depth consists of interbedded
sandstones with an increased strength (UCS = 50-80MPa). The thickness of the interburden varies between 70 and 100m. The interburden strata comprise interbedded weak to moderate strength units (UCS ~20-50MPa).

Figure 9.1 –Schematic map of the coalfields within the Sydney Basin © State of New South Wales through Department of Trade and Investment, Regional Infrastructure and Services.
### Table 9.1 – Stratigraphy of the Hunter Coalfield (Stevenson et al., 1998).

<table>
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<th>Period</th>
<th>Stratigraphy</th>
<th>Lithology*</th>
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<td>Silt, sand, gravel.</td>
<td></td>
</tr>
<tr>
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<td>Basalt</td>
<td></td>
</tr>
<tr>
<td>Jurassic</td>
<td>Basalt</td>
<td></td>
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<tr>
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<td>Hawkesbury Sandstone</td>
<td>Massive quartz sst and minor sltst.</td>
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<td>Terrigal Formation</td>
<td></td>
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<tr>
<td></td>
<td>Patonga Claystone</td>
<td>Sandstone, interbedded sandstone and siltstone, claystone.</td>
</tr>
<tr>
<td></td>
<td>Tuggerah Formation</td>
<td>Conglomerate and sandstone.</td>
</tr>
<tr>
<td></td>
<td>Widden Brook Congl.</td>
<td></td>
</tr>
<tr>
<td>Permian</td>
<td>Wollombi Coal Measures</td>
<td>Coal, claystone, siltstone, shale sandstone, conglomerate, and tuffaceous sediments.</td>
</tr>
<tr>
<td></td>
<td>Watts Sandstone</td>
<td>Medium to course sandstone</td>
</tr>
<tr>
<td></td>
<td>Denman Formation</td>
<td>Sandstone, siltstone, laminitie</td>
</tr>
<tr>
<td></td>
<td>Jerrys Plain Subgroup</td>
<td>Coal, clst, tuff, sltst</td>
</tr>
<tr>
<td></td>
<td>Archerfield Sandstone</td>
<td>Well sorted quartz-lithic sandstone</td>
</tr>
<tr>
<td></td>
<td>Vane Subgroup</td>
<td>Coal, sltst, lithic sst, shale, cgl.</td>
</tr>
<tr>
<td></td>
<td>Saltwater Ck Formation</td>
<td>Conglomerate, sandstone, siltstone.</td>
</tr>
<tr>
<td>Maitland</td>
<td>Mulbring Siltstone</td>
<td>Sltst, clst, minor fine grained sst</td>
</tr>
<tr>
<td>Group</td>
<td>Muree Sandstone</td>
<td>Fine to very coarse grained sst, cgl.</td>
</tr>
<tr>
<td></td>
<td>Branxton Formation</td>
<td>Conglomerate, sandstone, siltstone.</td>
</tr>
<tr>
<td>Greta Coal</td>
<td>Rowan Fm</td>
<td>Coal, siltstone, sandstone</td>
</tr>
<tr>
<td>Measures</td>
<td>Skeletar Fm</td>
<td>Pellet claystone, siltstone, chert</td>
</tr>
<tr>
<td>Dalwood</td>
<td>Gyarran Volcanics</td>
<td>Felsic to mafic volcanics and pyroclastics</td>
</tr>
<tr>
<td>Formation</td>
<td>Farley Formation</td>
<td></td>
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<tr>
<td></td>
<td>Rutherford Formation</td>
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<tr>
<td></td>
<td>Allandale Formation</td>
<td></td>
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<tr>
<td></td>
<td>Lochinvar Formation</td>
<td></td>
</tr>
<tr>
<td>Carboniferous</td>
<td>Tuff and ignimbrite interbedded with cgl, sst and shale.</td>
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</tbody>
</table>
Table 9.2 – Stratigraphy of the Wittingham Coal Measures (Stevenson et al., 1998).

<table>
<thead>
<tr>
<th>Wollombi Coal Measures</th>
<th>Denman Formation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mountain Leonard Formation</td>
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<tr>
<td></td>
<td>Malabar Formation</td>
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<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Mountain Ogilvie Formation</td>
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<td></td>
<td>Milbrodale Formation</td>
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<td></td>
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<tr>
<td></td>
<td>Mount Thorley Formation</td>
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<tr>
<td></td>
<td>Fairford Formation</td>
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<tr>
<td></td>
<td>Burnamwood Formation</td>
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<td></td>
<td></td>
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<tr>
<td></td>
<td>Vane Subgroup</td>
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<tr>
<td></td>
<td>Bulga Formation</td>
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<td></td>
<td>Saltwater Creek Formation</td>
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</tbody>
</table>

More detailed information about the geology of the site can be found in reports by MSEC (Mine Subsidence Engineering Consultants, 2008; Mine Subsidence Engineering Consultants, 2011) and SCT (Strata Control Technology, 2008), which can be sourced from the GlencoreXStrata website. The MSEC report (Mine Subsidence Engineering Consultants, 2011) includes contour plots of depth of cover to the Whybrow seam, Whybrow seam thickness, depth of cover to the Blakefield seam, Blakefield seam thickness, and interburden thickness.
9.2.2. Geometry of workings

All the longwalls in the Lower Whybrow seam are orientated at a bearing of approximately 014° off north. The layout of these longwalls is presented in Figure 9.2 with dimensions of each longwall presented in Table 9.3. Extraction began with LW1 and proceeded to LW6, after which extraction continued with LW_E1 through to LW_E7. The width of LW1 to 4 was 200m and the width of LW5 and LW6 was 260m. The ratio $W/H$ for all of these longwalls (given in Table 9.4) suggests that the recorded subsidence profiles should be characteristic of supercritical longwall panels. The lower Whybrow seam varied in thickness from 2.0 to 2.7m within the extent of the extracted longwalls (Mine Subsidence Engineering Consultants, 2008). The widths of the chain pillars that separated the longwall panels were 25-27m.

Table 9.3 – Geometry of the extracted longwalls in the Lower Whybrow seam.

<table>
<thead>
<tr>
<th>Longwall</th>
<th>Overall void length (m)</th>
<th>Overall void width incl. headings (m)</th>
<th>Solid chain pillar width (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Why LW1</td>
<td>3205</td>
<td>210</td>
<td>-</td>
</tr>
<tr>
<td>Why LW2</td>
<td>2930</td>
<td>210</td>
<td>25</td>
</tr>
<tr>
<td>Why LW3</td>
<td>2915</td>
<td>210</td>
<td>25</td>
</tr>
<tr>
<td>Why LW4</td>
<td>2785</td>
<td>210</td>
<td>25</td>
</tr>
<tr>
<td>Why LW5</td>
<td>2705</td>
<td>260</td>
<td>25</td>
</tr>
<tr>
<td>Why LW6</td>
<td>2035</td>
<td>260</td>
<td>25</td>
</tr>
</tbody>
</table>

Table 9.4 – Geometry of extracted longwalls in the Lower Whybrow seam.

