Bulk Solid Interactions in Belt Conveying Systems

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The University of Newcastle
Discipline of Mechanical Engineering

By

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BE (Mech) (Hons II/1)

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Doctor of Philosophy

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Dusan Ilic
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<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>cross sectional area at location considered</td>
<td>m²</td>
</tr>
<tr>
<td>a₁</td>
<td>distance to centre of gravity of discharging stream</td>
<td>m</td>
</tr>
<tr>
<td>A_{LRP}</td>
<td>loose repose cross sectional area profile</td>
<td>m²</td>
</tr>
<tr>
<td>aₙ</td>
<td>idler spacing</td>
<td>m</td>
</tr>
<tr>
<td>A_{idler}</td>
<td>cross sectional area at idler roll set</td>
<td>m²</td>
</tr>
<tr>
<td>A_{skirting}</td>
<td>cross sectional area within skirt plates</td>
<td>m²</td>
</tr>
<tr>
<td>A_{skirting}</td>
<td>cross sectional area profile</td>
<td>m²</td>
</tr>
<tr>
<td>A_{SSP}</td>
<td>surcharge angle cross sectional area profile</td>
<td>m²</td>
</tr>
<tr>
<td>A_{trough}</td>
<td>cross sectional area within belt trough</td>
<td>m²</td>
</tr>
<tr>
<td>a_V</td>
<td>maximum acceleration</td>
<td>m²</td>
</tr>
<tr>
<td>b</td>
<td>contact perimeter</td>
<td>m</td>
</tr>
<tr>
<td>B</td>
<td>horizontal length of bulk solid material at location considered</td>
<td>m</td>
</tr>
<tr>
<td>B_s</td>
<td>initial chute or stream width</td>
<td>m</td>
</tr>
<tr>
<td>B_w</td>
<td>belt width</td>
<td>m</td>
</tr>
<tr>
<td>c</td>
<td>standard edge distance</td>
<td>m</td>
</tr>
<tr>
<td>C</td>
<td>length of contact between bulk solid material and inclined side of belt</td>
<td>m</td>
</tr>
<tr>
<td>G_r</td>
<td>damping rate</td>
<td>N-s/m</td>
</tr>
<tr>
<td>e</td>
<td>coefficient of restitution</td>
<td>-</td>
</tr>
<tr>
<td>E</td>
<td>material bulk elastic stiffness</td>
<td>N/m³</td>
</tr>
<tr>
<td>F_{ai}</td>
<td>Coulomb passive friction force (internal)</td>
<td>N</td>
</tr>
<tr>
<td>F_{ia}</td>
<td>internal friction force in active stress state</td>
<td>N</td>
</tr>
<tr>
<td>F_{ip}</td>
<td>internal friction force in passive stress state</td>
<td>N</td>
</tr>
<tr>
<td>F_m</td>
<td>total force acting on centre roll (or centre section of belt)</td>
<td>N/m</td>
</tr>
<tr>
<td>F_{ma}</td>
<td>Coulomb vertical weight force in active stress state</td>
<td>N</td>
</tr>
<tr>
<td>F_{mp}</td>
<td>Coulomb vertical weight force in passive stress state</td>
<td>N</td>
</tr>
<tr>
<td>F_{ma}</td>
<td>vertical weight force in active stress state</td>
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<tr>
<td>F_{mp}</td>
<td>vertical weight force in passive stress state</td>
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<td>F_n</td>
<td>normal contact force</td>
<td>N</td>
</tr>
<tr>
<td>F_{n}</td>
<td>contact normal force</td>
<td>N</td>
</tr>
<tr>
<td>F_{n,adh}</td>
<td>adhesive force</td>
<td>N</td>
</tr>
<tr>
<td>F_{na}</td>
<td>total active normal force acting on inclined side of belt</td>
<td>N</td>
</tr>
<tr>
<td>F_{nap}</td>
<td>total passive normal force acting on inclined side of belt</td>
<td>N</td>
</tr>
<tr>
<td>F_{na}</td>
<td>normal component of active force on retaining wall</td>
<td>N</td>
</tr>
<tr>
<td>F_{pi}</td>
<td>Coulomb passive friction force (internal)</td>
<td>N</td>
</tr>
<tr>
<td>F_{s}</td>
<td>tangential or shear force</td>
<td>N</td>
</tr>
<tr>
<td>F_{sw}</td>
<td>force on wall or inclined side of belt during active stress state</td>
<td>N/m</td>
</tr>
</tbody>
</table>
$F_{sn}$ total normal force acting on wall or inclined side of belt (N/m)

$F_{sna}$ normal force on wall or inclined side of belt during active stress state (N)

$F_{sup}$ normal force on wall or inclined side of belt during passive stress state (N)

$F_{zp}$ force on wall or inclined side of belt during passive stress state (N/m)

$F_s' \text{ tangential force value at last direction reversal or previous time step}$ (N)

$F_{wa}$ resulting active force acting on the retaining wall (N)

$F_{wp}$ resulting passive force acting on the retaining wall (N)

$F_z$ total force acting on inclined side of belt acting over one idler pitch (N)

$F_{zn}'$ force per unit length on inclined side in active stress state (N/m)

$F_{zp}'$ force per unit length on inclined side in passive stress state (N/m)

$G$ shear modulus (N/m$^2$)

$h$ average depth or height of bulk solid material on conveyor belt (m)

$H$ height of bulk solid material at location considered (m)

$h_b$ half of trough depth or height (m)

$H_o$ initial stream height at location considered (m)

$h_f$ fall or drop height (m)

$h_s$ height or depth of material in contact with skirt plates (m)

$h_t$ height or depth of belt trough under skirts (m)

$I_r$ equivalent moment of inertia (kg·m$^2$)

$k$ spring stiffness (N/m)

$k^1$ distance between parallel parabolic curves (m)

$K_{1a}$ active stress pressure factor (-)

$K_{2a}$ active stress pressure factor (-)

$K_{1p}$ passive stress pressure factor (-)

$K_{2p}$ passive stress pressure factor (-)

$K_{1e}$ effective (total) pressure factor (-)

$k_{H2}$ Hertzian spring stiffness (N/m)

$k_s$ normal stiffness (N/m)

$k_{NL}$ loading spring stiffness (N/m)

$k_{NU}$ unloading spring stiffness (N/m)

$k_r$ rolling stiffness (N/m)

$k_s$ tangential or shear stiffness (N/m)

$K_s$ sag ratio (-)

$K_V$ vertical to normal pressure ratio (-)

$L$ particle size (m)

$L_b$ acceleration distance (m)

$L_{SL}$ length of contact between bulk solid and inclined side of belt (m)

$L_t$ length of the transition (m)
mass of particle (kg)

effective mass (kg)

percentage of bulk solid material cross-section located above centre idler roll (%)

pressure due to weight of wedge only, bound by, $\theta_a$ (Pa)

pressure due to weight of wedge only, bound by, $\theta_c$ (Pa)

pressure due to weight of wedge only, bound by, $\phi_i$ (Pa)

$P_{ma}$ average active normal pressure (Pa)

$P_{nsa}$ total active normal pressure (Pa)

$P_{np}$ total passive normal pressure (Pa)

impact pressure (Pa)

$q_m$ bulk solid material mass per metre (kg/m)

$Q_m$ throughput (kg/s)

R radius of curvature (m)

$R^*$ effective radius (m)

$R'$ equivalent radius (m)

s distance from initial location or point considered (m)

$s_{adh}$ adhesive distance at which adhesive force acts (m)

V velocity at location considered (m/s)

$v_b$ conveyor belt velocity (m/s)

$v_e$ exit velocity (m/s)

$V_{ex}$ horizontal component of exit velocity (m/s)

$V_{ey}$ vertical component of exit velocity (m/s)

$V_i$ velocity immediately prior to impact (m/s)

$V_n$ normal component of velocity (m/s)

$V_o$ initial velocity at location considered or velocity immediately following impact (m/s)

$V_{amin}$ minimum velocity to achieve required feeding rate (m/s)

$V_p$ component of velocity parallel to belt travel direction (m/s)

$V_s$ velocity at a distance from loading or initial point (m/s)

$V_x$ horizontal velocity (m/s)

$V_y$ vertical velocity (m/s)

t_{crit}$ critical time step (s)

$T_r$ total rolling resistance torque (N-m)

$T_{rk}$ $r,t+\Delta t$ mechanical spring torque (N-m)

$T_{rv}$ $r,t+\Delta t$ viscous damping torque (N-m)

$U_{nl}$ non-dimensional cross section area factor (-)

$W_a$ abrasive wear parameter (Pa-m/s)

$W_s$ skirt inside opening width (m)

$Z$ height of wall from base (m)
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_f$</td>
<td>vertical height of bulk solid material at idler junction</td>
<td>(m)</td>
</tr>
<tr>
<td>$z_0$</td>
<td>distance between particles</td>
<td>(m)</td>
</tr>
<tr>
<td>$Z_w$</td>
<td>height of inclined wall from base</td>
<td>(m)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>nominal inclination of conveyor belt</td>
<td>(degrees)</td>
</tr>
<tr>
<td>$\alpha_0$</td>
<td>angle of wrap from vertical</td>
<td>(degrees)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>idler roll inclination, inclination of side wall or belt</td>
<td>(degrees)</td>
</tr>
<tr>
<td>$\beta_{nPC}$</td>
<td>normal damping factor</td>
<td>(-)</td>
</tr>
<tr>
<td>$\beta_{tPC}$</td>
<td>tangential damping factor</td>
<td>(-)</td>
</tr>
<tr>
<td>$\chi$</td>
<td>chute width convergence angle</td>
<td>(degrees)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>effective angle of internal friction</td>
<td>(degrees)</td>
</tr>
<tr>
<td>$\delta_{max}$</td>
<td>maximum overlap during contact</td>
<td>(m)</td>
</tr>
<tr>
<td>$\delta_n$</td>
<td>normal overlap</td>
<td>(m)</td>
</tr>
<tr>
<td>$\delta_n'$</td>
<td>normal relative velocity</td>
<td>(m/s)</td>
</tr>
<tr>
<td>$\delta_o$</td>
<td>relative impact speed</td>
<td>(m/s)</td>
</tr>
<tr>
<td>$\delta_{res}$</td>
<td>residual overlap</td>
<td>(m)</td>
</tr>
<tr>
<td>$\delta_s$</td>
<td>tangential or shear overlap</td>
<td>(m)</td>
</tr>
<tr>
<td>$\delta_s'$</td>
<td>tangential relative velocity</td>
<td>(-)</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>inclination of conveyor belt transition</td>
<td>(degrees)</td>
</tr>
<tr>
<td>$\varepsilon_1$</td>
<td>effective inclination of outer belt edge</td>
<td>(degrees)</td>
</tr>
<tr>
<td>$\varepsilon_2$</td>
<td>belt plan transition divergence</td>
<td>(degrees)</td>
</tr>
<tr>
<td>$\epsilon_n$</td>
<td>normal coefficient of restitution</td>
<td>(-)</td>
</tr>
<tr>
<td>$\epsilon_{res}$</td>
<td>particle restitution coefficient</td>
<td>(-)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>kinematic angle of internal friction</td>
<td>(degrees)</td>
</tr>
<tr>
<td>$\phi_i$</td>
<td>internal angle of friction</td>
<td>(degrees)</td>
</tr>
<tr>
<td>$\phi_w$</td>
<td>wall friction angle</td>
<td>(degrees)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>conveyor surcharge angle</td>
<td>(degrees)</td>
</tr>
<tr>
<td>$\lambda_a$</td>
<td>active stress state pressure factor</td>
<td>(-)</td>
</tr>
<tr>
<td>$\lambda_p$</td>
<td>passive stress state pressure factor</td>
<td>(-)</td>
</tr>
<tr>
<td>$\mu$</td>
<td>coefficient of friction</td>
<td>(-)</td>
</tr>
<tr>
<td>$\mu_1$</td>
<td>friction coefficient between bulk solid material and belt</td>
<td>(-)</td>
</tr>
<tr>
<td>$\mu_2$</td>
<td>friction coefficient between bulk solid material and skirts</td>
<td>(-)</td>
</tr>
<tr>
<td>$\mu_{adh}$</td>
<td>adhesive fraction coefficient</td>
<td>(-)</td>
</tr>
<tr>
<td>$\mu_d$</td>
<td>dynamic coefficient of friction</td>
<td>(-)</td>
</tr>
<tr>
<td>$\mu_E$</td>
<td>equivalent coefficient of friction</td>
<td>(-)</td>
</tr>
<tr>
<td>$\mu_p$</td>
<td>coefficient of particle/particle sliding friction</td>
<td>(-)</td>
</tr>
<tr>
<td>$\mu_r$</td>
<td>coefficient of rolling friction</td>
<td>(-)</td>
</tr>
<tr>
<td>$\mu_s$</td>
<td>static coefficient of friction</td>
<td>(-)</td>
</tr>
</tbody>
</table>
\( \mu_{\text{shear}} \) coefficient of friction due to internal shearing (-)
\( \mu_w \) coefficient of particle/wall friction or bulk solid material to wall friction (-)
\( \nu \) Poisson’s ratio (-)
\( \nu_n \) normal damping coefficientv (-)
\( \nu_s \) tangential damping coefficient (-)
\( \theta \) angular position (degrees)
\( \theta_1 \) angle of impact (degrees)
\( \theta_a \) active state slip plane angle (degrees)
\( \theta_c \) angle to centre and top of material pile on belt (degrees)
\( \theta_p \) passive slip plane angle (degrees)
\( \theta_R \) angle of repose (degrees)
\( \rho \) bulk density (kg/m\(^3\))
\( \rho_\text{p} \) particle density (kg/m\(^3\))
\( \sigma_i \) consolidation pressure (Pa)
\( \tau \) shear stress (Pa)
\( \omega \) rotational velocity (rad/s)
\( \omega_{\text{rel}} \) relative angular velocity of the two particles in contact (rad/s)
\( \psi \) spoon or chute cut-off angle (to horizontal) (degrees)
\( \zeta \) damping ratio (-)
Abstract

In the field of bulk solid materials handling, belt conveyor systems play an important role in the productivity of mines, ports and processing plants. A vast range of bulk solid materials are handled and understanding the influence of their characteristics on the interactions throughout the system is critical to ensuring performance and reliability.

Integral areas of belt conveying systems include transportation, discharge, re-direction, transfer, loading and acceleration of bulk solid materials. In all of these areas, bulk solid interactions occur within the material itself, with the conveyor belt and the encompassing environment.

To date, techniques used to investigate bulk solid material interactions in belt conveying systems have been based on classical engineering mechanics and conventional continuum “lump” based analyses. These analyses traditionally rely on bulk solid material properties which are extracted from standardised tests and measurement procedures. Based on a determined set of parameters describing the characteristic nature of the bulk solid material handled, existing theoretical approaches are presented and applied in the analysis of the belt conveyor system. Testing procedures for the determination of these parameters are also presented.

In contrast, Discrete Element Modelling (DEM) analysis focuses on the intrinsic constituents (individual particles) that effectively make up the “lump” and allows a more detailed investigation into such interactions. Due to the dissociation between existing standardised tests and measurement of parameters required for DEM analysis, calibration techniques are developed from which a set of characterising parameters of the modelled bulk solid material may be obtained. A number of calibration tests implemented for the selection of characterising modelling parameters are applied and presented with results compared to laboratory and full scale tests.

Following selection of DEM parameters, an in-depth study of each area of bulk solid interactions in belt conveying systems is presented. The work embodied in this thesis will investigate and evaluate the existing theoretical approaches through the application of continuum mechanics and DEM, both individually and in combination. Laboratory experiments, full scale tests and site observations are used to validate the analyses presented.
Chapter 1 - Introduction

A typical belt conveying system consists of a number of important areas in relation to the bulk solid material movement. These areas, illustrated Figure 1-1, include loading, acceleration, transportation, discharge, re-direction and transfer and may be summarized as follows:

1. Transfer chute flow interactions and acceleration from one conveyor to another
2. Loading/acceleration zone (bulk solid material discharging from the loading chute and being accelerated up to the belt speed of the receiving belt) interactions
3. Bulk solid and conveyor belt interaction (as the granular material is transported on the conveyor belt)
4. Bulk solid interactions in the transition zone (immediately prior to discharge) and trajectory discharge (as material is discharged)

Each of the abovementioned areas provides an opportunity for particle/particle and particle/boundary type interactions which form the basis for the work presented in this thesis.

Thus far, the focus of the analysis of these systems has predominantly relied on classical engineering mechanics and continuum “lump” parameter analysis. Such analyses are based on readily obtainable bulk solid material flow properties from standardised tests. From these test results, theoretical approaches have been developed and a definition of design criteria for belt conveying system design in relation to bulk material interaction has been determined. Without the groundbreaking work of Behrens [5], Krause and Hettler [64], Spaans [90] and Roberts [80] it would not be possible to advance this knowledge and select specific components of this area of research to further investigate.

Research into the conveying of bulk solid materials has thus far focused on the quantification of resistances occurring along the length of a running conveyor which are of greatest influence. These resistances are termed the main resistances and include belt and bulk solid flexure resistance, rotational resistance of the idler rolls and the indentation rolling resistance of the conveyor belt. A good understanding of these resistances has been attained; however, emphasis on bulk solid material properties has not been investigated in great depth at the particle level.
To date, the analytical analysis of discharge trajectories and transfer chute flow interactions has been based on two dimensional accelerated flow conditions. Through significant progress, this approach may also be extended to analyse flow in three dimensions and an overview of transfer functionality can be obtained within a rapid timeframe. However, as material characteristics vary further from idealized free flowing materials and transfer chute geometries diverge from single, straight and ninety degree transfers, existing analysis techniques are becoming limited in their application.

In contrast, Discrete Element Modelling (DEM) was first developed and applied over 40 years ago by Cundall [18], and while this type of analysis has been restrained by computing power, we are now at a stage where analysis of real life practical systems is more thorough and advanced. Complex systems and systems consisting of a similar number of particles approaching those found in reality (in certain applications) are able to be analysed with good accuracy and in timeframes that are acceptable to researchers and industry.

Common bulk solid materials which are handled in belt conveying systems can vary profoundly in characteristics and include coal, iron ore, bauxite, gold ore, alumina, potash and wheat. Typical particle size diameters associated with these materials can range from ultra-fine particles, measured in microns, up to 400mm lumps associated with primary/secondary type crushed hard rock. However, it is not uncommon for a typical bulk solid material to consist of a combination of large and fine particles, making modeling the entire range of particles within the particle size distribution of a given sample more difficult, and often not possible, particularly at high throughput.

As an example of the difficulty in simulating a typical bulk solid material, consider coal conveyed on a 900mm wide belt at a throughput of 800t/h and a belt speed of 3.4m/s with a bulk density of 800kg/m$^3$ and a particle density of 1500kg/m$^3$. The number of 10mm diameter spherical particles required to fill the corresponding cross section for a 1.0m longitudinal length on the conveyor belt is in excess of 83,000 particles. For a diameter of 5mm the number of spherical particles required is in excess of 660,000 particles.