<table>
<thead>
<tr>
<th>Longwall</th>
<th>Extracted seam height(^{\wedge}) (m)</th>
<th>Average overburden depth(#) (m)</th>
<th>Width of longwall (m)</th>
<th>$W/H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Why LW1</td>
<td>2.5</td>
<td>155</td>
<td>210</td>
<td>1.35</td>
</tr>
<tr>
<td>Why LW2</td>
<td>2.6</td>
<td>150</td>
<td>210</td>
<td>1.4</td>
</tr>
<tr>
<td>Why LW3</td>
<td>2.6</td>
<td>150</td>
<td>210</td>
<td>1.4</td>
</tr>
<tr>
<td>Why LW4</td>
<td>2.6</td>
<td>145</td>
<td>210</td>
<td>1.45</td>
</tr>
<tr>
<td>Why LW5</td>
<td>2.5</td>
<td>145</td>
<td>260</td>
<td>1.79</td>
</tr>
<tr>
<td>Why LW6</td>
<td>2.5</td>
<td>140</td>
<td>260</td>
<td>1.86</td>
</tr>
</tbody>
</table>

\(\#\) Determined from contour plot MSEC334-09, \(^{\wedge}\) Determined from contour plot MSEC334-08

The longwalls in the Blakefield seam are orientated at a bearing of approximately 045° off north. The layout of these longwalls is presented in Figure 9.2. All the longwalls in the Blakefield seam were given the prefix BSLW to avoid confusion with the longwalls.
in the overlying seam. The width of extraction was 330m and 410m for BSLW1 and BSLW2 (Mine Subsidence Engineering Consultants, 2013), respectively, and the extracted seam height for both longwall panels was 2.8m. The width of the chain pillar separating the two longwall panels was 60m.

Figure 9.2 – Map of the layout of longwalls in the Lower Whybrow seam (prefix LW) and Blakefield seam (prefix BSLW). The map also shows the survey lines used to monitor the subsidence when extracting longwalls in the Lower Whybrow seam (denoted in pink) and longwall BSLW1 (denoted in green) (MSEC subsidence monitoring report for 2010).
9.3. Single-seam mining

9.3.1. Field measurements of subsidence

The survey lines that were used to record subsidence when mining the Lower Whybrow seam are shown in Figure 9.2 in a pink colour. Only the result for the survey line denoted as XLA shall be considered here. The average extracted seam height along this survey line \(T_1\) was 2.5m for LW1 and 2.6m for LW2, LW3 and LW4. These values were determined from the contour plots of the Whybrow seam thickness (Mine Subsidence Engineering Consultants, 2011) as accurate details on the extracted seam thickness were not available.

The total subsidence recorded along survey line XLA after the sequential mining of LW1 through to LW5 in the Lower Whybrow seam is presented in Figure 9.3. The crosses mark the locations of the survey pegs where the measurements were taken. The start of the survey line is in the east and finishes in the west. The survey line XLA appears to have been positioned at an orientation perpendicular to the length of the longwalls in the Lower Whybrow seam. Thus, the measurements along the survey line XLA present a cross-section of the subsidence bowl above each longwall panel. The maximum vertical subsidence above each of the longwall panels varies between 1.09m and 1.27m. This corresponds to approximately 42% to 49% of the extracted seam height of the first seam \(T_1\). This percentage subsidence of the extracted seam height is less than is typical of longwall panels in the coalfields in the Sydney Basin (Holla, 1985; Holla, 1987; Holla, 1991; Mine Subsidence Engineering Consultants, 2007). The subsidence results also clearly show that the extraction of longwall LW2 caused additional subsidence above the chain pillar between LW1 and LW2, as well as addition subsidence above longwall LW1. It has been suggested that this response in surface subsidence has been caused by strata compression (Mills, 1998; Mills et al., 2009).

Figure 9.4(a) presents the measured vertical subsidence above LW1, which has been normalized by the extracted seam height \(T_1\) of 2.5m. The horizontal distance is presented as distance from the centre of the longwall panel as a function of the longwall panel width \(W\). The maximum subsidence is 49% of \(T_1\) and occurred at the closest surveying peg to the centre of the longwall panel. The subsidence profile is relatively symmetrical about the centerline of the longwall panel. The subsidence above the rib-edges is 2.5% and 1.8% of \(T_1\), which is less than the magnitudes reported from other
The normalized incremental vertical subsidence profiles measured above LW2, LW3 and LW4 are similar to the normalized vertical subsidence profile measured above LW1, as shown in Figure 9.4(b). They differ in that the subsidence profiles above LW2, LW3 and LW4 are not symmetrical about the centerline of the longwall panel. The eastern side, which corresponds to the tailgate side of the longwall, is much steeper than the western side of the subsidence profile. The location of maximum subsidence has also moved towards the tailgate side of each longwall panel. This phenomenon of migration of the maximum subsidence point towards the tailgate side of the extracted panels has been observed previously and described in detail by Waddington and Kay (1998).

Figure 9.3 – Total subsidence recorded along survey line XLA line due to extraction of longwalls in the Lower Whybrow seam. The crosses mark the location of where subsidence measurements had been recorded.
Figure 9.4 – Normalised incremental vertical subsidence recorded along survey line XLA above (a) LW1 and (b) LW1 to LW4.
9.3.2. Numerical modelling method

The displacement-finite element method was used to attempt to match the subsidence recorded above the supercritical longwall panels in the Lower Whybrow seam. The normalised subsidence profiles presented in Figure 9.4(b) showed that the profile of LW1 was a reasonable approximation for the shape and magnitude of all the surveyed subsidence profiles above the other longwall panels extracted from the Lower Whybrow seam. Therefore, only the subsidence recorded above LW1 extracted from the Lower Whybrow seam is presented in the plots here for the purpose of comparing predicted single-seam subsidence profiles to measured subsidence profiles. The recorded subsidence profile along survey line XLA was deemed to be located far enough away from the start or end of LW1 that plain-strain conditions could be assumed for these analyses.

The analyses used modified versions of the Cavity model and Goaf model presented in the earlier Chapter, which investigated the prediction of subsidence above a single-seam supercritical longwall panel using numerical modelling (Chapter 7). Scale drawings of the models used here are shown in Figure 9.5, with the appropriate dimensions to suit the geometry of the subsurface strata of LW1. These models assume plane strain conditions and the analysis plane was orientated perpendicular to the length of the longwall panels. The overburden height \( H \) was defined as 155m, the longwall width \( W \) was 210m and the coal seam thickness was 2.5m. The constitutive laws used to represent the overburden were an isotropic elastic material, a transversely isotropic elastic material, an isotropic elastic material with equally spaced smooth horizontal interfaces, a ubiquitous joint material and an elastoplastic strain-softening material. All other specifications of the models were as per the description given in Chapter 7. The bulking ratio \( b \) of the caved goaf was assumed to be 1.2, which meant that the total height of the caved-goaf material was 15m. The constitutive laws used for the strain stiffening goaf were as per Equations 7.3 to 7.5. The magnitudes of parameters, initial tangent modulus \( E_i \) and dimensionless constant \( a \), were selected in accordance with Equation 8.4, which was derived in Chapter 8.
Figure 9.5 – Scale drawing of the geometry and material properties used in the numerical models to predict subsidence above LW1 in the Whybrow seam: (a) initial conditions for both the Cavity and Goaf models (b) final conditions for the Cavity model and (b) final conditions for the Goaf model.

9.3.3. Results

As a form of reference, the vertical subsidence predicted using the Cavity model with an isotropic elastic material for the overburden is presented in Figure 9.6. Three magnitudes of Young’s modulus for the overburden were considered, \( E_o = 10 \text{GPa} \), \( E_o = 1 \text{GPa} \) and \( E_o = 0.65 \text{GPa} \). The stiffest isotropic elastic material predicted a subsidence profile that matched the measured subsidence profile above the un-mined areas of the
coal seam either side of the longwall panel. However, the maximum subsidence at the centre of the longwall panel was only 5% of $T_1$. Reducing $E_o$ to 10% of 10GPa (i.e., 1GPa) increased the maximum subsidence to 31% of $T_1$. The Young’s modulus needed to be reduced to 0.65GPa to achieve a maximum subsidence of 47% of $T_1$. However, as previously noted (Coulthard et al., 1988; Kay et al., 1991), the use of an appropriately soft $E_o$ required to accurately predict the maximum subsidence recorded above supercritical longwall panels results in an overestimation of the subsidence over the rib-edges. For $E_o = 0.65GPa$, the subsidence above the rib-edge was 23% of $T_1$.

Figure 9.7 presents the results for the Cavity model with a transversely isotropic elastic overburden. The smallest independent shear modulus used in the previous Chapter was $G'/G_{iso} = 0.05$. Using this value together with $E_o = 10GPa$ predicts a maximum subsidence above the centre of the longwall panel of 37% of $T_1$. The maximum subsidence recorded for LW1 can be achieved using a transversely isotropic elastic overburden by either reducing the magnitude of $G'/G_{iso}$ to 0.0357 or reducing the magnitude of $E_o$ to 7.5GPa. For these cases the maximum subsidence is 49% and 48% of $T_1$ respectively. For all three cases, the subsidence above the rib-edges is between 6% and 8% of $T_1$. The subsidence profiles predicted by the transversely isotropic elastic overburden are narrower than for the isotropic elastic material, but not narrow enough to match the recorded field measurement profile.

Figure 9.6 – Normalised predicted vertical subsidence for LW1 using the Cavity model with an isotropic elastic overburden and the normalised measured vertical subsidence for LW1 along survey line XLA.
Figure 9.7 – Normalised predicted vertical subsidence for LW1 using the Cavity model with a transversely isotropic elastic overburden and the normalised measured vertical subsidence for LW1 along survey line XLA.

Figure 9.8 presents the results for the Goaf model with a transversely isotropic elastic overburden. To achieve the maximum subsidence recorded above LW1 of 49% of $T_1$, if $G'/G_{iso} = 0.05$, then $E_o$ needed to be reduced to 1GPa and parameters for the strain-stiffening goaf needed to be $E_i = 12$MPa and $a = 22.1$. However, the small magnitude of $E_o$ leads to a relatively wide predicted subsidence profile, with subsidence above the rib-edges of 11% of $T_1$. The other option for achieving the maximum subsidence recorded above LW1 was when $E_o = 10$GPa and the magnitude of $G'/G_{iso}$ was reduced to 0.005 and the parameters for the strain-stiffening goaf were $E_i = 3$MPa and $a = 40.6$. The maximum subsidence is 47% of $T_1$ and the predicted subsidence profile is a narrower profile. However, it seems unlikely that a magnitude of an independent shear modulus equal to 0.5% of $G_{iso}$ could be realistic. This issue needs further investigation and validation.
The Goaf model with a bedded overburden generated an even narrower subsidence profile, which was closer in shape to the measured subsidence profile above LW1 (Figure 9.9). This bedded material consisted of horizontal frictionless interfaces that were equally vertically spaced by a distance \((D)\). The overburden was defined by \(E_o = 10\text{GPa}\) and the parameters for the strain-stiffening goaf were \(E_i = 5\text{MPa}\) and \(a = 38\). The vertical spacing of the frictionless interfaces considered here was 7.5m and 5m. The maximum subsidence was 45% and 47% of \(T_1\), respectively. Interestingly, these subsidence curves are relatively similar in shape and magnitude. The bedded material tends to confine the subsidence above the width of the extracted longwall panel. In these cases presented here, the subsidence above the rib-edge of the longwall panel for the bedded overburden is approximately 5% of \(T_1\).
The Goaf model with a ubiquitous joint overburden generated a narrower subsidence profile than the measured subsidence profile for LW1 (Figure 9.10). The predicted maximum subsidence was equal to 41% of \( T_1 \), when the overburden was defined by \( E_o = 10\) GPa, \( \varphi = 20 \) degrees and the parameters for the strain-stiffening goaf were \( E_i = 5\) MPa and \( a = 38 \). When \( E_o \) was reduced to 1GPa, it was not an unexpected result to observe that the ground subsided more above the un-mined areas of coal either side of the longwall panel relative to the case where \( E_o = 10\) GPa. However, the reduction of \( E_o \) to 1GPa resulted in a reduction in the overall maximum predicted subsidence to 39% of \( T_1 \). This occurred because the overburden strata had deflected more elastically before the onset of plastic deformation. Therefore, less overall plastic deformation occurred in the case where \( E_o = 1 \) GPa leading to a reduced maximum predicted subsidence relative to the case where \( E_o = 10\) GPa. The magnitude of maximum predicted subsidence was closest to the field measured value when \( E_o = 10\) GPa and the stiffness of the strain-stiffening goaf was reduced to \( E_i = 2\) MPa and \( a = 42 \). The maximum predicted subsidence in this case was 46% of \( T_1 \) and the predicted subsidence above the rib-edges was 0.3% of \( T_1 \).
Figure 9.10 – Normalised predicted vertical subsidence for LW1 using the Goaf model with a ubiquitous joint overburden and the normalised measured vertical subsidence for LW1 along survey line XLA.

Figure 9.11 shows the results for the Goaf model with an elastoplastic strain-softening overburden. The parameters used for the strain-softening material were $c = 800\text{kPa}$ and this cohesion was reduced to zero over a plastic deviatoric strain of $\varepsilon_{pd,\text{max}} = 0.08$. Two magnitudes of friction angle were considered; $\phi = 10$ degrees and $\phi = 15$ degrees. The maximum subsidence for these two cases was 51% and 46% of $T_1$, respectively. As identified in Chapter 7, the implementation of the elastoplastic strain-softening material often has issues with numerical instability and sensitivity to mesh size (Pietruszczak et al., 1981). The analysis of the Goaf model for the prediction of the subsidence above LW1 was sensitive to the mesh size. The minimum size of the mesh for which a solution could be found was 8m. The points on the predicted subsidence curves in Figure 9.11 show the location of the nodes along the ground surface of the numerical model. This relatively large mesh size means that the predicted subsidence profile probably has poor accuracy and should not be considered as a reliable solution. These results show that a more robust implementation of strain-softening constitutive laws in DFEM is required before its potential in prediction of subsidence can be assessed.
Chapter 9 – Case Study: Subsidence above single-seam and multi-seam longwall mining in the Hunter Valley

The comparison of the subsidence predicted using DFEM with a range of material models for the overburden and caved goaf to the measured subsidence above LW1 at the South Bulga Complex gave results in agreement with those presented in Chapter 7. Both the isotropic elastic overburden and the transversely isotropic elastic overburden predicted subsidence profiles too shallow and as such overestimated the subsidence above the rib-edge of the longwall panel. The bedded material achieved the closest agreement to the measured subsidence profile without the additional complication of numerical instability or a mesh dependent solution. The representation of the extracted coal with a cavity in the analysis does not allow for the stress along the longwall floor to return to the overburden stress, which will be an important factor when predicting subsidence above the longwall in the Blakefield seam. The inclusion of the strain-stiffening goaf to represent the caved goaf achieved the desired vertical stress distribution along the longwall floor, as long was the overburden material did not bridge stress to the surrounding strata.