In stark contrast, typical conveying throughputs of various coal ship loaders at the port of Newcastle, Australia are in the order of 6000-8500t/h, being transported on conveyor belts which are 2000mm wide and running at approximately 6.0m/s. Other facilities include those handling iron ore at 20,000t/h in South America and belt conveyor systems in German lignite open pit mines with mass flow rates up to 40,000t/h. These numbers clearly indicate the scale of the systems which are being modeled.

Consequently, with the advantages and disadvantages of the different analysis approaches evident, it is important to utilize the entire array of analysis and characterisation techniques available in order to obtain a thorough understanding of the bulk solid interactions in belt conveying systems. Taking each of these separate components into consideration, the flow summary diagram below shows the approach that will be implemented in this work to achieve this aim.
Figure 1-2 - Bulk Solid Interactions in Belt Conveying Systems

The behaviour of bulk solid materials during transportation on a belt conveyor depends on a number of bulk solid material properties and system parameters. These include particle size distribution, moisture content, angle of repose, bulk density, friction between the bulk solid material and conveyor belt and also the internal friction and internal strength of the bulk solid material handled. Furthermore, properties of the belt conveyor such as idler roll inclination angle, belt stiffness, belt sag and belt inclination angle also play a significant role in influencing how a bulk material behaves on a conveyor belt. The relative interaction of these parameters governs the behaviour of the bulk solid material at any location in a belt conveying system.

The purpose of this research is to provide insight into bulk material behaviour and the development of design techniques to model bulk solid interactions in belt conveying systems. The two main areas of research are:

1. Predicting bulk solid material behaviour and cross sectional shape during loading, transportation and discharge, and;
2. To investigate the interaction between the bulk solid material and the conveyor belt during transportation

Chapter 2 outlines existing theoretical approaches in the analysis of bulk solid material interactions in the field of belt conveying. The main resistances of belt conveyors are discussed and an overview of different components which make up such systems is presented. Emphasis is placed on the component of the main resistance dealing with the interaction between the bulk solid material and the conveyor belt, material flexure resistance. Due to the difficulty in isolating the bulk solid material and the conveyor belt flexure, research into bulk solid material flexure resistance has thus far been limited to measuring reaction loads.
on idler rolls. The research contained herein aims to isolate and solely focus on the influence of the bulk solid material reaction loads on the belt. Also presented in Chapter 2 is an overview of the existing theoretical approaches regarding analysis of bulk solid interactions during discharge and subsequent flow through transfer chutes.

Chapter 3 presents an overview of the Discrete Element Modelling (DEM) normal, tangential force and rolling friction contact models including parameters describing contacts between individual particles and boundaries. Particle parameters referred to and used in the analysis of subsequent chapters are also presented.

An overview of existing standardised tests and procedures for the determination of bulk solid material flow properties is presented in Chapter 4. Experimental test results for a number of bulk solid materials which form the basis for analysis using existing theoretical methods are also presented. Similarly, the selection of DEM simulation parameters through a set of developed calibration tests is undertaken in view of the experimental test results obtained. Particle parameters selected for DEM analysis of subsequent chapters are specified.

Chapter 5 presents a comparison of experimental laboratory test results on a conveyor belt simulation test apparatus to analysis using existing theory and DEM simulations. Evaluation of results is presented with a focus on bulk solid material transverse behaviour, cross sectional shape and loading profile exerted on the belt during opening and closing of the troughed shape. Investigation of DEM parameters and their influence on exerted loads is also presented.

Results from on-site experimental tests on a re-circulating belt conveyor facility are compared to existing theoretical analysis in Chapter 6. To establish the deflection profile of the belt exhibited in reality, 3D laser scans of the facility were performed with varying belt sag. The opening and closing cycle of the belt cross section was modelled using DEM based on the belt deflection profile obtained and results are compared to experimental measurements and existing theory. In a similar manner to Chapter 5, investigation of the influence of parameters on loads exerted by the bulk solid material on the belt is also presented.

In Chapter 7, bulk solid material behaviour and the geometry of the cross section in the area of the last idler set to discharge is examined, modelled and compared to the existing theory. The influence of material behaviour on discharge trajectories is investigated and the transverse spread of the bulk solid material is modelled with DEM and compared to a theoretical approach. Results will show that the existing theory is limited in constraining the analysis to 2 dimensions and a new approach provides the extension of the analysis to 3 dimensions.

Chapter 8 provides an overview of the practical implementation of existing theoretical analysis and DEM to bulk solid material interactions during flow through transfer chutes. The application of both the
continuum “lump” parameter model in combination with DEM is presented via two case study examples. Analysis of bulk solid material and belt interaction during loading from the transfer chute and acceleration is also investigated. For both case study examples, outcomes of both approaches are compared to observations from site.

Conclusions from the work contained within this thesis including a summary of important results obtained and correlation to theoretical analysis of bulk solid interactions in belt conveying systems are presented in Chapter 9. Also presented is a concise indication of further application and work that should be undertaken.
Chapter 2 - Overview of Existing Theoretical Approaches

The existing theoretical approaches available for the analysis of interactions in belt conveying systems span across a number of different areas including conveying, discharging, re-directing and loading of bulk solid materials. For each of these areas, an overview of the key concepts and parameters used in the analysis of subsequent chapters is presented.

A brief overview of the main resistances of belt conveyors is presented from which bulk solid flexure resistance is explored in greater detail. Earth pressure theory from soil mechanics literature which forms the founding principle of this resistance is presented along with its application to belt conveying. The theoretical approach to analysing the geometrical shape of bulk solid material cross section during conveying and at discharge is also presented including the model for plotting discharge trajectory.

An overview of the continuum analysis approach used to assess bulk solid material interactions during the re-direction process through the application of transfer chutes is presented including bulk solid material parameters required for analysis. Also presented is the theoretical analysis approach to loading and acceleration of the bulk solid material from the transfer chute and onto the conveyor belt.

2.1 Bulk Solid and Conveyor Belt Interaction

2.1.1 Main Resistances

According to ISO 5048 [50] and DIN22101 [21], the motion resistance of belt conveyors can be broken down into five groups:

1. Main Resistances
2. Secondary Resistances
3. Special Main Resistances
4. Special Secondary Resistances
5. Slope Resistance

Main and secondary resistances occur on all belt conveyors, while special resistances are only present in certain types of installations. Main resistances are of a continuous nature during operation of the belt conveyor whereas secondary resistances are present in local areas. Slope resistances are dependent on the inclination or declination of the conveyor and occur either continuously along the entire length of the conveyor or arise in specific areas.

The main resistances of belt conveyors include the rotational resistance of the idler rolls, belt advancement resistance from the impression of idler rolls into the belt and recurring flexing of the belt and the bulk solid material, and can be summarized as:
1. Conveyor belt flexure resistance
2. Bulk solid flexure resistance
3. Rotating resistance of idler rolls (bearings and seals)
4. Conveyor belt indentation rolling resistance

The main component of research regarding the conveying of bulk solid materials in this thesis focuses on
the resistance which is influenced by bulk solid material interaction, i.e. bulk solid material flexure
resistance. Bulk solid material flexure resistance is extremely difficult to isolate and measure individually
in the case of a troughed conveyor belt. General practice to date has been to measure the total resistance
to motion and subsequently back calculate by subtracting other resistance components such as idler roll
indentation rolling resistance and idler rotating resistance. The remaining resistance consists of bulk solid
flexure resistance and belt flexure resistance of which the bulk solid flexure resistance is typically the
greatest magnitude, Spaans [90]. It is the investigation of the bulk solid flexure resistance which is the
focus of the research presented herein. The work presented is a continuation of prior examination by Ilic
et al. [42 - 49].

2.1.2 **Bulk Solid Material Flexure Resistance**

Investigation of bulk solid and conveyor belt interaction and the influence of bulk solid material
properties on the motion resistance of belt conveyors have previously been undertaken by a number of
authors including Spaans [90], Behrens [5], Roberts [83] and Wheeler [106]. These investigations have
typically consisted of classical theoretical analysis techniques and direct experimental measurements or a
combination of the two.

Spaans [90] investigated bulk solid material flexure resistance as a consequence of the sag of the belt
based on Krause and Hettler’s [64] approach. In the investigation, it was assumed that during closing (at
the idler set) a passive stress state is induced in the bulk solid material, while during opening (up to point
of maximum belt sag), an active stress state is induced in the bulk solid material cross section.

Wheeler [106] investigated bulk solid material flexure resistance by applying orthotropic plate mechanics
with a finite difference approach. However, rather than calculating the resultant normal force acting on
the conveyor belt due to the induced stress state, the analysis implemented a pressure distribution over the
surface area of the conveyor belt. The normal forces acting on nodes of a finite mesh were also calculated
based on Krause and Hettler’s theory [64].

Previous studies regarding energy losses associated with belt sag have also been undertaken by Mustoe
and Bin [70] by modelling sag energy losses using DEM, while Nordell [72] implemented a dashpot
(viscoelastic) analogy in the calculation of what is referred to as material “trampling loss”. The latter
states that once the conveyed bulk solid material has taken a shape, work is required in order for that
shape to change. As the belt travels from one idler set to another, the bulk solid material is forced into changing from a consolidated shape at the idler set to a relaxed shape in between the idler sets. Continuous work leads to a gradual loss in compression and in relaxation and the bulk solid material approaches a steady state. Some materials may consolidate with continued working or disturbance, thereby altering the dashpot properties. Energy losses due to trampling or bulk solid flexure become more severe when belt tension is low, due to higher belt sag between idler sets. Both of these studies indicate a direct increase in motion resistance (or power requirement) with an increase in the belt sag.

Harrison and Teo [36] indirectly obtained coefficients of rolling and material flexure resistance of belt conveyors based on the measured pulling force with the application of an inverted elastic bed apparatus. However, the relationship between the coefficients of material flexure resistance to bulk solid material properties was not investigated.

Hager and Hintz [35] analysed a wide range of belt conveyor systems and estimated the average contribution of each of the main resistance components, see Figure 2-1. That research indicated that for 1.0km long belt conveyor, the motion resistance attributed to the flexure resistance of bulk solid material is in the order of 18%.

![Figure 2-1 - Main Resistance Components from Hager and Hintz [35]](image)

More recently, Reicks [77] indicated that the losses attributed to the bulk solid flexure may exceed this value. During the study it was shown that for an 1800mm wide, fabric conveyor belt handling coal a loss of over 30% was indicated.

General comments regarding parameters which influence material flexure resistance can be found in Mulani [69]:

- Higher belt tension (lower sag) decreases the material flexure resistance
- Bulk solid materials with rough surface and/or interlocking shapes will give rise to higher friction and a higher resistance to belt motion. Smooth/rounded lumps or particles will produce a lower flexure resistance.
The quantity as well as magnitude of bulk solid material flexure resistance is proportional to the weight of the material (bulk density and solids/particle density).

Grabner, et al [29] conducted experimental tests on a 650mm wide belt apparatus with idler trough stations to measure normal forces on idler rolls due to a loaded belt (carrying sand), in a number of different configurations including: empty, trough-in, trough-out sections, horizontal curves and a straight travelling belt. For the straight belt, tests were conducted with a trough angle of 30°, and a tensile force of 10kN. An excerpt from this work showing the measured pressure profile on a 3-roll idler set from the weight of the bulk solid material and the belt is shown in Figure 2-2. These data show the normal force decreasing towards the edge of the belt. In Figure 2-2 it can be observed that the force acting on the inclined side rolls and the centre roll is approximately 30% and 70% of the total force respectively.

![Figure 2-2 - Measured Idler Loads from Grabner et al [29]](image)

Behrens [5] investigated the influence of a range of different factors on the “pressing resistance” (load resistance) of heavy belt conveyor systems. Studies were conducted to determine the influence of belt tension, belt and material weight, troughing angle, belt speed and idler spacing on the load resistance.

A 2200mm wide steel cord belt (ST2000) was used with a reinforced supporting frame consisting of offset idler supports mounted on a rail car which was driven back and forth. Tests were conducted with gravel (6-8mm size fraction) and also water. Normal force measurements on the idler rolls due to the bulk solid material and the belt indicated that the normal force on the middle idler roll could be accurately given by the weight force of the volume of material located directly above it. The normal force acting on the side idler roll was found to be greater than the weight force of the material volume.

Both the loads on the middle idler and side idler rolls proved greater at higher troughing angle. For testing with gravel, the measured normal forces on both side idler rolls, due to the bulk solid material and the belt were found to be 1.2 (for a 30° trough) and 1.9 (for a 45° trough) times greater than simply the weight of the material carried by the side idler rolls. The load resistance on an idler assembly of a loaded conveyor
belt was listed as composing of indentation rolling resistance, vibration bending resistance of the belt (i.e. belt flexure resistance) and the pressing resistance of the material (bulk solid flexure resistance).

With increased belt tension, sag deformation of the belt between the middle and side idler rolls was found to reduce, and the belt and bulk solid flexure resistance also reduced significantly. Clearly the nature of the experiment did not permit the loads on the idler rolls attributed to the belt and bulk solid material to be distinguished.

The research presented in Chapter 5 and Chapter 6 differs to the work of Behrens [5] and Grabner et al [29] by isolating and focusing solely on the determination of the bulk solid material flexure resistance via experimental and simulation techniques. The work presented herein regarding bulk solid material flexure resistance is related solely to the load exerted by the bulk solid material on the conveyor belt.

### 2.1.3 Bulk Solid Material Burden Profile on Belt

The behaviour of bulk solid material on a moving belt is influenced by the conveyor belt itself. As the conveyor belt advances over each carrying side idler roll set, bulk solid material being conveyed is agitated and the load profile of the material on the belt fluctuates. The variation in load profile is influenced by the layout of the conveyor, idler trough angle and configuration, belt inclination, belt tension and therefore sag, velocity and vibrations that arise from the movement of the belt and supporting idler rolls.

According to Roberts [83, 84] the amount of settling that occurs during transportation in typical belt conveying applications is stated to be in the range of 10-15%. The upper surface of the material burden cross section profile will vary from the angle of repose in an unsettled or loose state (during initial loading) to the conveyor surcharge angle in a settled or packed state (following material attaining belt speed and transport distance). In addition to the change in profile, the bulk density will increase by 10-15% and therefore the cross sectional area will reduce by 10-15%. The variation of bulk density in loose and packed states (at different consolidation pressures), the angle of repose, conveyor surcharge angle and the particle size distribution of the conveyed material are all parameters of significant importance. These factors are influenced by the flow property characteristics, the content of fines present and moisture content of the bulk solid material handled.

The influence of induced acceleration due to the vertical displacement of the conveyor belt resulting from the belt sag can be significant at high belt speeds and consolidation pressures under dynamic conditions will be higher than at the load point as noted by Roberts [83]. This is due to the acceleration of the material in the direction normal to the belt as the belt moves from the maximum sag position to the zero sag position at each idler roll set. It is assumed that the consolidation pressure is uniformly distributed throughout the mass and has the maximum value given by:
\[ \sigma_1 = \rho \cdot g \cdot h \left(1 + \frac{a_V}{g}\right) \quad \text{(2.1.1)} \]

Where \( h \) = average height of bulk solid on the conveyor belt and \( \rho \) = bulk density at load point corresponding to \( \sigma_1 \).

Maximum acceleration may be estimated as follows:

\[ a_V = \frac{2\pi^2 \cdot V^2 \cdot K_S}{a_s} \quad \text{(2.1.2)} \]

Where \( V \) = belt velocity, \( K_S \) = sag ratio and \( a_s \) = idler spacing.

As a bulk solid is transported along a belt conveyor, research by Roberts [83], CEMA [6], Waters and Mikka [104] has found that larger particles become more concentrated at the top of the bed and fine particles percolate to the belt surface. It is thought that most of this segregation occurs within a portion of the total conveyor length. It was also shown that beyond a certain conveying distance, bulk solid material will attain a steady state cross-sectional profile. For the purpose of this research, a parabolic cross-sectional profile of bulk solid material on the belt is assumed in correlation to previous work by Roberts [86] and ISO 5048 [50] unless noted otherwise. A typical bulk solid material cross section profile on a conveyor belt is as shown in Figure 2-3:

![Figure 2-3 - Bulk Solid Material Cross Section Profile](image)

The contact perimeter, \( b \), is given by:

\[ b = B + 2C \quad \text{(2.1.3)} \]

Where \( B \) is the length of contact between the bulk solid material in the horizontal section, and \( C \) is the length of contact on the inclined side of the belt

CEMA [6] calculates the standard edge distance, \( c \), for the determination of contact perimeter, \( b \), based on 100% material cross section loading by:

\[ c = 0.055 \cdot B_\omega + 22.86 \quad \text{(2.1.4)} \]
\[ b = 0.89 \cdot B_w - 45.72 \] (2.1.5)

Where \( B_w \) is the width of the belt (mm)

The contact perimeter (or usable width of the belt), \( b \), (m) is determined using the following criteria by ISO 5048 [50] and DIN22101 [21]:

For \( B_w \leq 2m \)
\[ b = 0.9 \cdot B_w - 0.05 \] (2.1.6)

For \( B_w > 2m \)
\[ b = B_w - 0.25 \] (2.1.7)

The conveyor surcharge angle, \( \lambda \), of a material is the angle to the horizontal which the surface of the material assumes when the material is deemed to have settled on a moving conveyor belt (previously defined in Figure 2-3). CEMA [6] states that this angle is usually 5 degrees to 15 degrees less than the angle of repose, though in some materials it may be as much as 20 degrees less. According to ISO 5048 [50], the conveyor belt surcharge angle can be approximated by 75% of the angle of repose.

Colijn [17] gives the following empirical relationship for the surcharge angle on a conveyor belt, \( \lambda \), based on the angle of repose, \( \theta_R \), of the material and the idler inclination angle, \( \beta \):

\[ \lambda = 1.11 \cdot \theta_R - (0.1 \cdot \beta + 18^\circ) \] (2.1.8)

The total cross-sectional area, \( A \) of material on the belt is described by ISO 5048 [50] as:

\[
A = \left[ B + (b - B) \cdot \cos(\beta) \right] \cdot \frac{\tan(\lambda)}{6} + \left[ B + \frac{(b - B)}{2} \cdot \cos(\beta) \right] \left[ \frac{(b - B)}{2} \cdot \sin(\beta) \right]
\] (2.1.9)

CEMA [6] gives the cross-sectional area at the idler roll set to be:

\[
A_{SC} = 2 \cdot B_w^2 \left[ A_{SCI} + \left( \frac{B}{2} \cdot (L_S) \cdot \sin(\beta) \right) + L_S \cdot \frac{\sin(\beta) \cdot \cos(\beta)}{2} \right]
\] (2.1.10)

Where:

\[
A_{SCI} = \left( \frac{B}{2 \cdot \sin(\lambda)} + \frac{\cos(\beta) \cdot L_S}{\sin(\lambda)} \right)^2 \left( \frac{\lambda \cdot \pi}{360} \cdot \frac{\sin(\lambda) \cdot \cos(\lambda)}{2} \right)
\] (2.1.10a)
2.1.3.1 Cross Sectional Profile by Roberts

The cross sectional profile is typically assumed to be parabolic as presented in ISO 5048 [50] and also Roberts [83]. The latter presents the profile as a function of the non-dimensional cross-sectional area factor, U, belt surcharge angle, \( \lambda \), idler inclination and the ratio of the length of material in contact with the inclined side of the belt, \( C \) and the length of contact between the material on the horizontal portion of the belt, which for a given belt width, can be approximated equal to the horizontal idler roll length, B.

\[
A = U \cdot b^2 \tag{2.1.11}
\]

Where \( U \) = non-dimensional cross-sectional area factor and \( b \) = contact perimeter

For a three-roll idler set, the cross sectional area factor is given by

\[
U = \frac{1}{(1+2 \cdot r)^2} \left[ r \cdot \sin(\beta) + \frac{r^2}{2} \sin(2\beta) + \frac{\tan(\lambda)}{6} \left[ 1 + 4 \cdot r \cdot \cos(\beta) + 2 \cdot r^2 \cdot (1 + \cos(2\beta)) \right] \right] \tag{2.1.12}
\]

Where \( r = \frac{C}{B} \)  
\( \beta \) = troughing angle  
\( \lambda \) = surcharge angle

The overall height, \( H \) of the bulk solid material on the belt at the idler set is given by:

\[
H = C \cdot \sin(\beta) + (B + 2 \cdot C \cdot \cos(\beta)) \cdot \frac{\tan(\lambda)}{4} \tag{2.1.13}
\]

2.1.4 Earth Pressure Theory

During transportation between successive idler sets along a belt conveyor, transverse stresses are induced in the bulk solid material due to the constant opening and closing of the troughed shape of the belt. The corresponding loads exerted by the bulk solid material on the belt influence the motion resistance of the conveyor, energy consumption (belt tension selection) as well as belt wear. This so called “bulk flexure” also influences loads on the idlers ultimately influencing idler life. Due to bulk solid material being transversely supported along the length of the conveyor by the belt sides, the approach in examining bulk flexure has been analogous to the application of soil mechanics with respect to retaining wall structures. An overview is now presented.

In granular materials, a small applied force produces a small elastic deformation. When forces reach a critical value, the bulk solid material will divide itself into two blocks. If elastic deformations are ignored, the concept of a rigid-plastic failure mode is introduced. It is assumed that the material divides itself into two rigid blocks, separated by a narrow plastic zone, with the plastic zone commonly about ten particle
diameters in width, which is often narrow compared with the typical dimensions of the system. Therefore, it is usually sufficient to assume that the plastic zone is a plane of negligible width and it is referred to as the yield, slip or failure plane, Nedderman [71].

### 2.1.4.1 Theory of Active Stress State

Rankine investigated the stress conditions corresponding to states of plastic equilibrium that can develop simultaneously throughout a semi-infinite mass of soil acted on only by gravity. These are known as Rankine states of plastic equilibrium (Terzaghi and Peck [93] and Terzaghi [94], Nedderman [71]). When a wall is being pushed outwards by a material which is failing internally it is termed to be in a Rankine “active stress” state. The active stress is the stress on a wall that is moving outward and the slip within the material is in a downward and outward direction as illustrated in Figure 2-4. For the active Rankine state, shear failure occurs along a slip plane angle, $\theta_a$, to the horizontal given by:

$$\theta_a = \frac{\pi}{4} + \frac{\phi_i}{2} \tag{2.1.14}$$

Where $\phi_i$ is the internal angle of friction of the bulk solid material

![Figure 2-4 - Rankine’s Active State](image)

The corresponding active stress pressure factor, $K_{1a}$, is given by:

$$K_{1a} = \frac{1 - \sin(\theta_a)}{1 + \sin(\theta_a)} \tag{2.1.15}$$

The total active force, $F_{sa}$, on the wall is given by:

$$F_{sa} = \frac{1}{2} \cdot K_{1a} \cdot \rho \cdot g \cdot Z^2 \tag{2.1.16}$$

And acts at a distance of $Z/3$ from the base of the wall

Reimbert and Reimbert [78, 79] investigated the behaviour of cohesion-less granular materials supported by retaining walls by assuming that maximum thrust on a retaining wall structure occurs when the minimum angle of internal friction is equal to the angle of repose. Based on this assumption, it was shown
that for cohesion-less granular materials the active state shear plane angle to the horizontal can be approximated by:

\[ \theta_a = \frac{\pi}{4} + \frac{\theta_R}{3} \]  

(2.1.17)

Where \( \theta_R \) is the angle of repose

This is illustrated in Figure 2-5 below.

![Figure 2-5 - Reimbert and Reimbert Active State](image)

Additionally, Terzaghi and Peck [93] and Terzaghi [94] state that for cohesive materials supported by a rough retaining wall, the shear slip surface can consist of a curved lower portion near the pivot point of the wall and a planar upper part corresponding to the active state pattern as described by Rankine. The slip surface starts as a curve at the bottom of the wall and blends into a plane towards the top of the mass at an angle that can be described by Eqn. (2.1.14) as illustrated in Figure 2-6.

![Figure 2-6 - Active State for Rough Retaining Wall](image)

The theory of Rankine considers that no shear force acts along the retaining surface, and as such the influence of friction on the wall is not considered.
Coulomb’s earth pressure theory, first published in 1776, considers a wedge of bulk solid between a retaining wall and a failure plane. The theory provides an equilibrium analysis of the forces acting on the wedge at the point of sliding in order to determine the forces acting on the wall. Coulomb derived the orientation of the failure plane as a function of the internal friction of the bulk solid and, from knowing the internal friction and friction between the wall and the bulk solid an equilibrium analysis allows the calculation of the force on the wall. A general solution for non-cohesive bulk solids inclined at a surcharge angle, $\lambda$, in contact with a retaining wall inclined at an angle $\beta$ with the horizontal, is shown in Figure 2-7 below.

The corresponding active stress pressure factor, $K_{2a}$, is given by:

$$K_{2a} = \left[ \frac{\sin(\beta + \phi)}{\sqrt{\sin(\beta - \phi) + \frac{\sin(\beta + \phi) + \sin(\phi - \lambda)}{\sin(\beta + \lambda)}}} \right]^2$$  \hspace{1cm} (2.1.18)

For the active stress state, the resulting force acting on the retaining wall, $F_{wa}$ is given by:

$$F_{wa} = \frac{1}{2} \cdot \rho \cdot g \cdot \frac{1}{\sin^2(\beta)} \cdot Z_w^2 \cdot K_{2a}$$  \hspace{1cm} (2.1.19)

The normal component of the active force, $F_{nwa}$ is given by:

$$F_{nwa} = F_{wa} \cdot \cos(\phi_w)$$  \hspace{1cm} (2.1.20)

2.1.4.2 Theory of Passive Stress State

Conversely to the active state of stress, if the walls are pushed slowly together the stress will rise to what is termed the Rankine “passive stress” state before failure occurs. The passive stress is that which would occur when a wall is moving inward causing inward and upward failure within the bulk solid material, as illustrated in Figure 2-8. Any horizontal stress between these two values will be stable and will cause no slip within the material, Nedderman [71].
For the passive Rankine state, shear failure occurs along a slip plane at an angle, \( \theta_p \), to the horizontal given by:

\[
\theta_p = \frac{\pi}{4} - \frac{\phi_i}{2}
\]  

(2.1.21)

The corresponding passive stress pressure factor, \( K_{1p} \), is given by:

\[
K_{1p} = \frac{1 + \sin(\phi_i)}{1 - \sin(\phi_i)}
\]  

(2.1.22)

The total passive force, \( F_{sp} \), on the wall is given by:

\[
F_{sp} = \frac{1}{2} \cdot K_{1p} \cdot \rho \cdot g \cdot Z^2
\]  

(2.1.23)

And also acts at a distance of \( \frac{Z}{3} \) from the base of the wall.

Coulomb’s method of wedges considers friction at the interface of the bulk solid material and the inclined wall. The passive stress state is shown in Figure 2-9:

The corresponding passive stress pressure factor, \( K_{2p} \), is given by:
For the passive stress state, the resulting force acting on the retaining wall, \( F_{wp} \), is given by:

\[
F_{wp} = \frac{1}{2} \cdot \rho \cdot g \cdot \frac{1}{\sin^2(\beta)} \cdot Z_w^2 \cdot K_{2p}
\]  \hspace{1cm} (2.1.25)

Similarly to the Rankine approach, for both the active and passive stress state, the resultant force, according to Coulomb, is assumed to act at a distance of \( Z_w/3 \) from the base of the wall.

### 2.1.5 **Earth Pressure Theory Applied to Belt Conveyors**

Due to bulk solid material being transversely supported along the length of the conveyor belt by inclined sides of the belt, the approach to examining bulk solid material behaviour on conveyor belts has been based on the aforementioned process of active and passive stress states during the opening and closing of the belt’s troughed shape.

#### 2.1.5.1 **Theory of Mulani**

Mulani [69] investigated the application of a number of different earth pressure classical methods to analyse the opening and closing phenomenon in belt conveyors. The analysis included the Rankine earth pressure theory and Coulomb method with Rebhann’s graphical analysis for a convex top surface rather than a straight inclined top surface.

In the analysis it is assumed that the active stress state acts for 2/3 of the idler spacing while the passive stress state acts over the remaining 1/3. Mulani [69] also assumed that the passive stress state is not entirely developed in the bulk solid material due to insufficient strain during conveying and a value of 85% of the calculated passive pressure factor, \( K_{1p} \), is used. The effective (total) pressure factor for the conveyor is expressed as:

\[
K_E = \frac{2}{3} \cdot K_{1a} + \frac{1}{3} \cdot 0.85 \cdot K_{1p}
\]  \hspace{1cm} (2.1.26)

For a horizontal conveyor, the total normal force (per unit length) due to the bulk solid material on the inclined side is given by:

\[
F_{sn} = \frac{1}{2} \cdot g \cdot \left[ \left( \frac{100 - \rho_c}{100} \right) \cdot q_m \cdot \cos(\beta) + K_E \cdot \rho \cdot Z_i^2 \cdot \sin(\beta) \right]
\]  \hspace{1cm} (2.1.27)
Where $P_c$ is the percentage of bulk solid material cross-section area located above the centre idler roll, $q_m$ is the material mass per metre and $Z_j$ is the height of the bulk solid material at the idler junction.

In the calculation of the Rankine active and passive pressure factors Mulani [69] uses the dynamic repose angle (or conveyor surcharge angle, $\lambda$) rather than the internal angle of friction $\phi_i$ of the bulk solid material. In addition to this discrepancy, a further disadvantage of the application of the Rankine theory is that friction between the bulk solid material and the belt is neglected. Mulani [69] postulates that the contraction and expansion of the bulk solid material during conveying acts to nullify the presence of net shear forces acting along the belt.

2.1.5.2 \textit{Theory of Krause and Hettler}

Krause and Hettler [64] applied the modified Coulomb earth pressure theory to calculate the normal forces acting on the idler rolls of a belt conveyor. The active slip plane angle, $\theta_a$, is illustrated in Figure 2-10 and the passive slip plane angle, $\theta_p$, is illustrated in Figure 2-11. Also shown in Figure 2-10 and Figure 2-11 is the angle from the idler junction to the centre of the top of bulk solid material pile $\theta_c$.

For the active stress state, the shear plane angle $\theta_a$ to the horizontal as a function of the idler inclination angle $\beta$, bulk solid internal friction angle $\phi_i$, belt and bulk solid friction angle $\phi_w$, and conveyor surcharge angle $\lambda$, is given by:
\[ \theta_a = \tan^{-1} \left[ \frac{\sqrt{\lambda \sin(\beta - \phi_w)}}{\sin(\beta + \lambda)} \right] \sin \lambda - \sin \phi \]

\[ \lambda_a = \frac{\sin(\beta + \phi)}{\sqrt{\sin(\beta - \phi_w) + \sin(\phi_w + \phi) \sin(\phi - \lambda)}} \]

Where the active pressure factor \( \lambda_a \) is:

\[ \theta_p = \tan^{-1} \left[ \frac{\sqrt{\lambda_p \sin(\beta + \phi_w)}}{\sin(\beta + \lambda)} \right] \sin \lambda + \sin \phi \]

\[ \lambda_p = \frac{\sin(\beta - \phi)}{\sqrt{\sin(\beta + \phi_w) - \sin(\phi_w + \phi) \sin(\phi + \lambda)}} \]

Similarly for the passive stress state, the slip plane angle \( \theta_p \) to the horizontal is evaluated by:

Where the passive pressure factor \( \lambda_p \) is given to be:

The above equations are valid provided that:

\[ \theta_a \geq \theta, \quad \theta_p \geq \theta, \quad \lambda \leq \phi \quad \text{and} \quad \beta \geq \phi_w \]

The force vector diagrams showing equilibrium of forces for the passive and active stress states are shown in Figure 2-12 and Figure 2-13.
The total normal force per unit length $F_{sn}$, acting on the inclined side due to the bulk solid material is:

$$F_{sn} = \frac{1}{2} \cdot \rho \cdot g \cdot L_s^2 \cdot \frac{(\lambda_a + \lambda_p)}{2} \cdot \cos(\phi_w) \quad (2.1.32)$$

Where the height of the retaining wall, $Z_w$, has been replaced by the length of contact between the bulk solid material and the inclined side of the belt, $L_s$, and the active and passive stress states are assumed to act over one half of the idler spacing.

2.1.5.3 **Sensitivity Analysis of the Krause & Hettler Approach**

The following sensitivity analysis was performed to highlight the influence of material properties on calculated active, $\theta_a$, and passive, $\theta_p$, slip plane angles and also more importantly to highlight the
shortcomings of the passive stress state calculations. Investigation of the condition \( \theta_a \geq \theta_c, \theta_p \geq \theta_c, \lambda \leq \phi \) and \( \beta \geq \phi \) was performed for a 100% loaded belt according to CEMA [6] for a 600mm wide belt. However, from geometry, the same would apply for all belt widths.

General trends obtained from the sensitivity analysis of active and passive stress state slip plane angles indicate that for the active angle determination:

- Increasing the wall friction angle, \( \phi_w \), leads to a decrease in the active state angle, \( \theta_a \)
- Higher belt surcharge angle, \( \lambda \), provides lower values of the active stress state angle, \( \theta_a \)
- For the same wall friction angle, \( \phi_w \), increasing the internal angle of friction, \( \phi_i \), leads to an increase of the active stress state angle, \( \theta_a \) (except when \( \phi_w = \beta \))
- When \( \phi_w = \beta \) (wall friction angle is equal to the idler inclination), the slip plane angle, \( \theta_a \), is equal to the bulk solid internal angle of friction i.e. \( \theta_a = \phi_i \)
- Low internal angles of friction, \( \phi_i \), and high wall friction angles, \( \phi_w \), provide lowest active stress state angle, \( \theta_a \)
- High internal angles of friction, \( \phi_i \), and low wall friction angles, \( \phi_w \), provide highest active stress state angle, \( \theta_a \)

General trends regarding the passive stress state angle indicate that:

- Increasing the wall friction angle, \( \phi_w \), leads to an decrease in the passive stress state angle, \( \theta_p \)
- Higher belt surcharge angle, \( \lambda \), provides higher values of the passive stress state angle, \( \theta_p \)
- For the same wall friction, \( \phi_w \), increasing the internal angle of friction, \( \phi_i \), leads to a decrease in the passive stress state angle, \( \theta_p \) (including when \( \phi_w = \beta \))
- Low internal angles of friction, \( \phi_i \), and high wall friction angles, \( \phi_w \), provide the highest passive stress state slip angle, \( \theta_p \)
- High internal angles of friction, \( \phi_i \), and high wall friction, \( \phi_w \), provides the lowest passive state angle, \( \theta_p \)

Furthermore, as the idler inclination, \( \beta \) decreases, the active stress state angle, \( \theta_a \), decreases and the passive state slip angle, \( \theta_p \), increases. Similarly, as the belt surcharge angle, \( \lambda \), decreases the active stress state angle, \( \theta_a \), increases and the passive stress state angle, \( \theta_p \), decreases. Figures 2-14 to Figure 2-19 below show active, \( \theta_a \), and passive, \( \theta_p \), stress state angles at different internal angles of friction, \( \phi_i \), with variation in the belt surcharge angle, \( \lambda \). Regarding the passive slip plane angle for an internal angle of friction of 30°, from Figure 2-15, it can be observed that the theory by Krause and Hettler would only be valid when the wall friction angle, \( \phi_w \), between the bulk solid material and belt is approximately 25° and lower. At higher internal angles of friction, \( \phi_i \), of the bulk solid material, as shown in Figure 2-17 and Figure 2-19, for Krause and Hettler theory to be applicable, the wall friction angle, \( \phi_w \), must be under 5-
Therefore it must be concluded that Krause and Hettler’s theory is only applicable to free flowing bulk solid materials, with low to moderate internal angles of friction, $\phi_i$, small belt surcharge angles, $\lambda$, and low friction angles between the bulk solid material handled and the belt i.e. wall friction, $\phi_w$.

Figure 2-14 - Krause and Hettler Active Angle Variation – $\phi_i = 30^\circ$

Figure 2-15 – Krause and Hettler Passive Angle Variation – $\phi_i = 30^\circ$
Figure 2-16 – Krause and Hettler Active Angle Variation – $\phi_i = 45^\circ$

Figure 2-17 – Krause and Hettler Passive Angle Variation – $\phi_i = 45^\circ$
The sensitivity analysis suggests that the theory presented by Krause and Hettler is not applicable to the majority of bulk solid materials that would be expected to be handled in everyday practical applications, particularly in the mining and minerals handling industry.

2.1.5.4 Theory of Spaans – Forces on the Rolls of a Troughed Idler Roll Station

Incorporating the work of Krause and Hettler [64], Spaans [90] states that the deformation pattern of the bulk solid material will be governed by either the passive or active stress state. Following the troughed idler station, the belt unfolds, commencing with the shape of the cross section of the troughed idler set and culminates in the curved profile of the unsupported belt surface at around half the idler spacing. When the belt approaches the idler set, a passive state of stress is induced in the bulk material due to the sides closing and as the belt opens upon leaving the idler, an active state of stress is induced in the bulk.
material. The force exerted on the inclined side of the belt, $L_{S_i}$, due to the pressure in the bulk material is calculated both for the passive and active stress state and assumed to act over one half of the idler spacing. For the active state, the normal force per unit length of bulk material on the belt is given by:

$$F_{za} = \left| \frac{\lambda_a \cdot L_{S_i}^2 \cdot \rho \cdot g \cdot \cos \phi_w}{2} \right|$$

(2.1.33)

Where the active state pressure factor, $\lambda_a$, is given by Eqn. (2.1.29) in Section 2.1.5.2.