9.4. Multi-seam mining

9.4.1. Field measurements

The survey lines used to monitor the subsidence above the longwalls in the Blakefield
seam are presented in Figure 9.12. Survey data was collected above BSLW1 prior to mining ceasing part way through the panel extraction because of an underground fire. There has been a more comprehensive collection of data set above the extraction of BSLW2 than above BSLW1. This Chapter shall only present the vertical subsidence results for survey lines DL, XL1, LOM and SBCP2 above longwall BSLW2. The extracted seam height in the second seam longwall BSLW2 ($T_2$) was 2.8m, which was the minimum height for the shearer being used.

![Figure 9.12 – Map showing the survey lines used to monitor the subsidence when extracting longwall BSLW2 (Mine Subsidence Engineering Consultants, 2013).](image-url)
The incremental subsidence recorded along survey line DL is presented in Figure 9.13(a). This survey line is orientated perpendicular to the length of longwall BSLW2. It also overlies the old workings of LW2 and LW3 in the Lower Whybrow seam. The relative location of the longwalls in the Lower Whybrow and Blakefield seams are schematically represented in the bottom section of Figure 9.13(a). Initial assessment of the shape of the incremental subsidence profile above BSLW2 is that it is quite different to the general shape of the incremental subsidence troughs recorded above the extracted longwall panels in the first seam (Figure 6.2). The subsidence profile is not parabolic in shape, with the maximum occurring above the centre of the extracted longwall panel. There appears to be a local reduced subsidence lying directly over the location of the remnant chain pillar that is present in the Lower Whybrow seam. There are also local maximums of subsidence either side of this chain pillar. These local maximums lie above the edges of the longwall goaf in LW2 and LW3. Aside from the variation in subsidence local to the chain pillar in the Lower Whybrow seam, the average subsidence at the bottom of the subsidence trough is approximately 2.2m. This occurs at approximately chainage 1150m to 1185m along the survey line, which is above a central section of BSLW2 and a central section of LW3 in the Lower Whybrow seam. Aside from the local variations, this average subsidence corresponds to a vertical subsidence of 79% of the extracted seam height of the second seam ($T_2$). This is in agreement with the observation that subsidence above the second seam is often approximately 80% in coalfields with strong rocks in the overburden strata (Schumann, 1987; Dynl, 1991; Sheorey et al., 2000; te Kook et al., 2008).

Figure 9.13(b) presents the measured incremental vertical subsidence above BSLW2 along survey line XL1. Survey line XL1 is positioned at an orientation approximately 7 degrees off being perpendicular to the length of the longwall panel BSLW2. The subsidence profile has some similar features but also some differences to the subsidence profile recorded along survey line DL. In general, there is a local subsidence minimum above the location of the chain pillar present in the Lower Whybrow seam and there are local subsidence maxima above the edges of the goaf in the Lower Whybrow longwalls. However, these variations in the subsidence trough are not as pronounced as were observed along the survey line DL. The subsidence above BSLW2, over areas other than the location of the local maximum and local minimum, is approximately 2.3m, which is 82% of $T_2$. The shape of the subsidence profile appears to be slightly wider.
than recorded along survey line DL, which can be easily observed because the horizontal scale is the same in Figure 9.13(a) and (b). This degree of elongation is probably an illusion because the survey line XL1 is orientated on a diagonal to longwall BSLW2. This was confirmed by projecting the results of XL1 onto a plane perpendicular to the length of the longwall panel and the subsidence trough for survey lines DL and XL1 where then of similar width.

The survey line LOM is positioned on a diagonal to the orientation of the length of BSLW2. The orientation of the survey line LOM is approximately 25 degrees off being perpendicular to the length of the longwall panel BSLW2. Figure 9.13(c) presents the measured incremental vertical subsidence above BSLW2 along survey line LOM. The recorded results agree with the observations made of the subsidence recorded along survey lines DL and XL1: there is a local subsidence minimum above the location of the chain pillar present in the Lower Whybrow seam; there are local subsidence maximums above the edges of the goaf in the Lower Whybrow longwalls; and over areas other than the location of the local maximum and local minimum the subsidence trough is approximately 2.2m deep, which is 79% of $T_2$.

The survey line SBCP2 is positioned on a diagonal to the orientation of the length of BSLW2 such that it lies directly above the chain pillar located in the Lower Whybrow seam, between LW2 and LW3. Figure 9.13(d) presents the incremental vertical subsidence above BSLW2 along survey line SBCP2. It should be noted that the shape of the subsidence profile on the right hand side of the figure is probably still in transition to achieving its final depth. The longwall face was underlying a point at a chainage of approximately 1100m along survey line SBCP2 at the time the measurements were being made. Unlike for the subsidence profiles recorded along the other three survey lines, there appear to be no local maximum or minimum in the subsidence profile. This result supports the hypothesis, made by previous authors and discussed in Chapter 8, that the mechanics of the strata in the previous workings are affecting the shape and magnitude of the subsidence profile above multi-seam longwall panels. Along this survey line, given there is probably relatively little change to the mechanics of deformation of the material in the old workings of the Lower Whybrow seam, the subsidence magnitude is relatively constant. The average depth of the recorded subsidence trough is approximately 1.85m, which is 66% of $T_2$. This is less than the 80% of $T_2$ that was recorded above the majority of the subsidence trough above
BSLW2 and closer to the subsidence typically recorded above single-seam longwall panels in the Sydney Basin.
Figure 9.13 – Incremental vertical subsidence, due to extraction of BSLW2, recorded along survey lines (a) DL line, (b) XL1 line, (c) LOM line and (d) SBCP2 line.
The subsidence measured above the four survey lines above BSLW2 clearly show that sag subsidence and strata compression are no longer the only two mechanisms governing the deformation of the subsurface strata and the associated ground surface movements. The magnitude of subsidence recorded along survey line SBCP2, which lies directly over the chain pillar present in the Lower Whybrow seam, is relatively constant at 66% of $T_2$. This result could be interpreted to indicate that the mechanics around the second seam longwall are similar to that which occurs when single-seam mining. Therefore, the bulking of the intermediate roof of the second seam and the behaviour of the interburden and overburden strata are similar to single-seam mining mechanics. If this idea is assumed to be correct, the extra subsidence above the second seam longwall would probably arise from remobilization and compaction of the caved goaf of the first seam. This proposal that the first seam goaf is remobilized and undergoes further compaction has been previously used to explain the unique shape of the subsidence profile above multi-seam longwall panels, as mentioned in Chapter 8 (Bai et al., 1995; Gale, 2004; Mine Subsidence Engineering Consultants, 2007).