A passive stress state will occur if the shape of the troughed cross section is reduced and the passive force exerted by the material on the inclined side is given by:

$$F_{zp} = \left| \frac{\lambda_p \cdot L_{S_i}^2 \cdot \rho \cdot g \cdot \cos \phi_w}{2} \right|$$

(2.1.34)

Where the passive state pressure factor, $\lambda_p$, is given by Eqn. (2.1.31) in Section 2.1.5.2.

Both the passive and active stress states of the bulk solid material are assumed to occur over one half of the idler spacing, with passive behaviour in the part approaching the trough and active behaviour in the part following. The total normal force, $F_z$, exerted by the bulk solid material on the inclined side is given by:

$$F_{z} = \frac{a_w}{4} \cdot \rho \cdot g \cdot L_{S_i}^2 (\lambda_a + \lambda_p) \cdot \cos \phi_w$$

(2.1.35)

Where, $a_w$, is the idler spacing or pitch.

On inspection, Eqn. (2.1.35) above is identical to Eqn. (2.1.32) found in Section 2.1.5.2. The pressure distribution is assumed to progress from zero at the intersection of the top of the surface of the bulk solid material burden and outer edges and the belt, up to a maximum value at the junction of the inclined side and horizontal bottom. The pressure surface is assumed to concentrate in an area at a distance of about $L_{S_i}/3$ from the idler junction.

The force on the bottom roll, $F_{m}$, is equal to the total load of the belt and the bulk material over the length of idler spacing, reduced by the vertical components of the forces exerted on the side rolls:

$$F_{m} = \rho \cdot g \cdot a_k \cdot A - 2 \cdot F_{za} \cdot \cos(\beta - \phi_w) - 2 \cdot F_{zp} \cdot \cos(\beta + \phi_w)$$
\[ F_m = \rho \cdot g \cdot a \cdot A - \frac{1}{2} \rho \cdot g \cdot a \cdot L_s^2 \cdot [\lambda_a \cdot \cos(\beta - \phi_w) + \lambda_p \cdot \cos(\beta + \phi_w)] \] (2.1.36)

Where \( A \) is the surface cross section of the bulk material on the belt

Spaans [90] states that the share of flexure resistance of the bulk material in the main resistance is larger if:

- The mass of the bulk solid material per unit length is larger
- The depth of the bulk solid material burden on the belt is larger
- The internal friction of the bulk solid material is higher
- The tensile force of the belt is smaller (greater sag)
- The elasticity modulus of the belt is lower

Research regarding bulk solid flexure resistance presented in this thesis will consist of applying the theoretical approach of Krause and Hettler, Mulani and Spaans to predict loads exerted by a number of bulk solid materials on the conveyor belt during transportation. Theoretical calculations are evaluated against predictions obtained using Discrete Element Modelling (DEM) simulations, experimental laboratory measurements and on-site tests. Furthermore, investigation of the influence of bulk solid material properties on loads exhibited is also undertaken. Finally, predictions of the bulk solid material cross sectional behaviour during conveying using the theoretical approach of CEMA and Colijn presented in Section 2.1.3 is also assessed.

2.2 Bulk Solid Interaction in the Transition Zone and at Discharge

Minimal research has been conducted on the influence of bulk solid material properties and conveyor belt transition geometry on the bulk solid material cross section at discharge. While CEMA [6] and Roberts [86] provide calculations for the approximate overall dimensions of the cross section and the influence of transition length on the inclination of the trajectory at discharge, separation of the cross section associated with cohesive materials and possible transverse spreading of free flowing materials has not been investigated in great detail. Conversely, analysis techniques regarding discharge trajectories have been conducted at great length, such as CEMA [6], Dunlop Conveyor Manual [25], Korzen [63], Golka [28] and Booth [9]. However, the majority of the work to date has focused on trajectory analysis in only one plane, i.e. the vertical plane along the length of the belt.

2.2.1 Material Profile at Discharge

Immediately prior to the point of tangency of the belt and head pulley, the material load shape of a troughed belt conveyor can be closely approximated by a segment of a circle, CEMA [6] as shown in Figure 2-20.
The cross sectional area, $A_{sc}$, is given by:

$$A_{sc} = \frac{2 \cdot b \cdot h}{3} + \frac{h^3}{2 \cdot b} \quad (2.2.1)$$

From continuity, the area of the circular segment is assumed equal to the cross sectional area of the bulk solid material burden present on the troughed portion of the conveyor belt. The distance to the centre of gravity of the material at discharge, $a_1$, and the height of the discharging stream, $h$, are tabulated based on the calculated cross sectional area at the idler trough and may be found in CEMA [6] for a number of belt widths and throughputs.

$$a_1 = -R + \frac{b^3}{12 \cdot A_{sc}} + h \quad (2.2.2)$$

Roberts [86] assumes a parabolic cross sectional profile as presented in Section 2.1.3.1 with the mean height, $h_a$, expressed as:

$$h_a = C \cdot \sin(\beta) + (B + 2 \cdot C \cdot \cos(\beta)) \cdot \frac{\tan(\lambda)}{6} \quad (2.2.3)$$

The height of the cross sectional profile at the head pulley is given as:

$$h = \frac{A}{(B + C)} \quad (2.2.4)$$

Where $A =$ cross-sectional area
The above equation typically overestimates the head height, since it is generally accepted that the bulk solid material is moving fast enough for only limited settling time to occur from last idler roll set to discharge. As a consequence, for high speed discharge, the head height of the bulk solid material stream can be assumed to be equal to the head height of the material burden bed depth, $H$, as calculated at the last fully troughed idler roll set prior to discharge, as shown in Section 2.1.3.1.

### 2.2.2 Discharge Trajectory

Following bulk solid interaction in the transition zone, during which the material burden moves from a troughed belt profile at the last idler roll set to a flat belt profile at the head pulley, the material stream will discharge through a given trajectory. Conveyor belt transition length, idler roll inclination and geometry, belt speed and head pulley diameter each influence the discharge trajectory. Similarly, properties of bulk solid materials which influence the trajectory and the cross section of the material stream at discharge include particle size, bulk density, moisture content, internal angle of friction, adhesion and friction between the bulk solid material and the belt.

A number of different models for the determination of discharge trajectories have been developed, including those presented by CEMA [6], Dunlop Conveyor Manual [25], Korzen [63], Golka [28] and Booth [9] amongst others. Experimental validation to assess the level of accuracy of these models, implementation of combinations of these models and differentiation between them have previously been studied in detail by Arnold and Hill [2], Huque and McLean [40,41] and Hastie [37].

While the abovementioned methods and guidelines have been developed, unfortunately, the general extent thus far has been limited to two dimensional analyses only. The exception is Golka [27,28] who proposes a coefficient of divergence for the flow commencing at the head pulley centreline, however, design guidelines based on bulk solid material properties are not provided.

Discharge trajectories fall into two categories; slow speed trajectories and high speed trajectories. For slow speed trajectories, the bulk solid material will wrap around the head pulley by some angle before discharging, while for fast speed trajectories, the material will discharge at the point of tangency between the conveyor belt and the head pulley.

Prior to any trajectory analysis being undertaken, the angle of discharge of the material stream from the conveyor must be evaluated. When transition geometry is not provided, the inclination of the belt transition, $\varepsilon$, may be found from CEMA [6] and design guides such as Apex Fanner [1] and Dunlop [25] based on one half the trough depth. In general, this means that the inclination at discharge is slightly higher than the inclination of the conveyor, $\alpha$, due to the transition from a troughed profile at the last idler roll set to a flat profile at the head pulley. The model at discharge is illustrated in Figure 2-21.
The determination of which trajectory category (i.e. slow speed or high speed) is chosen for analysis, is dependent on the belt velocity $V_b$ and head pulley radius, $R$. The condition for non-adhesive materials can be calculated by:

$$\frac{V_b^2}{(R + \frac{H}{2})g} \geq 1 \quad (2.2.5)$$

Where $R + \frac{H}{2}$ is the distance from the centre of the head pulley to a half of the height of the material stream.

If the above condition is satisfied, discharge will occur at the point of tangency between the conveyor belt and the head pulley. If the above condition is not satisfied then the bulk solid material will wrap around the pulley by an angle, $\alpha_B$, from the vertical which may be calculated by:

$$\frac{V_b^2}{(R + \frac{H}{2})g} = \cos(\alpha_B) \quad (2.2.6)$$

The discharge trajectory model as indicated in Roberts [86], is based on projectile motion and the general formula for the trajectory path is expressed as:

$$y = \frac{g}{2 \cdot V_o^2 \cdot \cos^2(\alpha + \varepsilon)} \cdot x^2 - \tan(\alpha + \varepsilon) \cdot x \quad (2.2.7)$$

Where $\alpha$ is the inclination of the belt conveyor, $\varepsilon$ is the inclination of the transition zone and, $V_o$ is the material stream velocity at discharge and is given by $V_o = V_b$.

Equation 2.2.7 can be used to graphically plot the parabolic trajectory at the bottom of the bulk solid material stream discharging from the conveyor belt. All subsequent material streamlines in the direction
from the belt surface (bottom of stream) towards the top of the discharging stream may be assumed parallel and can be graphically plotted by implementation of a curve parallel to the parabola as given by Max Stein [68]. The equations of the curve parallel to the curve given by \( y = f(x) \) are:

\[
X = x - k^1 \cdot \frac{y'}{\sqrt{1 + (y')^2}} \quad \text{and} \quad Y = y + k^1 \cdot \frac{y}{\sqrt{1 + (y')^2}} \quad (2.2.8)
\]

Where for Eqn. (2.2.8):

\[y' = A^* + 2 \cdot B^* \cdot x \quad (2.2.9)\]

And the coefficients \( A^* \) and \( B^* \) are given by:

\[
A^* = -\tan(\alpha + \varepsilon) \quad \text{and} \quad B^* = \frac{g}{2(V_0\cos(\alpha+\varepsilon))^2} \quad (2.2.10)
\]

The model outlined above is applied in the analysis presented in Chapters 7 and 8 by plotting material stream bottom, centre and top streamlines, with the velocity assumed to be identical for the entire bed depth. In practice however, the bulk solid material stream, specifically at large bed depths, will have a progressively faster discharge velocity with increasing depth from the belt surface.

Research presented in Chapter 7, presents an analysis of discharge trajectories in 3 dimensions, taking into account bulk solid material transverse spreading of free flowing materials and shearing, or breaking up of the cross section, typical of cohesive materials. Only high speed trajectories are analysed with emphasis placed on transverse bulk solid material behaviour in the third dimension and a comparison to trajectory discharge observed in DEM simulations is performed. It will be shown that bulk solid material behaviour at discharge is directly influenced by material characteristics and also interactions prior to this point resulting from the geometry of the discharge zone.

### 2.3 Transfer Chute Flow Analysis

Transfer chutes are normally employed in belt conveying systems when transferring bulk solid material from one belt conveyor to another, or alternatively discharging into storage vessels including silos, bins or ship holds. In other applications, transfer chutes are employed to accelerate bulk solid materials up to belt speed, as in the case of re-directing slow moving material stream from a hopper/feeder onto a fast travelling receiving belt.

Following discharge from the delivery or feed conveyor, bulk solid material is likely to go through an impact plate or curved hood and/or a subsequent transfer chute (loading spoon) generally comprised of wall lining materials such as stainless steel or ceramic tile. For impact resistant installations, alternative hard faced impact resistant liners are used. In other situations, when large hard rock type materials are
handled, the use of ledges (rock-box, dead-box) instead of hood and spoon type systems are implemented, where the mode of flow is dependent on the internal friction characteristics of the bulk solid material rather, than the friction between the material and the wall lining.

Pioneering work of Roberts [80 - 86], lead to the development of the lumped parameter model for bulk solid material flow through transfer chutes. In these types of accelerated flows, with shallow bed depth to chute width ratios, Roberts [80] verified experimentally that the greatest total frictional work is due to the bulk solid material sliding on the chute bottom. Chiarella et al [14], Charlton et al [13] and Roberts [80,81,85] also investigated optimum transfer chute profiles such as constant radius, cycloid, convex and parabolic, with the aim of minimizing transit time and accelerating the material stream by breaking up the path in a number of discrete segments respectively. Korzen [62] also investigated the dynamics associated with bulk solid material and impact plate interaction. While, Scott and Choules [88] provide an overview of the use of impact plates handling hard-rock mining products in conveyor transfers.

The chute flow lumped parameter continuum models for hoods (curved impact plates) and spoons (curved loading chutes) are shown in Figure 2-22 below. From the work of Roberts [80 - 86] the equations of motion are summarized below.

For the hood:

\[-\frac{dV}{d\theta} + \mu_E \cdot V = \frac{g \cdot R}{V} (\cos \theta + \mu_E \cdot \sin \theta) \quad \text{for} \quad \frac{V^2}{Rg} \geq \sin \theta \quad (2.3.1)\]

For the spoon:

\[\frac{dV}{d\theta} + \mu_E \cdot V = \frac{g \cdot R}{V} (\cos \theta - \mu_E \cdot \sin \theta) \quad (2.3.2)\]

Where \( R \) = radius of curvature of chute, \( \mu_E \) = equivalent friction and \( \theta \) = angular position
The throughput, $Q_m$, is given by:

$$Q_m = \rho \cdot A \cdot V \tag{2.3.3}$$

Where $V$ is the velocity at the location considered.

By maintaining continuity throughout the flow, i.e., Eqn. (2.3.1) and (2.3.2) may then be applied to analyse flow at specific areas throughout the transfer. Calculation of the approximate bed depth can be assessed from cross sectional area calculations relating throughput and velocity for a given bulk density. Solutions of the equations of motion take into account the variation of the equivalent friction along the chute as a function of stream velocity. For a converging width chute:

$$\mu_E = \mu \cdot \left(1 + \frac{C_2}{V \cdot B^2}\right) \tag{2.3.3}$$

Where

$$C_2 = K_V \cdot B_0 \cdot H_0 \cdot V_0 \cdot (1 + \frac{\tan \chi}{\mu}) \tag{2.3.4}$$

$v_0$ = initial velocity, $H_0$ = initial stream bed depth

$B = B_0 - 2 \tan \chi \tag{2.3.5}$

$b_0$ = initial chute width, $\chi$ = chute width convergence angle and

$s$ = distance down the chute, for a curved chute this would be the curved length.

Numerical analysis is necessary in order to evaluate the equations of motion where equivalent friction and/or the radius of curvature vary. Solutions of equations of the form $\frac{dy}{dx} = f(x, y)$ with initial conditions $y(x_o) = y_o$ may be solved by advancing $x$ in steps of $h$ so that $x_{n+1} = x_n + h$ where $x_n$ is the $n^{th}$ value of $x$ at which we have found $y$.

The analysis used in the following section was based on the $4^{th}$ Order Runge Kutta method where:

$$k_1 = h \cdot f(x_n, y_n)$$

$$k_2 = h \cdot f\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$
\[ k_3 = h_i \cdot f \left( x_n + \frac{h_i}{2}, y_n + \frac{k_2}{2} \right) \]

\[ k_4 = h_i \cdot f \left( x_n + h_i, y_n + k_3 \right) \]

\[ y_{n+1} = y_n + \frac{(k_1 + k_2 + k_3 + k_4)}{6} \]

Thus a knowledge of \( x_n \) and \( y_n \) allows progression to \( x_{n+1} = (x_n + h_i) \) and \( y_{n+1} \) by evaluating \( k_1, k_2, k_3 \) and \( k_4 \).

For a constant radius chute:

\[ \frac{dV}{d\theta} = f(\theta, V) \text{ with } V(\theta_o) = V_o \]

Also, for a straight inclined chute, assuming constant equivalent friction, \( \mu_E \), the equation of motion of the material stream at a distance, \( s \), down the incline can be expressed as:

\[ V = \sqrt{V_o^2 + 2 \cdot a \cdot s} \]  \hspace{1cm} (2.3.6)

Where the acceleration, \( a \), is:

\[ a = g \cdot (\cos \theta - \mu_E \cdot \sin \theta) \]

And, \( \theta \) is the inclination of the chute from the vertical

For the free-fall section of the transfer, for example in between a curved impact plate and a loading chute, the velocity of the free falling bulk solid material can be approximated by:

\[ V = \sqrt{V_o^2 + 2 \cdot g \cdot h_f} \]  \hspace{1cm} (2.3.7)

Eqn. (2.3.7) neglects air resistance which is a reasonable assumption for most bulk solid materials. The impact model used to determine velocity at impact, \( V_i \), and immediately following impact, \( V_o \), is given by:

\[ \frac{V_o}{V_i} = \cos(\theta_1) \cdot (1 - e) \cdot \mu \cdot \sin(\theta_1) \]  \hspace{1cm} (2.3.8)

Where; \( e \) is the restitution factor with a range of \( 0 < e < 1 \) and \( \theta_1 \) is the incidence angle of impact.
Nominally, when impact incidence angles are maintained low, completely inelastic impact and no rebound is assumed i.e. $e = 0$. From the above equation it can be observed that when the impact angle is significantly high (large value of $\theta$), impact energy is completely lost i.e. the velocity after impact will be zero.

The continuum method is generally applied in the analysis of straight and ninety degree chute configurations characterized by thin stream rapid flow in which the bulk solid stream bed depth is lower than the chute width and where impact incidence angles are low. These cases result in a smooth transition from the material trajectory or free fall to flow through the chute. The application of this method allows for a rapid approximation of overall flow through the transfer to be determined. From an analysis of velocity profiles within the transfer configuration, cross sectional area profiles can also be determined from which approximate bed depths can be calculated for a given width of the bulk solid material stream. Generally, a rectangular cross section is assumed in the analysis. Bulk solid material properties that are used in the analysis are bulk density, wall friction angle, internal angle of friction, angle of repose and belt surcharge angle; all of which can be measured directly in the laboratory.

In the analysis of transfer chute flow interactions presented as two case studies in Chapter 8, the differential equations shown in Eqn. (2.3.1) and Eqn. (2.3.2) are used to calculate velocity profiles for hood and spoon type flow. For flow in straight inclined chute areas of the transfer, Eqn. (2.3.6) is applied. In areas of the transfers where the bulk solid material stream is in free fall, material stream velocity immediately prior to impact is calculated using Eqn. (2.3.7) and the velocity immediately following impact with the chute, spoon or flowing surface is calculated by applying Eqn. (2.3.8). The velocity profiles obtained at various locations within the transfer are then used to calculate the corresponding material stream bed depths. A comparison of the results obtained using continuum analysis is made to DEM simulations and feedback received from site, including visual observations.

### 2.4 Loading/Acceleration Zone

Typically, bulk solid materials are loaded onto the conveyor belt in a loose and unpacked state, with a load profile governed on the lower surface by the profile of the conveyor belt, and an upper profile governed by the material’s angle of repose, (CEMA [6]). As the belt advances, transverse motion between the idler roll sets prompts the bulk solid material to settle into an equilibrium state. Following the settling process, the upper surface at the edges may then be approximated by the conveyor surcharge angle rather than the angle of repose. During settlement, segregation of the material will also occur with the larger lumps migrating towards the upper surface of the bed of material and the fines percolating towards the top surface of the belt.