Therefore, the subsidence recorded at the surface may result from a component of subsidence trough formation above the second seam longwall and another component due to compaction of the first seam caved goaf. However, it is not clear what the relative contribution of the remobilization and compaction of the first seam goaf is to the overall recorded subsidence. As an upper limit, it could be postulated that all the subsidence greater than that which was recorded above the first-seam chain pillar is due to compaction of the first seam goaf. This has been schematically shown on the results from survey line DL in Figure 9.14.

Other possibilities for the sub-surface geomechanics cannot be discounted, as it is not yet possible to say definitively if the above proposal is valid. The difficulty in discerning the appropriate mechanism is the lack of field information on the sub-surface ground movements. Another possibility could be that the subsidence trough generated by displacements around the second seam longwall could be in the order of approximately 80% of $T_2$. This could come about because of the fractured overburden strata being more compliant or that the caved goaf of the second seam does not bulk as much as for the first seam. This would mean that subsidence greater than approximately 80% of $T_2$ would be due to compaction of the first seam caved goaf compaction and there would need to be a degree of bulking over or under the first seam.
Chapter 9 – Case Study: Subsidence above single-seam and multi-seam longwall mining in the Hunter Valley

chain pillar. Since it is not yet possible to determine which mechanism is accurate both will be considered in the numerical analyses described below.

![Diagram showing subsidence due to trough formation and strata compression](image)

**Figure 9.14** – Proposed contributions from trough formation and strata compression of the first seam goaf to the subsidence recorded along survey line DL above longwall BSLW2.

### 9.4.2. Numerical modelling method

The displacement-finite element method was again used to attempt to match the incremental subsidence recorded in the field above the second seam supercritical longwall panel BSLW2. The incremental subsidence recorded above BSLW2 along survey line DL is presented in the plots here for the purpose of comparing predicted subsidence profiles to measured subsidence profiles. Since this survey line is orientated perpendicular to the length of the panel and it is located far enough away from the start or the end of the panel, plain-strain conditions were deemed appropriate for the analyses. Further, the subsidence results recorded along the survey lines did not indicate that there were significant contributions arising from three-dimensional displacements in the sub-surface strata.

Modified versions of the multi-seam Cavity model and Goaf model presented in Chapter 8 have been used here. Scale drawings of the models used are shown in Figure
9.15, with dimensions to reflect the geometry of the excavations of BSLW2 and overlying LW2 and LW3. The overburden height ($H$) was defined as 155m, the interburden thickness ($B$) was 80m, the width of longwall BSLW2 ($W$) was 410m, and the coal seam thickness of the Blakefield seam ($T_2$) was 2.8m. The original width of the panels in the overlying Lower Whybrow seam were 210m, however, because the survey line DL crosses LW2 and LW3 on a diagonal the width of LW2 and LW3 was 242m in the cross section used in the model. The extracted height in the overlying Lower Whybrow seam ($T_1$) was 2.5m.

![Figure 9.15 – Scale drawings of the geometry and material properties used in the numerical models to predict subsidence above BSLW1 in the Blakefield seam, with: (a) initial conditions for both the Cavity and Goaf models (b) final conditions for the Cavity model and (b) final conditions for the Goaf model.](image)

Figure 9.15 – Scale drawings of the geometry and material properties used in the numerical models to predict subsidence above BSLW1 in the Blakefield seam, with: (a) initial conditions for both the Cavity and Goaf models (b) final conditions for the Cavity model and (b) final conditions for the Goaf model.
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The multi-seam Goaf model was considered with two magnitudes of bulking ratio for the second seam. As mentioned in Section 7.2.2, \( b = 1.2 \) has typically been used in numerical analyses of single-seam mining cases where the overburden consists primarily of sandstone but it is not yet clear what governs the magnitude of the bulking ratio. For \( b = 1.2 \), the caved goaf height (\( h_g \)) would correspond to 16.8m given that the extracted seam height in the Blakefield seam is 2.8m. It has been suggested that the caved goaf in the second seam longwall may not bulk as much as the first seam caved goaf (Ma et al., 1984). Therefore, a magnitude of bulking ratio of 1.1 was also considered in the analyses here. In this case, the caved goaf height (\( h_g \)) in the Blakefield seam would correspond to 30.8m when \( b = 1.1 \). The constitutive laws used for the strain stiffening goaf were as per Equation 7.3. The magnitudes of parameters, initial tangent modulus \( E_i \) and dimensionless constant \( a \), for the second seam goaf were selected in accordance with Equation 8.4, which was derived Chapter 8. The parameters used for the first seam strain-stiffening goaf were \( b = 1.2 \), \( E_i = 5 \text{MPa} \) and \( a = 38 \), as these magnitudes predicted appropriate subsidence profiles, as presented in Section 9.3.3.

The overburden and interburden were modelled as a bedded material, which consisted of horizontal frictionless interfaces that were equally vertically spaced by a distance (\( D \)) of 5m. All other specifications of the multi-seam Cavity and Goaf model were as per the description given in Chapter 8.

9.4.3. Results

Figure 9.16 presents the predicted incremental subsidence above the extracted BSLW2 using the multi-seam Cavity model with a bedded overburden and interburden. As observed for the single-seam mining case, the bedded material effectively confines the predicted incremental subsidence to within the width of extracted longwall panel. The subsidence above the rib-edge of the longwall panel for the bedded overburden is approximately 6% of \( T_2 \). The shape of the predicted profile is similar to those determined for the single-seam case, where there is a single trough. The bottom of the trough is relatively flat and is approximately 0.4 \( W \) wide. The maximum predicted subsidence, of 101% of \( T_2 \), is located 0.1\( W \) off the centre of BSLW2. As expected, DFEM did not predict the variation in the subsidence local to the chain pillar and edges.
of the caved goaf present in the first seam workings. This is because the distribution of stress and associated strain did not change significantly in the area of the first seam working and overburden in response to the second seam extraction.

Aside from the local maximum and minimum in the subsidence profile above the edges of the caved goaf and chain pillar present in the first seam workings, the measured depth of the subsidence trough that formed above BSLW2 was approximately 79% of $T_2$. The predicted depth of the subsidence trough using the Cavity Model is much larger than this measurement, at 100% of $T_2$. This shows that the multi-seam Cavity model significantly overestimates the incremental subsidence above an extracted longwall in the second seam. There is a need to include the strain-stiffening goaf into the second seam as well as the first seam of the model.

![Diagram](image_url)

*Figure 9.16 – Normalised predicted incremental vertical subsidence for BSLW2 using the Cavity model with a bedded overburden and the normalised measured vertical subsidence for BSLW2 along survey line DL.*

Figure 9.17(a) presents the predicted incremental subsidence for the extraction of BSLW2 using the multi-seam Goaf model with a bedded overburden and interburden. Two magnitudes of bulking ratio were considered for the second seam goaf. For $b = 1.2$, to achieve a subsidence profile similar in shape to the field measurements the strain-stiffening parameters needed to be $E_i = 4$MPa and $a = 40.6$. For $b = 1.1$, to
achieve a subsidence profile similar in shape to the field measurements the strain-
stiffening parameters needed to be $E_i = 5\text{MPa}$ and $a = 76$. The maximum predicted
subsidence is 88% and 91% of $T_2$ for $b = 1.1$ and $b = 1.2$, respectively. For both of the
bulking ratios considered here, the predicted subsidence profiles are very similar in
shape and magnitude, with the subsidence above the rib-edges at 5% of $T_2$. The
subsidence predicted above areas of caved longwall goaf present in both seams is
approximately 79% of $T_2$ and matches the measured subsidence. However, the
predicted subsidence located above the chain pillar and edges of the caved goaf present
in the first seam workings does not match the recorded subsidence measurements. The
maximum predicted subsidence occurs above the chain pillar present in the first seam
workings. The results presented in Figure 9.17(a) would support the proposal
mentioned above, that 80% of $T_2$ of the subsidence profile is due to the formation of the
second seam subsidence trough. The displacement-finite element method is able to
approximately match the general trough profile, but not the local variation close to the
chain pillar.

The other proposal has also been considered, where the component of the subsidence
profile due to subsidence trough formation above the second seam constitutes
approximately 66% of $T_2$. Given the DFEM is not able to replicate displacements
associated with remobilization and compaction of the first seam goaf, only the
subsidence due to trough formation is replicated in the following prediction. The
predicted subsidence profile is shown in Figure 9.17(b). The desired solution was
obtained by using the same properties for the first seam goaf as the second seam goaf,
i.e., $b = 1.2$, $E_i = 5\text{MPa}$, $a = 38$. The predicted subsidence curve is a reasonable shape
for a supercritical longwall panel, except it does include a maximum in the subsidence
profile directly above the Lower-Whybrow chain pillar. This occurred because the
chain pillar in the first seam still attracts more vertical stress than the adjacent strain-
stiffening goaf as the isotropic elastic overburden layers redistribute stress to
surrounding strata.
Figure 9.17 – Normalised predicted incremental vertical subsidence for BSLW2 using the Goaf model with a bedded overburden and the normalised measured vertical subsidence for BSLW2 along survey line DL.
9.5. Discussion

The results from this case study support all the findings presented in the Chapter 7 and Chapter 8. The subsidence profile predicted using the bedded overburden and the ubiquitous joint overburden closely matched the recorded subsidence above single-seam longwall panel LW1. Both of these materials predicted magnitudes of maximum subsidence and the subsidence above the rib-edge closest to the measured subsidence profile above LW1. The predicted subsidence profile with the bedded overburden appears to overestimate the vertical displacements above the edges of the longwall goaf, while the ubiquitous joint overburden appears to under-estimate the vertical displacements above the edges of the longwall goaf. Review of the results from physical models (e.g., Whittaker et al., 1989) shows that the strata in the area between the break angle and the angle of draw can undergo some degree of bulking. It is possibly this increase in volume of the strata in this area that may not be adequately represented by the bedded overburden. On the other hand, the ubiquitous joint overburden in the area between the break angle and the angle of draw behaves as a stiff isotropic elastic material, which results in an underestimation of the subsidence. Therefore, the ubiquitous joint material is only good at replicating the displacements associated with the failure of the block directly above the extracted longwall, which is trapezoidal in shape.

The displacement-finite element method (DFEM) was able to predict reasonably well the basic subsidence trough created above single-seam and multi-seam longwall panels. However, the DFEM was not successful in predicting the variations of subsidence above chain pillars in the first seam. If as proposed by previous authors (Bai et al., 1995; Gale, 2004; Mine Subsidence Engineering Consultants, 2007) the source of this local variation in subsidence is due to rearrangement of caved goaf material in the first seam, further investigations should be conducted to identify if other numerical methods would be able to match the location variations in maxima and minima in the measured subsidence above multi-seam longwall panels. The discrete element method might seem the most appropriate method to consider first, however, it should be noted that a reasonable representation of the stratigraphy and discontinuities around the first seam longwall would be required to achieve a plausible solution. A reasonable level of detail about the stratigraphy and discontinuities is often difficult to obtain for most real-life cases.
An advantage of the DFEM method used in this Chapter here is the lack of site specific geological information required to achieve a solution that is close to the measured results. The method is highly dependent on the DFEM using a continuum material, which requires the coal measure strata to be approximated to a homogeneous smeared material. One of the integral aspects to obtaining accurate predictions of the magnitude of the general subsidence trough above either the single-seam or multi-seam case is the selection of the magnitudes of the parameters is the strain-stiffening goaf material. Further study is required to get a better understanding on what physical properties of the rock mass affect the magnitude of the strain-stiffening goaf material and how to select these parameters in future predictions of subsidence.

**9.6. Conclusion**

In this case study, the subsidence measured above single-seam and multi-seam extraction using the longwall method of mining has been compared to predictions of subsidence using the displacement-finite element method. The best agreement between the measured single-seam subsidence data above LW1 at Blakefield South colliery was achieved by using a strain stiffening caved goaf material with an overburden defined by either the bedded material or the ubiquitous joint material. The multi-seam model with a strain-stiffening caved goaf in both seams and a bedded overburden and interburden was able to match the general shape of the trough that formed above the extraction of longwall BSLW2. However, the displacement-finite element method was not able to predict the variation in the subsidence profile local to the chain pillar in the first seam, because it was not able to replicate the remobilisation and compaction of the first seam goaf. The significance of this Chapter for multi-seam mine designers is that the theoretical findings presented in Chapter 7 and Chapter 8 have been validated to an acceptable degree against real-life measurements.
CHAPTER 10. CONCLUSIONS

The primary objective of this Thesis was to address some of the key geotechnical issues affecting multi-seam coal mining projects in Australia, particularly those that may have resulted from a prior lack of understanding of the associated geomechanics. There were three main foci of the research presented here: to quantify the effects of extraction of the first seam using the longwall method on the in situ stresses in the underlying strata; to assess the prediction of failure of the overburden and interburden of a multi-seam mine using appropriate failure criteria; and to accurately predict the subsidence typically observed above single-seam and multi-seam supercritical longwall panels in Australia.

10.1. In situ stresses prior to multi-seam mining

The effect of the fixed and controllable variables on the vertical and horizontal stress redistribution in the strata underneath an extracted supercritical longwall panel was investigated. The research was conducted primarily by performing finite element continuum analyses coupled with Wilson’s equations (Wilson, 1983) for the vertical stress distribution in the vicinity of a single longwall panel after it has been mined. This analysis method removed the uncertainty of using a full model of the coal measure strata to predict the stress redistribution in the overburden strata. The key finding was that the abutment angle, overburden depth, pillar width and anisotropic behaviour of the coal measure strata most influence the change to the in situ vertical stress in the lower coal seam.

The horizontal stress redistribution was modelled by assuming that the stiffness of the caved, fractured and deformed zones of the goaf was zero. This conservative approach predicted the maximum change in horizontal stress to be 11% of the initial in situ horizontal stress for an isotropic elastic material and this maximum occurred in the floor of the first seam longwall. The relative change in horizontal stress was independent of the stress ratio, $K$, of the initial in situ stress state. Larger equivalent extracted widths led to smaller maximum changes in horizontal stress, but the horizontal stress change extended deeper into the underlying strata.