Loading of bulk solid materials is influenced heavily by the effectiveness of the transfer chute design, which in turn is governed by the physical geometry of the transfer and bulk solid material properties. The use of skirt plates is generally employed in the loading area to prevent material spillage during loading.
and acceleration. During loading, CEMA [6] states that there are at least three different cross section conditions that are to be considered, a loose repose profile given by cross sectional area, $A_{LRP}$, followed by a profile governed by the skirting boards, cross section $A_{SP}$, and finally a steady state condition governed by the conveyor belt surcharge angle and given by cross sectional area $A_{SSP}$. This is illustrated in Figure 2-23.

![Figure 2-23 – Varying Cross Section Zones during Loading](image)

CEMA [6] also states that, the cross section at loading, $A_{LRP}$ can be evaluated by assuming that the belt surcharge angle, $\lambda$, is equal to the angle of repose, $\theta_R$. The cross section within the skirting, $A_{SP}$, is as shown in Figure 2-24.

![Figure 2-24 - Cross Section within Skirting, CEMA [6]](image)

An important application of loading chutes is to direct the flow of bulk solid materials onto belt conveyors, the task is illustrated in Figure 2-25 and Figure 2-26.

![Figure 2-25 – Loading Velocity Components at Exit](image)
According to Roberts [87] the primary objectives are to:

- Match the component of exit velocity $V_p$ as close as possible to the belt speed
- Reduce the vertical component of the velocity normal to the belt, $V_n$, in order to reduce the abrasive wear
- Load the belt in a central manner so that the load is evenly distributed and to avoid belt mis-tracking
- Ensure reliable flow without spillage or blockages, while maintaining self-cleaning of the chute

The impact pressure, $P_{vi}$, is given by:

$$P_{vi} = \rho \cdot V_n^2$$  \hspace{1cm} (2.3.9)

The wear parameter, $W_a$, expressing the rate of abrasive wear may be established as follows:

$$W_a = \mu_1 \cdot \rho \cdot V_n^2 (V_b - V_p)$$  \hspace{1cm} (2.3.10)

Where $\mu_1 = \text{friction coefficient between the bulk solid material and conveyor belt}$, $V_b = \text{belt speed}$, $V_n = \text{normal velocity component}$, $V_p = \text{velocity component parallel to belt travel direction}$

Frictional drag due to lateral pressure of the bulk material against the plates is assumed hydrostatic, and for each wall is given by $\frac{K_v P \cdot \rho \cdot g \cdot h}{2}$, where $K_v$ is the pressure ratio coefficient for the walls of the skirt plates and is generally $0.4 < K_v < 0.8$. This is illustrated in Figure 2-27.
The height of the bulk solid material in contact with the skirt plates, $h_s$, within the acceleration zone varies inversely with the velocity $V_s$ at a distance, $s$, from initial loading point, along the length and in the direction of the receiving conveyor belt. For a constant throughput, $Q_m$, it follows that:

$$ h_s = \frac{Q_m}{\rho W_s V_s} \quad (2.3.11) $$

An approximate calculation for the acceleration of the material based on the work of Roberts [87] shows that the acceleration of the material is non-uniform and that the velocity as a function of distance along the belt is non-linear. Assuming block like motion of the bulk solid material, an analysis of the forces due to the belt driving the material forward between the skirt plates allows the acceleration to be calculated from:

$$ a = \mu_1 \cdot g - \mu_2 \cdot g \cdot K_v \cdot \frac{h_s}{W_s} \quad (2.3.12) $$

Where $\mu_1$ is the friction between the material and the belt, $\mu_2$ is the friction between the material and the skirt plates and typically lie in the range of $0.5 < \mu_1$ and $\mu_2 < 0.7$.

Substituting $h_s$ from Eqn. (2.3.11) gives:

$$ a = g \cdot \left( \mu_1 - \mu_2 \cdot \frac{K_v Q_m}{\rho W_s^2 V_s^2} \right) \quad (2.3.13) $$

From the above equation, it can be observed that, following initial loading at a given velocity, and maintaining constant $K_v$, friction between bulk solid material, belt and skirt plates and the width between the skirt plates, the acceleration will increase in a non-linear manner. This is due to the head height of the bulk solid material burden decreasing with an increase in velocity and implies that a minimum value of the bulk solid material stream velocity is required to achieve the required feeding at the rate of $Q_m$ onto the belt. The critical condition will be at the point of entry where $s = 0$ and $V_s = V_\infty$ when $a = 0$:

$$ V_{\text{omin}} = \frac{\mu_2 K_v Q_m}{\mu_1 \rho W_s^2} \quad (2.3.14) $$

**Figure 2-27 – Skirting Pressure Profile**
Loading and acceleration of the bulk solid material stream on the conveyor belt is inevitably related to interactions encountered throughout the transfer chute. As such, loading and acceleration investigated in Chapter 8 forms part of two transfer chute analysis case studies. Following a velocity analysis of material stream flow through the transfer configurations, including loading velocity, impact pressure is calculated by applying Eqn. (2.3.9) and the depth of the bulk solid material is approximated. Acceleration at loading is then calculated using Eqn. (2.3.12) from which the distance for the material stream to attain belt velocity is determined. A profile of the abrasive wear parameter, \( W_a \), on the receiving belt is estimated by assuming that the normal component of velocity, \( V_n \), will decrease linearly from the point of loading to the location along the belt at which belt speed is attained. The above-mentioned calculations are compared to corresponding analysis results predicted by DEM simulations. Cross sectional material stream profiles at loading and during acceleration, obtained from DEM simulations, are also compared to those theorized by CEMA [6].

### 2.5 Summary

A number of available existing theoretical approaches have been presented regarding bulk solid material interactions in belt conveyor systems. These approaches span across a number of integral areas including loading, acceleration, transportation, discharge, re-direction and transfer.

Determination of physical bulk solid material properties via experimental tests required for the application of the theoretical analysis will be presented in subsequent chapters. In a similar manner, an overview of the Discrete Element Modelling (DEM) contact models and parameters required for analysis, including the calibration procedure and selection of DEM parameters, will also be presented.

Theoretical analysis based on the existing approaches will be evaluated against DEM analysis, experimental laboratory tests and experimental measurements. Methodology will be proposed which implements existing continuum model theory in parallel with DEM and highlights how the use of both of these analysis tools in a combined effort provides very useful outcomes.
Chapter 3 - Discrete Element Modelling (DEM)

Initial conceptions of Discrete Element Modelling (DEM) emerged in the field of rock mechanics over 40 years ago for the analysis of blocky rock systems. This work was performed by Cundall [18] and soon after adapted for soils (Cundall and Strack [19]). Research since then has produced a significant number of varying implementation techniques of the original methodology, consisting of alternative contact models and applied by a number of different software codes.

In recent years, computer simulations based on the Discrete Element Method (DEM) have become increasingly popular in the analysis and visualization of bulk solid interactions in belt conveying systems. Research regarding transfer chutes (Dewicki [20], Donohue et al [24], Hastie [37], Hustrulid [38-39], Ilic et al [44], Nordell [73], Katterfeld [55 - 59], Kessler [61]) is by far the most documented. Other applications of DEM in the general field of bulk solid materials handling include flow in hoppers and mixers, (Zhu [111]), dragline filling, sampling bias, segregation in splitter bins, screw conveying, vibrating screens, grinding, rod and sag mills and landscape modelling on real topography (Cleary [15]), bucket and chain conveyors (Katterfeld [59]).

In contrast to classical continuum methods presented in Chapter 2, DEM considers individual particles. The interaction of an assembly of particles or particles and their environment (boundaries) are treated as a dynamic process and during one calculation cycle, contact forces and displacements are found by tracing every particle in the flow. Forces acting upon each particle or boundary, and the resulting acceleration by the application of Newton’s second law are calculated alternately. By integration over a very small time step the particle velocity and updated particle positions are determined from the given acceleration.

The research contained within the scope of this thesis focuses on the implementation of two DEM software codes: PFC3D 4.0™ by Itasca and Rocky by Granular Dynamics International (GDI). Modelling of common materials exhibiting characteristics typical to that present in the bulk solid materials handling industry is presented, with a focus on bulk solid interactions in belt conveying systems. Normal and tangential contact force models implemented for the calculation cycle in each of the DEM analysis codes, including parameters describing the contacts between individual particles and boundaries are presented. A summary of input parameters which are to be used in calculations later in the thesis is presented and key simulation parameters are specified. Also presented is a brief overview of the modelling procedure adopted in subsequent chapters, regarding bulk solid and conveyor belt interactions, and transfer chute bulk solid material flow analysis.

3.1 Overview of Models for DEM Analysis

Particles simulated with DEM are typically assumed to remain geometrically rigid (no deformation of shape) with deformation accounted for in the contact force models. The particles are allowed to overlap
and the overlap characteristics are used to determine the contact force. Typically the contact forces are resolved into a normal force and a tangential force, with the normal force being independent of the tangential force. As a first step in the calculation, particles and boundaries are generated in the system and contacts between them are identified to calculate the contact forces. Depending on the contact force, and boundary conditions, the position and velocity of each particle are updated.

The total force acting on a particle when two particles are in contact is equal to the summation of the total normal force, $F_n$, and the total tangential force, $F_s$, acting at that contact. The normal and tangential contact force models for both PFC3D™ and Rocky are presented in the following section, by way of illustrating contact between two particles (spheres). A similar procedure is also adopted for particle/boundary contacts; however, the properties of the wall are generally assigned to the properties of the particle.

Specifics regarding different types of contact models available and their constituents theorized and investigated by Cundall [18,19], Walton [96-100], Wassgren [101-103], Di Renzo and Di Maio [22], Tsuji [95], Schäfer et al [89], Luding [67] and Kruggel-Emden et al [65] have been collated. From that research, a generalised overview of the contact models incorporated in DEM analysis in this thesis is presented.

3.1.1 PFC3D Model

The contact model used in PFC3D™ is a coupled Hertz-Mindlin (no tension) model (PFC3D Manual [74]). The normal contact force model is based on a damped Hertzian spring, and a diagram illustrating contact between two particles (spheres) with radius, $R_i$ and, $R_j$, of mass, $m_i$, and mass, $m_j$, is shown in Figure 3-1.

![Figure 3-1 – Normal Contact Force Model (PFC3D)](image)

The normal force, $F_n$, is given by:
\[ F_n = -k_{HZ} \cdot \delta_n + v_n \cdot \dot{\delta}_n \]  

Where:

\[ k_{HZ} = \text{Hertzian spring stiffness} = \frac{2G \sqrt{R'}}{3(1-\nu)} \sqrt{\delta_n} \]  

\[ R' = \text{equivalent radius} = \frac{2R_i R_j}{R_i + R_j} \]  

\[ G = \text{Shear Modulus (input by the user)} \]  

\[ \nu = \text{Poisson’s ratio (input by the user)} \]  

\[ \delta_n = \text{normal overlap} \]  

\[ \delta_n = d\delta_n/dt \text{ (normal relative velocity)} \]  

\( v_n \) is the normal damping coefficient and is given by from Rahman et al [76]:

\[ v_n = 2 \beta_{nPFC} \cdot \sqrt{k_{HZ} \cdot m^*} \]  

Where \( m^* = \frac{m_i \cdot m_j}{m_i + m_j} \) = effective mass

The damping factor, \( \beta_{nPFC} \), is specified by the user. In all simulations performed with PFC3D in this thesis the damping factor has been assumed to be, \( \beta_{nPFC} = 0.9 \).

The tangential contact force model implemented in PFC3D is based on a damped spring in series with an incrementally slipping friction element and is illustrated in Figure 3-2 below:

Figure 3-2 – Tangential Contact Force Model (PFC3D)

The tangential force, \( F_t \), acting on particle \( i \), is determined in an incremental manner and is given by:
\[ |F_{\text{on},i}| = \min \left( |F'_s| + k_s \cdot |\delta_s - \delta'_s| + v_s \cdot \dot{\delta}_s \cdot \mu |F_n| \right) \]  \hspace{1cm} (3.4)

Where:
\[ \delta_s = \text{tangential overlap} \]

\[ k_s = \text{tangential stiffness} = \frac{2 \cdot (G^2 + \gamma - 1) R^2 \cdot \mu_s \cdot \frac{1}{2}}{1} \cdot |F_n|^{1/3} \]  \hspace{1cm} (3.5)

\( F'_s \) is the tangential force value at last direction reversal, (Wassgren [103]) or in other words, the tangential force calculated for the previous time step, (Schäfer et al [89])

\[ v_s = 2 \cdot \beta_{\text{IPFC}} \cdot \sqrt{m \cdot k_s} = \text{tangential damping coefficient} \]  \hspace{1cm} (3.6)

\( \beta_{\text{IPFC}} \) is the damping factor and is specified by the user (Rahman et al [76])

\( \delta_s = d \delta_s/dt = \text{tangential relative velocity} \)

\( \mu \) is equal to static, \( \mu_s \), or dynamic, \( \mu_d \), friction depending if sliding at the contact takes place or not. The total tangential force, \( F_n \), is limited by the Coulomb frictional limit, \( \mu_s F_n \), at which the point the contact shears (breaks) and the particles begin to slide over each other (Cleary [15]).

The contact model used in PFC3D does not distinguish between sliding and dynamic friction. Specification of the friction coefficient in this contact model is only related to particle/particle, \( \mu_p \), and particle/boundary, \( \mu_w \), friction, which are directly input by the user.

### 3.1.2 Rocky Model

The normal force contact model implemented in Rocky is a partially latching, or hysteretic linear spring model, originally introduced by Walton and Braun [96]. The model is based on a specified stiffness during loading and a different stiffness during unloading at the contact. The stiffness of the contact during loading is lower than the stiffness during unloading. A diagram illustrating contact between two spheres with radius \( R_i \) and \( R_j \), of mass \( m_i \) and mass, \( m_j \), is shown in Figure 3-3 and a brief overview is presented below, as found in Wassgren [102].
The normal force \( F_n \) is given by:

\[
F_n = \begin{cases} 
-k_{NL} \cdot \delta_n & \delta \geq \delta_{\text{max}} \\
-k_{NU} \cdot (\delta_n - \delta_{\text{res}}) & \delta_{\text{res}} \leq \delta \leq \delta_{\text{max}} \\
0 & 0 \leq \delta \leq \delta_{\text{res}} 
\end{cases}
\]

(3.7)

Where:

\( k_{NL} \) = loading spring stiffness
\( k_{NU} \) = unloading spring stiffness \((k_{NU} \geq k_{NL})\)
\( \delta_n \) = normal overlap
\( \delta_{\text{res}} \) = residual overlap
\( \delta_{\text{max}} \) = maximum overlap during contact
\( \mathbf{n} \) = unit vector normal to the contact plane and pointing from particle \( i \) toward particle \( j \)

Energy dissipation is due to the spring force hysteresis and the hysteretic stiffness, \( k_{NL} \) and \( k_{NU} \), model the strain hardening of the material due to plastic deformation. The residual overlap, \( \delta_{\text{res}} \), represents the permanent plastic deformation of the contact:

\[
k_{NL} \cdot \delta_{\text{max}} = k_{NU} (\delta_{\text{max}} - \delta_{\text{res}})
\]

(3.8)

\[
\delta_{\text{res}} = \delta_{\text{max}} (1 - \frac{k_{NL}}{k_{NU}})
\]

(3.9)

\[
\delta_{\text{max}} = \dot{\delta}_0 \sqrt{\frac{m^*}{k_{NL}}}
\]

(3.10)

\[
\varepsilon_n = \frac{k_{NL}}{k_{NU}}
\]

(3.11)

\[
\delta_{\text{res}} = \dot{\delta}_0 \sqrt{\frac{m^*}{k_{NL}} (1 - \varepsilon_n^2)}
\]

(3.12)
Where:

\[ \delta_o = \text{relative impact speed} \]
\[ m^* = \text{effective mass} \]
\[ \varepsilon_n = \text{normal coefficient of restitution (input by the user)} \]

The loading stiffness in Rocky is calculated by multiplying group stiffness to particle size. In this manner, the group stiffness is roughly equivalent to bulk elastic modulus of the simulated material (and not individual particles) based on work done by Bathurst and Rothenburg [4]. The loading stiffness is directly input by the user.

\[ k_{NL} = E \cdot L \] \hspace{1cm} (3.13)

\[ k_{NU} = \frac{E \cdot L}{\varepsilon_{res}^2} \] \hspace{1cm} (3.12)

Where:

\[ E = \text{material bulk elastic stiffness} \]
\[ L = \text{particle size} \]
\[ \varepsilon_{res} = \text{particle restitution coefficient} \]

For more information regarding the normal force contact model, refer to Walton et al [96 - 100] and Wassgren [101, 102].

The tangential contact force model implemented in Rocky is based on a linear spring in series with a sliding friction element (without damping) and is illustrated in Figure 3-4 below.
The tangential force, $F_s$, acting on particle $i$, is given by:

$$F_{s,\text{on } i} = \min(k_S \cdot \delta_s, \mu|F_n|)$$  \hspace{1cm} (3.13)

Where:
- $\delta_s$ = the tangential (shear) overlap
- $\mu$ is equal to static, $\mu_s$, or dynamic, $\mu_d$, friction depending if sliding is or is not taking place for the contact
- $k_S$ = shear stiffness

From Wassgren [103], the stiffness ratio may be expressed as:

$$\frac{k_s}{k_n} = \frac{1-v}{1-2v}$$  \hspace{1cm} (3.5)

For comparison of simulations results obtained from Rocky to those from PFC3D, Rocky simulations have been conducted under the assumption that the static friction, $\mu_s$, is equal to the dynamic friction, $\mu_d$. Hence, the specification of a single particle/particle, $\mu_p$, and particle/boundary, $\mu_w$, coefficient of friction is referred to throughout this thesis.

### 3.1.3 Normal Contact Force – Adhesive Force

Cohesion in granular materials occurs by the attraction or adhesion of fine particles due to the presence of moisture. The fine particles act as a form of bonding agent between the coarse particles and generally an increase in the attractive forces or internal strength of the granular material is exhibited with higher

![Figure 3-4 – Tangential Contact Force Model (Rocky)](image-url)
cohesion. At present, the application of PFC3D 4.0 to the simulation of materials exhibiting cohesion is in a state of review, and subsequently the modelling of cohesion has been limited to Rocky only.

Previous work regarding the implementation of a cohesion model in PFC3D has been explored by Gröger et al [32] focusing on modelling cohesion with parameters related to surface tension and liquid bridge theory. Luding et al [67] also investigated the implementation of the adhesion model in parallel with the plastic, hysteretic non-linear contact force model.