The degree of bedding in the interburden was predicted to impact significantly both the vertical and horizontal stress distributions. A more highly bedded material, which would be represented by a material with a lower independent shear modulus, would
cause the vertical stresses imposed onto the chain pillars to be transferred deeper into the underlying strata. The vertical stress changes would change more rapidly with horizontal distance than for an isotropic elastic material, which can lead to more significant changes to mining conditions. Highly bedded material causes the maximum change in horizontal stress to be larger than for the isotropic case and to dissipate to zero at a much shallower depth.

The significance of this section of the Thesis is that it informs designers of multi-seam mines about which parameters have the largest influence on the pre-mining vertical and horizontal stresses induced in strata underlying those seams already mined. The effect of the identified parameters will have implications for the design of ground support measures for the lower seam pillars and roadways. The implications of the findings are that there is potential for large variations in the predicted pre-mining vertical stress while generally smaller changes to the predicted pre-mining horizontal stress can be expected. The rapid changes in vertical stress with horizontal distance in transverse isotropic strata behaviour are likely to be reflected in more sudden changes in rock mass response. Both the vertical and horizontal stress changes need to be considered in light of each other to assess the overall stability of a second seam.

### 10.2. Prediction of overburden and interburden failure

Three independent methods were used to evaluate the roof collapse of underground rectangular cavities for a range of geometries and rock properties. The rock mass strength was described by the Hoek-Brown failure criterion and two forms of the Mohr-Coulomb failure criterion. The results were presented as stability charts to enable designers of underground openings to predict rapidly the safe widths of underground cavities. The magnitudes of the stability factors $N$ for both forms of the Mohr-Coulomb failure criterion were all much lower than for the Hoek-Brown failure criterion and further investigation is required to identify which failure criterion predicts conditions for roof collapse more accurately for coal measure strata.

The results from the finite element upper bound and lower bound showed that the friction angle governed the shape of the failure surface for the linear forms of the Mohr-Coulomb failure criterion. Subsequently this governed the critical ratio of cavity width to overburden depth ($W/H_{\text{crit}}$) corresponding to boundary of subcritical failure and supercritical failure. This boundary between subcritical and supercritical failures of the
overburden was included on the stability charts. The prediction of the ratio \((W/H)_{\text{crit}}\) that best matched the measurements made in the field was for a friction angle \(\varphi\) of approximately 30 degrees, which agrees well with values often used by practicing engineers. The Hoek-Brown criterion overestimates the critical cavity width, probably because in the range of tensile and very low confining stresses it corresponds to effective friction angles that are very large. The results obtained by imposing a tension cutoff clearly show that the stability of the cavity and mechanism of failure of the overburden are controlled mainly by the tensile strength.

The significance of the findings is that they provide multi-seam mine designers with information about the consequences of using either the Hoek-Brown or the Mohr-Coulomb failure criterion to represent the strength of the coal measure strata when they are represented as smeared homogeneous materials.

### 10.3. Prediction of subsidence above single-seam and multi-seam longwalls

The predictions of vertical subsidence profiles above single-seam and multi-seam longwall panels where compared with observations typically observed in coalfields in New South Wales, Australia. A selection of constitutive laws was used to represent the mechanical laws of the coal measure strata, varying in complexity from simple (e.g., an isotropic linear-elastic material) to very sophisticated (e.g., strain-softening elastoplastic material). A strain-stiffening goaf was also included in some models to allow for the generation of vertical stress on the longwall floor, and subsequently onto the interburden, in some cases allowing it to return to the original overburden stress.

The best agreement between the measured single-seam and multi-seam subsidence data was achieved by using a strain stiffening caved goaf material with a bedded overburden and interburden. The results show that more sophisticated and numerically taxing constitutive laws do not necessarily lead to more accurate results when the predictions are compared to field measurements. The bridging nature of an elastic overburden prevented the prediction of the return of overburden stress levels along the longwall floor. This caused the prediction of the subsidence above multi-seam staggered longwalls to be governed by the chain pillar in the first seam, which effectively behaved as a concentrated or point load acting at the midspan of the interburden. The displacement finite element method was not able to predict the variation in the subsidence profile local to the chain pillar in the first seam, because it was not able to
replicate the remobilisation and compaction of the first seam goaf.

The case study compared the subsidence measured above single-seam and multi-seam longwall extraction in the Hunter Valley, Australia to the predictions of subsidence using the displacement-finite element method. The analyses supported the findings presented in previous Chapters.

10.4. Future work

The general approach for the research presented in this thesis has involved addressing the fundamentals of the problem before adding additional complexity. This led to several assumptions in the analysis method that could be further investigated in future studies. For all three focus areas considered here, plane strain conditions were assumed in the analyses. In each case it was deemed that the three-dimensional effects were negligible. Further investigations could be conducted to verify this or else quantify the contribution of three-dimensional effects to stability and subsidence around a longwall panel. Subsidence predictions in three dimensions could probably also consider the progressive nature of the longwall mining process.

The prediction of subsidence above both single-seam and multi-seam longwall panels only considered the use of the finite element method. Additional research could be conducted to consider other numerical methods to assist in predicting subsidence. The results presented in Chapter 8 indicated that the continuum method was not able to predict the variation in subsidence recorded in the field local to the first seam chain pillar. It would be of particular interest to see if the discrete element method is able to predict subsidence closer to what is typically recorded above multi-seam longwall panels.

The limited availability of data from the field prevented some of the results presented in this thesis from being validated. It would be useful to compare the predicted in situ vertical and horizontal stress prior to undermining, presented in Chapters 3 and 4, to field measurements of in situ stress. Further, stability charts determined from the Hoek-Brown failure criterion showed significant variation in the stability number $N$ to the results obtained using the linear-form of the Mohr-Coulomb failure criterion. A comparison of the predicted stability numbers to cases of collapse in the field would help identify which failure criterion better represents the strength of coal measure strata.
These three areas of proposed further research highlight the limitations of the methods adopted in this thesis. More general research in the area of geomechanics of multi-seam mining could be also conducted to assess the geotechnical effects when the long axes of the longwall panels in different seams are not parallel. In this situation, it is expected that three dimensional effects together with the mechanical behaviour of the goaf in the first seam (including caving) are likely to play an important role.

10.5. **Concluding remarks**

This thesis has focused on three key geotechnical issues that are posing the biggest uncertainties associated with the geomechanics of multi-seam longwall mining. This is from both a mine design and approval perspective. The research presented here has typically consisted of parametric studies to provide detailed analyses of the advantages and limitations of various aspects of these geotechnical issues. The thesis provides multi-seam mine designers with a comprehensive set of results from which informed decisions can be made for many projects.
APPENDIX A – EQUIVALENCE OF HOEK-BROWN AND MOHR-COULOMB CRITERIA

For comparison purposes, it is often convenient to express both the Hoek-Brown and the Mohr-Coulomb failure criteria in terms of shear and normal stress components at failure. For the Hoek-Brown criterion, the relationships of the normal and shear stresses at failure to the principal stresses were derived by Balmer (1952) and for completeness are provided here in Equations (A1.1) and (A1.2):

\[
\sigma_n' = \frac{\sigma_1' + \sigma_3'}{2} - \frac{\sigma_1' - \sigma_3'}{2} \left( \frac{d\sigma_1'/d\sigma_3'}{d\sigma_1'/d\sigma_3' + 1} \right) \tag{A1.1}
\]

\[
\tau = \left( \sigma_1' - \sigma_3' \right) \sqrt{\frac{d\sigma_1'/d\sigma_3'}{d\sigma_1'/d\sigma_3' + 1}} \tag{A1.2}
\]

where

\[
\frac{d\sigma_1'}{d\sigma_3'} = 1 + am_b \left( \frac{m_b \sigma_3'}{\sigma_{ci}} + s \right)^{a-1}
\]
APPENDIX B – DETAILS OF REGRESSION ANALYSIS FOR PARAMETERS A AND B

Two regression methods were used to determine the values of parameters $A$ and $B$ (Equation (5.1)), which in turn are necessary to obtain an upper bound to the stability number $N$, according to the method of Fraldi and Guarracino (2009). The first method used 8 increments for the values of $\sigma_3$ in the range from 0 to 25% of $\sigma_{ci}$, which is the approach recommended by Hoek (2000). The values obtained using this method for a value of $GSI$ of 20, $m_i$ of 20, $\sigma_{ci}$ of 100 MPa and $D$ of zero were $A = 0.4691785$ and $B = 0.7349077$. When the curve corresponding to Equation (5.1) is compared to the Hoek-Brown criterion plotted in terms of shear and normal stresses, it appears to be a good fit over the full range of normal stresses used in the regression. However when focussing on the low normal stress range, this curve overestimates the failure envelope, as seen in Figure A.1. The low normal stress range corresponds to both tensile and very low compressive minor principal stresses. Indeed, this regression method appeared to provide a relatively poor fit of Equation (5.1) to the Hoek-Brown shear/normal stress curve in the low stress range.

To improve solution accuracy, it was necessary to identify the confining stress range most likely to occur at failure for this particular problem. It is only possible to do so reliably if there are field data or another independent method of analysing the problem. Fortunately, the numerical approaches used to solve this problem are also able to provide ‘independent’ information of this type.

The principal stresses computed at all the finite element nodes located within the cover depth above the cavity were available from the FE UB analysis. Results of this type are plotted in Figure A.2 for the problem defined by $W/H = 2$, $m_i = 20$ and $GSI$ ranging from 50 to 90. Stress points for each of the $GSI$ cases show a congregation in the tensile minor principal stress range ($\sigma_t < \sigma_3 < 0$). There are also stress points with compressive minor principal stresses, but only a very limited number are located near the Hoek-Brown failure surface.

The large density of stress points in the tensile confining stress range suggest that a regression analysis to evaluate values of the parameters $A$ and $B$ might be more appropriate over the range of $-\sigma_t$ to zero, with 10,000 increments. The regression analysis was conducted using a specially prepared MATLAB script. For values of $GSI$
of 20 and \( m_i \) of 20, the resulting values of \( A \) and \( B \) were \( A = 0.59928982 \) and \( B = 0.76741635 \). Figure A.1 shows the curve fit to Equation (5.1) using these values of \( A \) and \( B \) over the normal stress range used in the second regression method. It is noted that this second regression method produces a better curve-fit than the first. However, it should also be noted that the values for \( A \) and \( B \) determined by the second regression method should only be used for this problem where confining stresses range from \(-\sigma_i\) to zero. If these particular values of \( A \) and \( B \) were to be used for problems where values of \( \sigma_3 \) are predominantly larger than zero, it is likely that the fit to the Hoek-Brown criterion would be poor over the stress range most appropriate for these problems.

![Figure A.1 - Hoek-Brown failure criterion (Hoek et al., 2002) and curve-fit of Equation (5.1) using two regression methods to calculate parameters \( A \) and \( B \).](image-url)
Figure A.2 – Principal stresses in the cover at failure predicted by FE-LB analysis
APPENDIX C – FAILURE MECHANISMS FOR MOHR-COULOMB FAILURE CRITERION

The critical failure mechanisms for the smallest magnitude of friction angle considered in Chapter 6 did not all consist of roof collapse, as schematically shown in Figure 6.1. The curve for \( \phi = 20 \) degrees in Figure 6.2 was limited to results obtained for mechanisms associated with roof collapse only. The curve was truncated at magnitudes of the ratio \( W/H \) equal to 0.7 and 3.5. The complete set of results obtained from the UB and LB formulations for \( \phi = 20 \) degrees are presented in Figure A.3. This Figure also shows the plots of plastic dissipation at small and large magnitudes of ratio \( W/H \).

For a deep underground opening, where \( W/H = 0.7 \), that is surrounded by a material with \( \phi = 20 \) degrees, the predicted failure mechanism is a subcritical roof collapse (Figure 6.2). For magnitude of ratio \( W/H \) less than 0.7 the predicted collapse mechanism is more complex than the roof failing as single block. For \( W/H = 0.3 \) the plot of plastic dissipation shows that failure occurs in the sides as well as the roof of the cavity (Figure A.3). For \( W/H = 0.1 \) the plot of plastic dissipation shows that the failure occurs in the roof, sides and floor of the cavity (Figure A.3). Both of these failure mechanisms are multi-block failures and have been previously described by Yamamoto et al. (2001) for circular underground openings and a surface surcharge load.

A wide underground opening located at a shallow depth, where the ratio \( W/H \) is greater than 3.5, that is surrounded by a material with \( \phi = 20 \) degrees, is also predicted to consist of a multi-block failure mechanism (Figure A.3). This form of multi-block failure mechanism for wide underground openings located at a shallow depth was also observed for the other magnitudes of friction angle considered in Chapter 6 but at larger values of \( W/H \).
Figure A.3 – FE-LB predictions of the degree of plastic dissipation showing the failure surfaces as dashed lines.
APPENDIX D – SUBROUTINE FOR IMPLEMENTING STRAIN-STIFFENING GOAF MATERIAL

C*******************************************************************
subroutine usdfld(field, statev, pnewdt, direct, t, celent,
1 time, dtime, cmname, orname, nfield, nstatev, noel, npt, layer,
2 ksp, kstep, kinc, ndi, nshr, coord, jmac, jmatyp, matlayo,
3 laccfla)
  include 'aba_param.inc'
character*80 cmname, orname
character*3 flgray(15)
dimension field(nfield), statev(nstatev), direct(3,3),
1 t(3,3), time(2)
dimension array(15), jarray(15), jmac(*), jmatyp(*),
1 coord(*)
C    ------------------------------------------------------------------
C    USER CODE START
REAL Es, Esec_in, Esec_E2, Esec
Es=10000000.
Esec_in=STATEV(3)
CALL getvrm('E', array, jarray, flgray, jrcd, jmac, jmatyp,
&           matlayo, laccfla)
E1=-array(1)
E2=-array(2)
E3=-array(3)

IF (E2<=0.) THEN
  E2=0.0000001
END IF

C Calculate Secant Young's modulus
Esec=5000./15./E2*(EXP(E2*15.)-1.)
IF (Esec>Es.) THEN
  Esec=10000000.
END IF
IF (Esec<Esec_in) THEN
  Esec=Esec_in
END IF

FIELD(1)=Esec
STATEV(1)=Esec_E2
STATEV(2)=E2
STATEV(3)=Esec
STATEV(4)=Esec_in

RETURN
END SUBROUTINE usdfld
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