The cohesion model used in Rocky is also in a state of development and at present it is based on a simple model implementing an on/off switch. For two particles in contact, the normal adhesive force is calculated by an adhesive fraction coefficient, \( \mu_{adh} \), multiplied by the minimum weight of the two corresponding particles and is initiated when the distance between particles, \( z_n \), is smaller than a user defined adhesive distance, \( s_{adh} \). When the distance between the particles exceeds the adhesive distance specified, adhesive force acting on the particles is set to zero (off). An illustration is shown in Figure 3-5.

\[
F_{n,adh} = \mu_{adh} \cdot \min(m_i, m_j) \cdot g; \quad \text{for } z_n < s_{adh} \tag{3.6}
\]

\[
F_{n,adh} = 0; \quad \text{for } z_n > s_{adh} \tag{3.7}
\]

Where:
- \( \mu_{adh} \) = adhesive fraction (input by the user)
- \( m_i, m_j \) = mass corresponding to particle \( i \) and particle \( j \)
- \( s_{adh} \) = adhesive distance (input by the user)
3.1.4 Rolling Friction

The influence of particle shape on flow characteristics of granular materials has been identified as of importance in particular applications such as mixing rates, strength of materials subjected to internal failure and flow patterns in hoppers, (Cleary [16]). However, modelling of particle shape in DEM generally results in a longer time for simulation completion due to an increased number of contact points and subsequently a greater number of associated calculations. For this reason, rolling friction is incorporated into models consisting of spheres as a way of modelling shape, (Wensrich [105]). The DEM modelling simulations used to analyse bulk solid material interactions in this thesis, will solely focus on modelling mono-sized spherical particles with the incorporation of rolling friction as a way of modelling shape.

Similarly to the normal and tangential contact force models, a number of different rolling friction models have also been investigated for implementation into the contact mechanics of particles in discrete element simulations. A review and assessment of the application of different models is presented by Ai et al [3]. The rolling friction model implemented both in PFC3D and Rocky is based on the Model Type C, elastic plastic spring-dashpot model as described in Ai et al [3] and refined by Wensrich and Katterfeld [105]. In the Model Type C, the total rolling resistance torque, \( T_r \), consists of two components: a mechanical spring torque \( T_{r,t+\Delta t}^k \) and a viscous damping torque \( T_{r,t+\Delta t}^v \)

\[
T_r = T_r^k + T_r^d
\]  

(3.8)

The incremental elastic component of the Model Type C rolling resistance can be described by:

\[
T_{r,t+\Delta t}^k = \begin{cases} 
\vec{T}_{r,t}^k - k_r \vec{\omega}_{rel} \Delta t & \text{provided} \ |\vec{T}_{r,t} - k_r \vec{\omega}_{rel} \Delta t| < \mu_r R^* |\vec{F}_n| \\
\mu_r R^* |\vec{F}_n| & \text{otherwise}
\end{cases}
\]

(3.9)

Where:

- \( k_r \) = rolling stiffness
- \( \mu_r \) = coefficient of rolling friction
- \( \vec{F}_n \) = contact normal force
- \( \vec{\omega}_{rel} \) = relative angular velocity of the two particles in contact
- \( R^* \) = effective radius

\[
T_{r,t+\Delta t}^v = \begin{cases} 
-C_r \vec{\omega}_{rel} & \text{provided} \ |\vec{T}_{r,t}^v| < \mu_r R^* |\vec{F}_n| \\
0 & \text{otherwise}
\end{cases}
\]

(3.10)

The damping rate, \( C_r \), is proportional to critical damping and damping ratio, \( \zeta \):

\[
C_r = \zeta \cdot 2 \sqrt{k_r I_r}
\]

(3.11)
Where:
\[ I_r = \text{equivalent moment of inertia} \]

### 3.1.5 Stiffness of Contacts

Currently, it is not possible to calibrate the particle stiffness for the majority of applications in the field of bulk solid interactions in belt conveyor systems. Gröger [31] recommends that particle stiffness be selected as high as the overall computational time allows it. Research suggests that the stiffness of the contacts does not vary significantly for moderate solids fractions, and sheared systems with soft springs produce stresses similar to those that would be generated if smaller particles were used, (Katterhagen et al [60]).

The influence of particle stiffness on measured force values due to a stream of particles impacting with an impact plate has also been studied by Katterfeld et al [59], for stiffness parameters in the range of 1e6 to 1e9 N/m². The results showed that the average absolute normal force decreases with increasing stiffness until a value of 1e8 N/m². A comparison between the calculated average normal force between the upper and lower limits indicated a variation of only ~6%.

Ji et al [52] made comparisons of experimental and DEM simulation of rapidly sheared glass spheres in an annular shear cell for stiffness values in the range of 8.5e5 N/m² to 8.5e8 N/m². In this study it was observed that particle stiffness had little influence (~4%) on the measured normal force. For both PFC3D and Rocky results presented in this thesis, a particle stiffness of 1e7 N/m² has been selected based on research outcomes mentioned above and according to further analysis presented by Wensrich [105] and Rahman et al [76].

### 3.1.6 Coefficient of Static and Dynamic Friction

The coefficient of static friction, \( \mu_s \), may be defined as the friction at which a particle will commence sliding and the coefficient of dynamic friction, \( \mu_d \), may be defined as the friction of a particle during sliding. In general, the dynamic friction is lower than static friction, and Korzen [63] gives the following relationship:

\[
\frac{\mu_d}{\mu_s} = 0.7 - 0.9 \quad (3.12)
\]

Grima [33, 34] identified the variation between static and dynamic friction for polyethylene pellets on three different lining materials. For all results reported, the variation in static and dynamic friction coefficients was lower than 0.03.
For all results presented herein, $\mu_{pp}$ denotes the particle to particle sliding friction and similarly, $\mu_{pw}$ denotes the particle to wall sliding coefficient of friction. In order to reduce the number of variables selected for analysis, the dynamic and sliding friction coefficients for both particle to particle, and particle to wall interactions have been assumed equal.

### 3.1.7 Poisson’s Ratio

Poisson’s ratio is a measure of the elastic stress to strain behaviour of materials. According to Wassgren [103], common materials lie in the range $0 < \nu < \frac{1}{2}$ and for many of them $\nu = 0.3$. Other researchers have also based their investigation on this value, (Zhou et al [107 - 110], Li et al [66], Wensrich [105]). In this thesis a Poisson’s ratio value of 0.3 has been assumed in all analysis performed and presented both for PFC3D and Rocky simulations.

### 3.1.8 Co-efficient of Restitution

The co-efficient of restitution, $e_{res}$, is a measure of the energy dissipated during a contact and may be express as the ratio of the velocity of a particle after impact to the velocity prior to impact:

$$e_{res} = \frac{V_o}{V_i}$$  \hspace{1cm} (3.13)

Where:

- $V_o$ = velocity immediately following impact
- $V_i$ = velocity immediately prior to impact

The coefficient of restitution is an extremely difficult parameter to measure for bulk solid materials. Researchers have used a bouncing ball type test to quantify it, either by dropping a single particle or a layer of particles onto a wall or a bed of particles, (Johnstone [52], Grima [33]). Grima [34] gave an approximation of the coefficient of restitution for coal by selecting two similarly shaped flat particles, and dropping one onto the other. From this technique a coefficient of restitution that was reported was in the order of 0.5-0.6.

The coefficient of restitution is much easier to measure for a single impact between a particle and a wall compared to impact with another single particle. As such, coefficients of restitution are measured for impact between a particle and a wall and then assumed as the coefficient of restitution for particle/particle contact.

In PFC3D, the coefficient of restitution is related to the coefficient of damping (PFC3D Manual [74]). In Rocky, the coefficient of restitution is input by the user. The results from DEM simulations presented in this thesis do not investigate the influence of the coefficient of restitution. All simulations were performed with the restitution coefficient equal to 0.3.
3.1.9 Time Step

The time step in DEM simulations determines the calculation time. The smaller the time step, the longer the calculation time. In order to minimize calculation time in simulations, typically low stiffness values are applied.

The critical time step is proportional to the mass, \( m \), of the particle and the stiffness of the contact, \( k \), (PFC3D Manual [74]), and may be expressed as:

\[
t_{\text{crit}} \propto \frac{m}{\sqrt{k}}
\]  

(3.14)

From the above equation, it can be observed that, for a constant particle mass, a larger stiffness would result in a smaller time step, therefore increasing the total simulation time. Also, for a constant stiffness, the higher the mass of the particle (density) the smaller the time step.

3.2 Discrete Element Modelling Simulation Procedure

The key parameters implemented in the contact models are loading stiffness (Shear Modulus, \( G \), or Bulk Elastic Stiffness, \( E \)), Poisson’s ratio, \( \nu \), coefficient of restitution, \( \varepsilon_{\text{res}} \), coefficient of particle/particle sliding friction, \( \mu_p \), particle rolling friction, \( \mu_r \), coefficient of particle/wall friction, \( \mu_w \), particle adhesive fraction, \( \mu_{\text{adh}} \), adhesive distance, \( s_{\text{adh}} \) and the normal and tangential damping factors, \( \beta_{\text{nPFC}} \) and \( \beta_{\text{tPFC}} \). The parameters which are assumed in all simulations, unless specified otherwise are summarized in Table 3-1 below.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shear Modulus, ( G ) (N/m(^2))</td>
<td>1e7</td>
</tr>
<tr>
<td>Bulk Elastic Stiffness, ( E ) (N/m(^2))</td>
<td>1e7</td>
</tr>
<tr>
<td>Poisson’s Ratio, ( \nu )</td>
<td>0.3</td>
</tr>
<tr>
<td>Coefficient of Restitution, ( \varepsilon_{\text{res}} )</td>
<td>0.3</td>
</tr>
<tr>
<td>Normal Damping Factor, ( \beta_{\text{nPFC}} )</td>
<td>0.9</td>
</tr>
<tr>
<td>Tangential Damping Factor, ( \beta_{\text{tPFC}} )</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Other parameters used in the simulations were selected based on the calibration testing results which will be presented in Chapter 4. The general procedure implemented for these calibration test simulations, and the laboratory simulations presented in Chapter 5, is presented in the next section. Also presented is the general modelling procedure implemented regarding the processing of analysis obtained from DEM simulations in Chapter 6, 7 and 8.

The main difference to the procedures adopted between PFC3D and Rocky involves the different functionality of each software package. PFC3D is programming based while Rocky consists of a
graphical user interface. During the course of research presented in this thesis, PFC3D was the DEM simulation analysis tool readily available for analysing bulk solid interactions in belt conveying systems. However, as Rocky software became available in early 2011, the extent of research presented has effectively increased due to the nature of implementation and also reduction in calculation time.

3.2.1 **Discrete Element Modelling (DEM) using PFC3D 4.0™**

For PFC3D simulations, spherical particles were packed in a known geometry by implementing the theory of Donohue [23]. The approach involves the repeated addition of random particles in a constrained volume, known as sequential addition. By optimizing an objective function governed by the gravitational and contact force acting on each particle, an assembly of particle co-ordinates is created. Particle assemblies within all boundary geometries simulated with PFC3D and reported in this thesis have been generated using this concept. This includes the angle of repose and slump plane angle tests presented in Chapter 4, and the 600mm wide belt experimental facility presented in Chapter 5.

For each particle that is generated in DEM an identification number is stored. In the simulation model, each of the particles is assigned parameters such as particle friction, $\mu_p$, rolling friction, $\mu_r$, loading stiffness and other parameters identified in Section 3.1 as specified by the user. Similarly, for each wall (boundary) element that is loaded into the simulation model, a wall identification number is assigned. Each of the boundary elements within a specified range are assigned parameters such wall friction, $\mu_w$. Also, any of the boundary elements may be assigned a motion, such as translation, rotation and oscillation, or a combination of these. The motion may also be constrained to the surface only, and the walls may or may not physically translate, or rotate. Within the simulation, once a particle is in contact with the boundary element that is assigned a specific surface motion, the particle itself will be subjected to the motion, governed by the contact model used.

Contacts that are created in the simulations are assigned contact identification numbers. The inbuilt contact detection calculation cycle specific to each software searches through all of the contacts occurring in the simulation. Over an output time step (specified by the user), the contact attributes for each particle/particle and each particle/boundary are output via text files. Typical attributes of the contacts which are accessible include x, y, z co-ordinates, velocity, normal force and power.

For angle of repose simulations presented in Chapter 4, a single layer of particles was initially generated and acted as the floor, on top of which the cylinder and assembly of particles was placed. Co-ordinates of the particle positions and radius values were assigned according to the dense packing algorithm of Donohue [23] and were imported into the DEM simulation. This procedure was implemented to minimize the influence of the friction of the floor surface. The boundaries which make the cylinder were assigned a vertical upward velocity of 0.02m/s until the entire material flowed out of the cylinder and formed a conical heap. Following 10.0s of simulation time, with the particles at rest, the positions of the stationary particles were exported, from which they were analysed with the numerical analysis package Matlab. An
algorithm procedure identical to that described by Wensrich [105] was implemented to measure the resulting angles of repose, in four cut-plane directions.

A similar procedure was adopted for the slump plane angle test; however, after filling the geometry with densely packed particles, one side of the square apparatus was opened, following which the particles were allowed to flow out over a period of 10.0s. At the end of the simulation, particle co-ordinates were analysed with Matlab. A straight line of best fit was produced matching the top surface of the simulated material, of which the angle is reported as the slump plane angle, or internal angle of friction.

For the 600mm wide belt laboratory simulations presented in Chapter 5, the inclined sides of the belt are assigned a periodic rotational velocity about a centre of rotation (junction between the inclined and horizontal sides). During the simulations the transverse cross sectional behaviour of the bulk solid material was observed and normal forces analysed.

Analysing the x, y, z co-ordinates and the corresponding identification numbers of the contacts between the balls and boundaries allows for normal forces to be obtained for all contact points in a specified area of the boundary. Breaking up the boundary geometry into desired elements of known dimensions, all contacts within any geometrical limits can be obtained. Subsequently, normal pressure is obtained by summation of normal forces for all contacts acting over a known area.

3.2.2 Discrete Element Modelling (DEM) using Rocky (GDI)

The procedure implemented in the DEM simulations using Rocky is almost identical in nature to that described in Section 3.2.1. A brief overview is summarized below, regarding simulation results presented in Chapter 4 to Chapter 8.

Regarding angle of repose simulations (presented in Chapter 4) spherical particles were allowed to settle under gravity over a time period of 20.0s, following which, the cylinder was assigned a vertical velocity of 0.02m/s in the upwards direction. Upon completion of the simulation, with particles at rest, particle co-ordinates were exported and analysed with Matlab. Similarly, for the slump plane angle tests, spherical particles were allowed to settle under gravity over a time period of 20.0s, following which, one side of the square apparatus was opened. The particles were then allowed to flow out over a period of 15.0s following which the co-ordinates of the stationary particles were exported for analysis with Matlab. A straight line of best fit was determined for the top layer of the pile, of which the angle reported is the slump plane angle.

For simulation of the experimental 600mm and 1050mm wide belt test facility in Chapter 5, a number of smaller “analysis” boundaries were selected and normal force during oscillation was analysed. Vertical boundary walls are also present at each end of the belt. In order to negate the influence of interaction with the end walls, the analysis boundaries are located in the middle of the trough and for the 600mm belt
width simulations were conducted on the boundaries as illustrated in Figure 3-6. The number of analysis boundaries on the inclined side is 10, and the number of analysis boundaries located on the horizontal section is 5. All analysis boundaries have a longitudinal length of 50mm. Each of the 10 analysis boundaries on the inclined side has a width of 13mm which corresponds to the total length of the inclined side (L₄) of 130mm. The width of each of the analysis boundaries on the horizontal section of the belt is 22mm, giving a total width of 110mm. This corresponds to half of the width of the horizontal section.

![Figure 3-6 – Analysis Boundaries: 600mm wide Belt Interaction Simulations](image)

The analysis boundaries for the 1050mm wide belt interaction simulations are shown in Figure 3-7. In this analysis, the number of boundaries on the inclined side is 26, and the number of boundaries located on the horizontal section of the belt is 19. The longitudinal length of each boundary is 50mm. The width of each analysis boundary on the inclined side of the belt is 10mm, which corresponds to the total length of the inclined side of 260mm. For the analysis boundaries located on the horizontal section of the belt, the width of each is also 10mm, corresponding to a total length of 190mm (half of the width of the horizontal section).

![Figure 3-7 – Analysis Boundaries: 1050mm wide Belt Interaction Simulations](image)

For the belt interaction simulations presented in Chapter 5, the inclined sides of each belt simulated, including the analysis boundaries, were assigned a periodic rotational velocity. The centre of rotation was located at the junction point between the inclined and horizontal sections of the belt. Normal pressure values from the simulations were investigated for the first 10.0s of simulation.
In contrast to the particle loading procedure in PFC3D, particles were generated and allowed to settle under gravity, following which they were profiled by the use of a “scrape” boundary translating across the top surface of the material pile. This is illustrated in Figure 3-8 below.

Figure 3-8 – Manual Profiling of the Bulk Solid Material Cross Section

Regarding simulation of the 600mm wide belt, a parabolic cross sectional shape of the top surface was modelled, 120mm high with a conveyor surcharge angle of 13°. For simulations with the 1050mm wide belt, a trapezoidal cross-sectional profile of the bulk solid material was modelled with a height of 250mm and a conveyor surcharge angle of 15°.

For the belt deflection simulations presented in Chapter 6, each of the analysis boundaries are assigned periodic translational and rotational velocities based on the total displacement of the belt in going from the closed profile to the open profile. Due to symmetry, boundaries were employed at each end of the belt and only one half of the belt was simulated, as shown in Figure 3-9. This procedure also aided in reducing the calculation time due to a reduction in the total number of particles simulated. As the bulk solid material movement due to the boundary translation and rotation is only in one plane, the relative movement of adjacent particles in the transverse direction is minimal (if any).

Figure 3-9 – Frictionless Boundaries

Only one half of the 600mm wide belt was modelled and images of an empty and loaded belt taken from the simulation are presented in Figure 3-10 and Figure 3-11. The height of the bulk solid material pile was 120mm (centre of belt) and the conveyor surcharge angle was 13°.
Analysis boundaries were placed along the entire width and longitudinal length of the belt section simulated. The length of the simulated section of the belt was 50mm and the width dimensions, horizontal and vertical displacements, velocities in the x and y direction and the rotational velocities are all presented in Chapter 6.

Results presented in Chapter 7 regarding transition zone analysis were conducted in a similar manner to the above. Following the generation and shaping of the particles, the inclined sides of the belt were rotated outwards for a total duration of 0.5s, until the sides were horizontal. From the DEM simulations, x, y, z co-ordinates and the cross sectional profile of the simulated particles were assessed.

For modelling of discharge trajectories in Chapter 7 and transfer chute flow analysis presented in Chapter 8, boundaries were assigned surface velocities (without translation). In this manner, the cyclic, continuous movement of the conveyor belt and typically static transfer chute geometry is maintained while the calculation time is reduced.

In Chapter 8, for the stacker transfer chute case study considered, specification of analysis boundaries was initiated in the impact plate (hood), loading chute (spoon) and the receiving belt as illustrated in Figure 3-12. A similar procedure was also adopted for the other case study considered, with the remainder of boundary geometry detailed in Chapter 8.
3.3 Summary

An overview of the DEM contact models for both PFC3D and Rocky has been presented. Also presented was a general description of the modelling procedure adopted in Chapter 4 to Chapter 8. Based on earlier research findings, a selection of parameters has been assumed and is presented in Table 3-1. Other parameters such as coefficient of particle/particle sliding friction, \( \mu_p \), particle rolling friction, \( \mu_r \), coefficient of particle/boundary friction, \( \mu_w \), coefficient of particle adhesive fraction, \( \mu_{adh} \), and adhesive distance, \( S_{adh} \), are all selected in the calibration process presented in Chapter 4.

Both of the software codes used in the modelling process and their respective contact models differ in their constituents; however, they have been used as a complementary tool in the analysis of bulk solid interactions in belt conveying systems in subsequent chapters. Strict comparison between the two contact models and software codes is outside the scope of this thesis.
Chapter 4 - Characterisation and Calibration of Bulk Solid Materials

In the field of bulk solids materials handling, flow properties of bulk solid materials form the principal basis for efficient and reliable design of handling facilities such as bins, hoppers, feeders, stockpiles, mechanical conveyors and transfer chutes. Accurate characterisation of these properties is of utmost importance in the understanding of material behaviour in these systems, including bulk solid interactions in belt conveying systems.

Typical properties of commonly handled bulk solid materials usually have a macroscopic character and include bulk density, wall friction angle, internal angle of friction and angle of repose, all of which can be directly measured in the laboratory. These properties allow for characterization of bulk solid materials based on the relevant nature of the content of the material sample and exhibited stress/strain behaviour. In contrast, properties of a material simulated with DEM are defined by the contact model used, and are of a microscopic nature. They include shear or Young’s modulus, bulk elastic stiffness, co-efficient of restitution, damping factor, particle/particle, particle/boundary friction, Poisson’s ratio, rolling/sliding friction and particle density.

Due to the difficult and impractical nature of measuring the aforementioned microscopic parameters (apart from particle density) the implementation of specific calibration type tests is required. These calibration tests provide a link between “real” material properties and simulation parameters and include angle of repose (rolling friction), internal friction (rolling friction, cohesive strength) and wall friction (sliding/dynamic friction) tests in which microscopic simulation parameters are adjusted to “fit” macroscopic behaviour.

A brief overview of characterisation techniques for physical testing of bulk solid material properties is outlined and the materials used for analysis are presented. From these tests, a set of properties for each of the bulk solid materials is obtained which form the basis for the calibration of DEM parameters. An overview of the calibration process is presented, including the application of three calibration tests. From the results, a set of representative parameters for each of the bulk solid materials tested is specified. The DEM parameters selected from the calibration process are implemented in the analysis contained within subsequent chapters.

4.1 Bulk Solid Material Properties

When a bulk solid material flows, it does so by the process of internal shear (yield) or boundary shear. Internal shear occurs within the mass of the bulk solid material itself and boundary shear occurs at the boundary wall. Standard tests for the determination of the flow properties of bulk solid materials include the Jenike shear, uni-axial (unconfined), bi-axial and tri-axial shear tests, torsional or ring shear, direct
shear wall friction, inverted wall friction and compressibility test amongst others. Other tests for the
determination of bulk solid material characteristics not indicative of flow properties include the angle of
repose test, transportable moisture limit test, particle size distribution test, moisture content, particle
density test, surface roughness and dust extinction moisture test. There are advantages and disadvantages
to all of these tests, however, a brief description of the tests used in this research work is provided
including:

1. Jenike direct shear test
2. Compressibility test
3. Jenike style direct wall friction test
4. Inverted direct wall friction test
5. Particle density test
6. Angle of repose test

In addition to the above tests, in order to quantify the properties of the coarse materials above a 4mm size
fraction, additional testing was also performed with the use of a slump plane angle test, complete draw
down test and an inclining wall friction test.

4.1.1 Jenike Direct Shear Test

During the flow of bulk solid materials with varying particle sizes, large particles move in their entirety
while shearing occurs across the fine particles. For this reason, the internal strength of bulk solid
materials is dependent on the content of the fine particles present. In order to determine the strength of a
full size sample of a bulk solid material containing particles of mixed sizes, a requirement for the Jenike
direct shear test is that the fines are screened to a -4mm size fraction, following which testing can be
performed.

The direct shear test apparatus was designed and built by Jenike [51] in 1953. It typically consists of a
shear cell of circular cross section, a gravity vertical loading system and an automated shearing force
device as shown in Figure 4-1. Generally, the cell used has an inside diameter of 95mm, giving the cell
cross-sectional area of 1/140m², (Roberts [84]). The shearing force (advancing in the horizontal plane) is
provided by means of an electro-mechanically driven loading system at a rate of 2.5mm/min. A record of
shear force versus normal load is measured.
The bulk solid material sample is sheared at a number of different normal loads, and from the resulting major and minor Mohr circles, a series of yield loci is generated, shown in Figure 4-2.

From the yield loci, the following properties of the bulk material may be determined:

1. Flow Function - measure of the material's strength as a function of the major consolidating pressure
2. Effective Angle of Internal Friction, $\delta$ - slope angle of the line formed from the origin and tangent to the major Mohr circle
3. Kinematic Angle of Internal Friction, $\phi_i$ - slope angle of the line at the point of tangency of the Instantaneous Yield Locus (IYL) and the Mohr circle
Extrapolation of the Yield Locus to the shear stress axis is a measure of the bulk solid materials cohesive stress at zero consolidation (normal stress).

The Flow Function allows for the bulk solid material to be classified according to its level of internal strength. In general, the Flow Function of a free flowing non-cohesive bulk solid material will coincide with the horizontal axis with zero, or very low, strength across a range of increasing loads. Also, for free flowing non-cohesive bulk solid materials, such as dry sand or grain, the internal angles of friction, \( \delta \) and \( \phi \), are equal. The Flow Function of a cohesive bulk solid material may be horizontal, inclined, or curved and will show strength at small loads and quite possibly an increase in strength with increased loads. This is the reason that cohesive bulk solid materials tend to retain their shape, even when not stressed.

4.1.2 **Slump Plane Angle Test**

For coarse materials above a size fraction of 4mm, the accurate implementation of the Jenike direct shear test (in the 95mm cell) is not possible due to mechanical interlocking and rotation of individual particle grains. The internal angle of friction for coarse materials was approximated with the use of a shear plane slump test. This test involved loosely pouring each material to be tested inside a 300mm square testing apparatus with clear Perspex sides. Once the container was full, one side of the box was removed (opened) and bulk solid materials flowed out. When in a final settled state, the resulting angle of the pile was then related to the bulk solid material internal angle of friction \( \phi_i \).

Some debate may arise regarding the validity of these tests to be related to the internal angle of friction, however, for free flowing materials the angle of repose is directly related. Research by Carrigy [11], refers to this angle as the critical angle of repose, or the angle of initial yield, and therefore the test is arguably a valid approach. Results presented later in this section will show that the slump test is also applicable to cohesive materials under low consolidation stress, as is directly related to the bulk solid interactions investigated in this thesis.

4.1.3 **Draw Down Test**

The internal angle of friction was also approximated by measuring the draw-down angle of a rathole, or reclaim crater, of a small scale stockpile of the tested materials. Typically, in order for this test to be related to the internal angle of friction, the opening at the base of the stockpile is required to be significantly large. For the tests performed in this study, the overall diameter of the stockpile was approximately 300mm, with the circular gate at the base measuring 100mm in diameter. The stockpile was formed on a raised floor, by gently tilting a container of the bulk solid material from a height of 300mm over the opening which was initially closed. Following the formation of the stockpile, the gate was opened and the internal angles of the crater measured with an electronic level.
4.1.4 **Wall Friction Test**

The wall friction test is similar in nature to the Jenike direct shear test; however, the cell base is replaced with a wall lining material, as illustrated in Figure 4-3. With the shear load advancing horizontally, the normal load is gradually reduced and a shear plane is established at the bulk solid and the wall lining material boundary. Normal stress versus shear stress is measured and recorded from which the Wall Yield Locus (WYL) is generated.

![Figure 4-3 – Jenike Type Direct Wall Friction Test](image)

For any normal stress, the friction angle between the bulk solid and wall lining material is determined by:

\[ \phi_w = \text{wall friction angle} = \tan^{-1} \left( \frac{\text{shear stress}}{\text{normal stress at boundary}} \right) \]

This is illustrated in Figure 4-4 below:

![Figure 4-4 – Wall Yield Locus (WYL), Cohesion and Adhesion](image)

For cohesive bulk solid materials the WYL is often convex upward in shape and when extrapolated to intersect the shear stress axis at \( \tau_o \), the wall friction angle, \( \phi_w \), will decrease with an increase in normal stress. At low consolidation pressure, the wall friction angle is limited by the effective angle of internal friction, \( \delta \), which is defined as the upper bound limit for, \( \phi_w \). At low normal pressure, the bulk solid material will fail internally by shear rather than by sliding along the wall boundary.
4.1.5  Inverted Wall Friction Test

The inverted wall friction tester utilizes a piston and ring assembly larger than the Jenike shear cell (300mm diameter), allowing particle sizes up to 35mm to be tested. The process is similar in nature to the Jenike shear tester, where the piston applies a normal load to the bulk solid within the ring assembly, which is then sheared across a wall sample while load cells measure the normal and shear forces on the wall sample. A photograph and a diagram illustrating the process are shown in Figure 4-5 and Figure 4-6 below.

Figure 4-5 – Inverted Wall Friction Test Apparatus

Figure 4-6 – Inverted Wall Friction Test Diagram [84]

Adhesion and cohesion can be directly measured with this apparatus as gravity is compensated due to the inverted design.

4.1.6  Compressibility Test

The bulk density of a bulk solid material is determined using a compressibility tester. The unit consists of a 63.5mm diameter and 19mm deep cell, which is filled with a -4mm sample of the bulk material. Variable normal loads are applied to the sample by means of a lid and weight carrier, and the compression of the sample is measured with a displacement transducer. Knowing the sample volume, mass and applied load allows the relationship between bulk density and the consolidation pressure to be determined. A diagram of the apparatus is presented in Figure 4-7.
In addition to the compressibility test above, which was only performed for the materials which could be screened to a -4mm size fraction, a loose poured bulk density test was also performed. This test involved filling a container of known volume with each of the bulk solid materials tested and a calculation was performed based on the measured mass to obtain the loose poured bulk density. Loose poured bulk density was used in the calculation of the Krause and Hettler theoretical approach for the determination of active and passive stresses presented in Chapter 5 and Chapter 6.

### 4.1.7 Particle Solids Density Test

A simple method for measuring the particle density is to use a nitrogen displacement pycnometer which measures the relative density of the solid particles in nitrogen within a small volume. In this way, the true particle volume of a sample of known mass is obtained from:

\[
\rho_p = \frac{\text{mass}}{\text{particle volume}}
\]  

\[(4.1)\]

### 4.1.8 Angle of Repose Test

Determination of the angle of repose of granular materials warrants significant debate as it is implemented extensively within the field of Discrete Element Modelling of granular bulk solid materials. A wealth of literature exists regarding this topic and researchers have been debating the accurate characterisation and representation of the angle of repose of granular materials for some time.

The angle of repose of a material is the natural angle formed by gravity discharge of the material and measured from a horizontal base, (Roberts [84]). The angle of repose has also been referred to as the angle (to the horizontal) at which loose granular material will stand when piled, or dumped, and has been equated with the angle of internal friction of granular material in the loosest state, (Carrigy [11]). The size of the control volume, particle size and shape, moisture content, type of test performed (including
time to conduct the test) and surface characteristics of individual particles all influence the angle of repose. An overview of some valuable research work conducted to date is now presented.

Grasselli and Herrmann [30] performed experiments in a rotating cubic cell to determine the angle of repose for various dry granular materials and also the influence on the effects of rigid walls (control volume) on the angle of repose. The size of the particles tested in the apparatus was between 80-400µm and the overall physical size of the test apparatus was in the order of 100mm cube. A ratio of wall separation to particle diameter in excess of 180 was found to produce angles of repose values indistinguishable from the same angle that would be obtained with a free pile. Burkalow [10] on the other hand investigated particle sizes in the range of 25-75mm in a confined box from which a ratio of particle diameter to wall separation of 1:6 proved negligible when quantifying angles of repose.

The characteristic length, representing a distance over which the confining walls influence the angle of repose was found to increase as particle size was decreased. Grasseli and Herrmann [30] state that the presence of walls can considerably increase the angle of the heap (10-20%) which is attributed to force lines (similar to arching) from packing supported by the wall, making bridges, and providing increased stability.

Kalman et al [54] investigated the effect of particle to surface (base) co-efficient of friction on the angle of repose with coloured alumina spheres. It was observed that the friction of the base surface (for a conical pile) influences the angle of repose, with higher friction surfaces of free flowing materials governed by the internal angle of friction. Higher piles were observed to produce more accurate and lower characterising angles when compared to smaller piles. Dynamic effects associated with conical pile tests were observed to lead to the collapse of the pile following the addition of a critical number of particles. This was also observed by Carrigy [11] and Burkalow [10].

Burkalow [10] observed that due to the inconsistencies in pile formation arising from the effect of dynamic slumping effects, the angle of the conical pile may not be uniform all around the cone, since slumping usually takes place on only one section of the pile at a time. A more static type two dimensional approach to the test was applied. After forming a conical pile on a base, a part of the base was withdrawn and the entire mass slumped in one movement, attaining the angle of repose along its entire length. As the impact of the falling particles was eliminated the slope was observed to be of greater uniformity and stability. Burkalow [10] also conducted experiments with dry artificial materials with closely graded and carefully sieved dry sand and found that the angle of repose varied inversely with the size of the particles. However, the influence of size was observed to be relatively weak and variations in size were attributed to lower differences in the angle of repose when compared to the influence of shape and surface characteristics. The influence of particle size was observed to be reversed in imperfectly sorted materials, for which, the angle of repose was observed to be greater for the coarser grains.
Carrigy [11] and Burkalow [10] observed that shape was of a stronger influence. Departure from the spherical form and increasing roughness of the surface of the particles led to an increase in the angle of repose.

In view of the above discussion, four different types of angle of repose tests were performed:

1. Raising Funnel (70mm diameter opening outlet, 50mm high with a 250mm diameter funnel giving a total height of 165mm)
2. Raising Funnel (100mm diameter opening outlet, 75mm high with a 350mm diameter funnel giving a total height of 475mm)
3. Raising Cylinder (50mm diameter, 150mm high)
4. Raising Cylinder (100mm diameter, 300mm high)

To determine the angle of repose, a quantity of the material to be tested is carefully poured through a funnel or cylinder onto a rough surface or bed of the same material, in such a way as to form a symmetrical cone. Initially, the funnel or cylinder is placed on the prepared surface and filled with the bulk solid. The apparatus is then raised slowly, ensuring the choke-flow is maintained. An illustration of the procedure can be found in Figure 4-8. The angle between the horizontal plane and the slope of the cone is measured on four sides of the cone. This procedure is repeated three times and the final angle of repose is determined as an average value of all recorded angles.

![Figure 4-8 – Angle of Repose Test][84]

The process was recorded with a video camera, with a height measurement device in place to record the lifting rate which was monitored to be approximately constant at 20mm/s. The results were averaged across all tests, from which a range of characterising angles of repose are shown in Table 4-2.

4.1.9 Inclining Wall Friction Test

The inclining wall friction test apparatus consists of a 350 x 450mm wall lining material clamped on a tilting table which rotates about a central axis. The speed of rotation is variable up to 0.1 rad/s. A Jenike shear cell ring is placed on the wall sample, which is filled with the bulk solid material to be tested. A
metal sensor is present at the top end of the cell ring. As the wall and bulk solid material is tilted, separation between the metal sensor and the cell ring terminates the test, allowing for a direct measurement of the angle at which the bulk solid material commences sliding down. The angle of sliding is correlated to the theoretical angle of wall friction, $\phi_w$. Consolidation may also be applied to the bulk solid material sample within the cell, and can either be left on during the test or taken off (following consolidation). It is worthy of noting that, if the consolidation load is left on during the test, the influence of any additional force component arising also needs to be considered. A photograph of the test apparatus is shown in Figure 4-9 and a 3D model is shown in Figure 4-10.

From the rotational velocity, a representation of the time of rotation versus wall friction (inclination of the test plate) was determined from the rotational speed and the results are plotted in Figure 4-11 below. Knowing the time when the material commences to slide, determination of the sliding angle of friction, $\phi_w$, or coefficient of friction, $\mu_w$, can be obtained.
4.2 Bulk Solid Materials Tested and Experimental Results

Five bulk solid materials were selected for testing, including gravel, riversand, magnetite, coal and iron ore. The gravel and magnetite materials contained particles over 3mm and below 25mm, and would be considered as average bulk solid materials with a free flowing to moderate nature. The riversand material contained particles in the 2-5mm size range and would be classified as an easy to handle free flowing material. Coal and iron ore samples exhibited a range of particle sizes, with a considerable content of -4mm size particles and would be classified as moderate-to-difficult to handle cohesive bulk solid materials. Also, from observation, the magnetite material exhibited particles which were furthermore from spherical.

4.2.1 Jenike Direct Shear Test Results

Coal and iron ore samples were tested in the Jenike shear cell apparatus for internal angle of friction. The results are shown in Figure 4-12 and Figure 4-13 below.
4.2.2 Slump Test Results

The coarse materials, namely gravel, magnetite and riversand contained particles above a size fraction of 4mm, and therefore were not able to be tested in the Jenike shear cell. The internal angle of friction for these materials was approximated with the application of the slump test. Typical results obtained from the slump plane tests are shown in Figure 4-14. Furthermore, the slump test was also performed for the coal and iron ore materials, with typical results shown in Figure 4-15.

Comparing results shown in Figure 4-15 above to the results obtained with the Jenike shear test in Figure 4-12 and Figure 4-13, it is reasonable to state that for the consolidation stress and materials investigated, the slump plane test can be used as an indication of the internal angle of friction. Results obtained from the Jenike test, slump plane test and the draw down test were averaged and a range of the internal angles of friction measured is presented for each material in Table 4-2. Internal angles of friction estimated from
slump tests were used in the selection of input parameters applied in DEM simulations and existing theoretical investigation of belt loading profiles presented in Chapter 5 and Chapter 6.

### 4.2.3 Wall Friction Test Results

The inverted wall friction tester was used for determining wall friction characteristics of the riversand, gravel and magnetite materials on a sample of rubber belt. This data was also implemented in the analysis undertaken in Chapter 5 and Chapter 6. Testing with the inverted wall friction apparatus produced linear angles of $26^\circ$, $30^\circ$ and $32^\circ$ degrees for the riversand, magnetite and gravel materials respectively.

The Jenike type wall friction tester was used determine the wall friction angle between the coal and iron ore with the same belt sample. The results from the experiments are shown in Figure 4-16.

![Figure 4-16 – Jenike Wall Friction Results](image)

### 4.2.4 Compressibility Results

Compressibility testing results for coal and iron ore as a function of consolidation load are shown in Figure 4-17.
Inclining Wall Friction Test Results

The results obtained from the experimental laboratory testing with the inclining wall friction apparatus are summarized in Table 4-1. Each material was consolidated with a normal load in the range of 0-10kPa following which the normal load was removed and rotation commenced.

Table 4-1 – Inclining Wall Friction Summary

<table>
<thead>
<tr>
<th>Load (kPa)</th>
<th>Coal $\phi_w$</th>
<th>Gravel $\phi_w$</th>
<th>Magnetite $\phi_w$</th>
<th>Riversand $\phi_w$</th>
<th>Iron Ore $\phi_w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>34°</td>
<td>29°</td>
<td>28°</td>
<td>24°</td>
<td>32°</td>
</tr>
<tr>
<td>2.5</td>
<td>34°</td>
<td>31°</td>
<td>28°</td>
<td>25°</td>
<td>34°</td>
</tr>
<tr>
<td>5</td>
<td>34°</td>
<td>30°</td>
<td>30°</td>
<td>24°</td>
<td>35°</td>
</tr>
<tr>
<td>7.5</td>
<td>34°</td>
<td>28°</td>
<td>29°</td>
<td>25°</td>
<td>32°</td>
</tr>
<tr>
<td>10</td>
<td>35°</td>
<td>27°</td>
<td>27°</td>
<td>24°</td>
<td>30°</td>
</tr>
</tbody>
</table>

Referring to Figure 4-16 and results presented above, good overall correlation can be observed for the coal and iron ore materials.

Summary of Parameters from Experimental Tests

A summary of the test results obtained with the each of the previously described procedures is presented in Table 4-2 below. Bulk solid material properties shown form the basis for the calibration of the DEM parameters presented in the following section.
Table 4-2 – Measured Bulk Solid Material Properties

<table>
<thead>
<tr>
<th>Material</th>
<th>$\phi_w$</th>
<th>$\phi$</th>
<th>$\theta_b$</th>
<th>$\rho_p$ (kg/m³)</th>
<th>$\rho$ (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>coal</td>
<td>33°</td>
<td>55°-65°</td>
<td>36°-41°</td>
<td>1350</td>
<td>800</td>
</tr>
<tr>
<td>gravel</td>
<td>32°</td>
<td>40°-50°</td>
<td>32°-35°</td>
<td>2589</td>
<td>1640</td>
</tr>
<tr>
<td>magnetite</td>
<td>30°</td>
<td>40°-50°</td>
<td>32°-37°</td>
<td>3665</td>
<td>2070</td>
</tr>
<tr>
<td>riversand</td>
<td>26°</td>
<td>30°-40°</td>
<td>27°-32°</td>
<td>2582</td>
<td>1590</td>
</tr>
<tr>
<td>iron ore</td>
<td>34°</td>
<td>55°-65°</td>
<td>34°-40°</td>
<td>3753</td>
<td>2000</td>
</tr>
</tbody>
</table>

$\phi_w =$ wall friction angle, $\rho_p =$ particle density, $\rho =$ loose poured bulk density

4.3 Calibration of DEM Simulation Parameters

Due to the high computational effort required for calculations associated when modelling the behaviour of bulk solid materials, idealisations of the DEM simulation model are necessary in order to minimize the calculation time. Particle number, shape, weight, as well as the particle stiffness influence the computation time significantly and therefore determine the degree of idealization. These so called idealised simulation models force the calibration of simulation parameters to provide realistic behaviour of the simulated bulk material. Importantly, the common trend in the modelling of typical applications is to select large particles and relatively low stiffness in order to minimise the simulation time. However, regarding the latter a low stiffness can lead to erratic behaviour, (Katterhagen et al [60]). In order to reduce the simulation time, the physical size of the calibration test apparatus is invariably linked to the maximum particle size modelled.

In reality, well established tests such as the Jenike shear cell is the preferred device for the determination of internal friction of bulk solid materials, however, this technique is limited to materials in the sub 4mm size fraction. This effectively means that even if this test is replicated via DEM simulations, a form of scaling procedure would be required in order to correlate the results obtained for fine particles to larger ones. Grima [33, 34] has explored this avenue with some validated results, including the novel selection of a swing arm type and a large wall friction testing apparatus.

An extensive array of DEM calibration testing procedures exists, with much debate on-going regarding the most practical method to be implemented. To make the calibration process more complex, DEM modelling packages are not ubiquitous and a number of dissimilar models are being implemented by various research groups. Each parameter of the contact models presented in Chapter 3 may, or may not, influence the results of the particular application. Past research has indicated the influence of a range of parameters regarding bulk solid material behaviour in particular applications. As such, the list of parameters presented in Chapter 3, Table 3-1 has been assumed in the calibration simulations presented herein and also in the analyses undertaken in further chapters.

A brief overview regarding prior research findings on the influence of particle parameters selected in the simulations is outlined. Focus is on the angle of repose type test which by far has had the most frequent
application. Subsequently, a similar procedure and calibration technique is adopted for the selection of additional particle parameters not identified in Table 3-1.

Zhou et al [108, 110] performed a numerical investigation of the angle of repose of mono-sized spheres. In these studies it was observed that the angle of repose is insensitive to particle density, Poisson’s ratio, damping coefficient and loading stiffness (Young’s Modulus) under the conditions and parameters simulated. However, it was observed that the angle of repose is influenced by the coefficient of particle sliding friction, particle size, coefficient of particle rolling friction and the overall control volume of the test. Sliding and rolling friction of the particles were observed to lead to an increase in the angle of repose. Contrary to this observation, an increase in particle size was observed to lead to a decrease in the measured angle. A smaller ratio of container thickness to particle size was observed to lead to an increase in the angle of repose. It is important to note that, the research presented was based on rolling friction model Type-A, (Ai et al [3]) and is different to the model used in DEM simulations in this thesis.

Li et al [66] investigated the influence of sliding friction between spherical particles and between particles and walls while ignoring the effect of rolling friction. A study of the angle of repose of spherical particles was performed in a rectangular 2D plane container and a 3D conical axi-symmetric pile. In this study, it was observed that the friction of the control volume influenced the angle of repose. In order to prevent the influence of the dimensions of the control volume on the angle of repose, the control volume geometry was selected to be 5-6 times larger compared to the maximum particle size simulated. An important observation in this work identified the influence of the kinematic nature of the conical axi-symmetric pile (3D) test compared to the 2D plane test, with greater angles observed in plane strain.

Consequently, the calibration process and methods presented below are focused on the attainment of characterising DEM parameters influencing the sliding, rolling, and internal strength of bulk solid materials modelled with mono-sized spheres. The parameters sought in this study are the coefficient of particle sliding friction, $\mu_p$, coefficient of rolling friction, $\mu_r$, adhesive fraction, $\mu_{adh}$, and the adhesive distance, $s_{adh}$, between particles. Three different types of calibration tests were performed, and include the raising cylinder angle of repose, the shear plane angle or slump test and the inclining wall friction test. All simulations were performed with 5mm and 10mm mono-sized spherical particles.

The angle of repose test involved filling a cylinder of known dimensions with each bulk solid material modelled and lifting at a slow controlled rate (20mm/s). Two different types of cylinder, $\phi = 50$mm (150mm high) – PFC3D simulations only, and $\phi = 100$mm (300mm high) – PFC3D and Rocky simulations were used. An image taken from the Rocky simulation is presented in Figure 4-18.
The resultant angles of the pile (to the horizontal) were measured using a Matlab algorithm procedure adopted from that described by Wensrich and Katterfeld [105]. The measured angle was used to quantify the simulated bulk solid material angle of repose \( \theta_R \). For each combination of particle sliding, \( \mu_p \), and rolling, \( \mu_r \), friction, four measurements were conducted on different lengths of the top surface of the conical pile, at 60\% and 40\% of the radius, at 70\% and 30\% of the radius, at 80\% and 20\% of the radius and at 10\% and 90\% of the radius. An illustration of the measurement process is presented in Figure 4-19.

Similarly, the shear plane angle calibration test involved filling a 300mm cube and opening one side, allowing the modelled bulk solid material to flow out. The plane angle is then measured to the horizontal once the material has come to rest. The shear plane slump test was implemented to determine the internal shear plane angle, or the angle of internal friction, \( \phi_i \). For DEM simulations using the Rocky software, the slump test was performed with 5mm and 10mm spherical particles. Simulations of the slump test using PFC3D were only performed with 10mm particles. A typical image of the slump test taken from the Rocky simulation is shown in Figure 4-20.
Measurements were taken based on the length of the stationary top surface of the particles located above the entire horizontal length of the cube bottom and for 40% of the pile located above the middle of the horizontal length. This is illustrated in Figure 4-21 below.

The measurements presented in Figure 4-22 to Figure 4-35 were taken at 10.0s following opening of the side. The calibration procedure involved simulating a large number of tests in an incremental procedure by modifying particle sliding friction, $\mu_p$, and particle rolling friction, $\mu_r$. Modelling with Rocky also included the incremental variation of parameters regarding adhesion between the particles. Simulations were performed with a wall friction coefficient, $\mu_w = 0.5$, which was taken as a median value.

4.3.1 PFC3D Calibration Test Results

For the angle of repose tests conducted with PFC3D, the raising cylinder was nestled above a single layer of constrained particles (150mm and 300mm diameter footprint) in order to negate the influence of the friction of the floor surface. Particles were packed using the method of Donohue [23] as mentioned.
previously. The smaller cylinder was used for the simulation of 5mm mono-sized spherical particles and the larger cylinder used for the 10mm particles.

### 4.3.1.1 Angle of Repose Simulation Measurements

Figure 4-22 and Figure 4-23 below show the results of the angle of repose calibration simulations based on a particle density, \( \rho_p \), of 2590 kg/m\(^3\) with 10mm and 5mm mono-sized spheres.

In general, from the above two graphs it can be observed that maintaining a constant ratio of diameter of the testing apparatus to the maximum particle size simulated, similar trends are observed. The ratio of the
diameter of the cylinder to the maximum particle size was 10:1 in both sets of simulations. Furthermore, from Figure 4-22, it can be observed that above a particle friction, $\mu_p$ of 0.5, the changes in angle of repose for a constant rolling friction are not as significant as for lower particle friction values as shown in Figure 4-23.

### 4.3.1.2 Slump Test Simulation Measurements

Figure 4-24 and Figure 4-25 show the results of the slump test modelled using the PFC3D software. The influence of particle density, within the particle friction range of $0.5 < \mu_p < 0.9$ is plotted in Figure 4-25 from which it can be observed that the particle density does not play a crucial role in the measured angle. The ratio of test apparatus length to maximum particle diameter was 30:1. Also, in similar observation to the angle of repose calibration tests, results from the slump plane tests indicate attainment of a steady state maximum slump plane angle, in this case reaching approximately $60^\circ$. The influence of rolling friction is observed to decrease at higher particle friction.

![Figure 4-24 - PFC3D Slump Test](image)

(10mm Spheres - $\rho_p = 2590\text{kg/m}^3$, $\mu_w = 0.5$)
Figure 4-25 – PFC3D Slump Test

(10mm Spheres - 1350kg/m³ ≤ ρₚ ≤ 3665kg/m³, 0.5 < μₚ < 0.9, μ_w = 0.5)

A selected number of images showing x-y co-ordinates obtained from the PFC3D calibration simulations presented in Figure 4-22 to Figure 4-25 above can be found in Appendix A.

4.3.2 Rocky Calibration Tests

Rocky parameters that were investigated include particle friction, rolling friction and adhesion between the particles. Calibration tests performed include angle of repose (raised cylinder) and slump plane angle tests. Additionally, a brief investigation of sliding and rolling friction was also conducted by simulating the inclining wall friction test.

For all tests, particles were loaded by gravity and allowed to settle for a period of 3.0s, following which the test commenced. For the raising cylinder simulations, this involved the cylinder being displaced in a vertical direction at a velocity of 20mm/s. For the slump plane angle tests, the simulation involved opening one side of the test box and allowing the particles to flow out. For the inclining wall friction simulations, rotation of the stationary particles on top of the wall sample was performed at angular rotational speed of 0.1rad/s.

4.3.2.1 Angle of Repose Calibration Measurements

Angle of repose calibration tests performed with Rocky software only modelled the 300mm x 100mm diameter cylinder and did not include the layer of particles located on the base. Simulations were performed using both 5mm and 10mm spherical particles. The particle density was based on gravel and riversand (2590kg/m³) and the coefficient of wall friction, μ_w of the floor and the cylinder was set to 0.5. This value was selected based on a median value indicative of the frictional properties that would be
observed in practice. The ratio of diameter of the cylinder to maximum particle size was 10:1 and 20:1 for simulations with 10mm and 5mm particles respectively.

Figure 4-26 and Figure 4-27 below show the results for the 10mm and 5mm spheres respectively. The results from these two graphs indicate similar overall trends. At low particle and low rolling friction values, the angle of repose is lower compared to high particle and rolling friction values. In a similar manner to calibration simulations performed with PFC3D and discussed in Section 4.3.1, the rolling friction values towards the top of the range simulated (i.e. $\mu_r = 0.9$), resulted in steady state maximum angles of repose in the range of 46-48°.
4.3.2.2 *Slump Test Measurements*

Figure 4-28 and Figure 4-29 below show the results of the slump plane test for the 10mm and 5mm particles respectively, based on the particle density of gravel and wall friction coefficient of 0.5.

Results indicate low angles of repose for smaller particle and rolling friction values, and higher angles of repose, with an increase in particle and rolling friction. Above particle friction of 0.5, rolling friction was observed to be of less influence. A steady maximum value of around 50° is approached with an increase in particle friction, $\mu_p$. 
In a similar manner to results obtained with PFC3D, a selected number of images showing x-y co-
ordinates obtained from the Rocky calibration simulations presented in Figure 4-26 to Figure 4-29 above 
can be found in Appendix A.

4.3.2.3 Influence of Adhesion Between Particles

The influence of adhesion parameters was investigated with Rocky only. It is important to note that 
accurate modelling of cohesion is on-going work across both software codes. Research in this area has 
been conducted by Gröger et al [32] focusing on modelling cohesion with parameters related to surface 
tension and liquid bridge theory, while Luding et al [67] implemented the adhesion force in parallel with 
the plastic, hysteretic non-linear contact force model. To gain an understanding of the adhesion influence 
on particles, the physics of granular media at a microscopic level needs to be investigated, including 
capillary condensation and the formation of the liquid bridges adhering on the surface of the particles 
(Bocquet et al [7, 8] and Fraysse et al [26]).

For granular bulk solid materials, adhesion between the particles is directly related to the content of the 
fines present and also the moisture content. In experiments conducted on the influence of moisture 
content on a coal heap, Standish et al [91] argued that this is also dependent on the particle size 
distribution. Increasing the moisture content increases the angle of repose to a maximum value, and any 
进一步 increase in moisture content is likely to decrease the angle of repose. This effect is similar to bulk 
solid material trends in the Jenike shear cell. Generally, greatest strength of cohesive granular materials is 
exhibited in the 60-80% of the saturated moisture content range, and following this peak a reduction in 
strength occurs as water is adsorbed to the surface of the particles.

On tests performed with crushed quartz, Carrigy [11] noted that below a certain size content (under 
approx. 80µm), quartz powders were found to not behave like granular materials due to possible increase 
in surface area to volume ratio which magnifies the accumulation of static electricity or thin molecular 
layers of adsorbed moisture. The influence of humidity and the capillary forces arising between particles 
was also noted as being of importance as particle size decreases, Grasselli and Herrmann [30]. Carstensen 
and Chan [12], communicated the relationship between cohesion, particle size and angle of repose of 
mono disperse powders. The latter investigated the influence of particle interlocking on the frictional and 
cohesive tendency of fine particles and state that the cohesive force is proportional to the diameter of the 
particle.

Complexities associated with the incorporation of cohesion have thus far prevented a universal model 
agreed upon by all researchers. Consequently, the focus of this research is on the implementation of the 
existing model available, in this case the normal adhesive force model implemented in Rocky. The results 
presented for the 10mm diameter particles modelled with adhesion are for an adhesive distance, s_{adh}, of 
1.0mm and adhesive fraction coefficient, μ_{adh}, of 0.6. The results of these simulations are presented in 
Figure 4-30 and Figure 4-32. Results presented in Figure 4-31 and Figure 4-33 are for 5mm particles
modelled with adhesion applying an adhesive distance, $s_{adh}$, of 0.5mm and adhesive fraction coefficient, $\mu_{adh}$, of 1. The particle density simulated was based on a median particle density (2590kg/m³) and the wall friction coefficient, $\mu_w$, of the cylinder, floor base and slump test cube walls were maintained at 0.5.

Figure 4-30 – Rocky Angle of Repose with Adhesion (300mm x φ100mm cylinder)

(10mm Spheres - $\rho_p = 2590$kg/m³, $\mu_{adh} = 0.6$, $s_{adh} = 1.0$mm $\mu_w = 0.5$)

Figure 4-31 – Rocky Angle of Repose with Adhesion (300mm x φ100mm cylinder)

(5mm Spheres - $\rho_p = 2590$kg/m³, $\mu_{adh} = 1$, $s_{adh} = 0.5$mm $\mu_w = 0.5$)
When comparing results presented in Figure 4-30 to Figure 4-33, to results without adhesion, it becomes clear that adhesion between the particles leads to an increase in the angle of repose and slump plane angle across the range of parameters simulated. The influence of adhesion at the low end of the particle and rolling friction values simulated indicate only a slight increase in the angles of repose and slump plane angles. The influence of adhesion is more pronounced with an increase in rolling friction, with an increase in angles of repose and slump plane angles in the order of 10-20° observed for particle and rolling friction coefficients of 0.9.
A selected number of x and y co-ordinates used for the measurement of angles of repose and slump plane angle tests with the adhesive force are shown in Appendix A. Images obtained from Rocky simulations show that the inclusion of the attractive force between particles leads to irregular piles with non-uniform top surfaces. This effect is dependent on the volume (height of the pile) of the material sample simulated, dynamic effects of slumping and also the geometry of the cylinder or control volume of the testing apparatus. Another important parameter of influence is the particle size relative to geometry of the testing apparatus. Therefore, the angles of repose presented above are strictly applicable for the dimensions of the control volume chosen in the simulations, and different angles of repose may be obtained with different geometry of the test apparatus, cylinder vertical velocity and/or particle size.

4.3.2.4 **DEM Calibration of 30mm Particles**

When modelling bulk solid interactions in belt conveying systems such as discharge trajectories and transfer chute flow, the physical geometry is typically of larger dimensions than can be modelled with 5 or 10mm particles. The calculation time due to the large number of particles which would be associated with these particle sizes would be impractical. Consequently, larger particles are selected in order to minimize the calculation time. For example, the typical size range of particles modelled for belt conveyor systems of geometrical dimensions corresponding to 2000mm wide belts, at capacities in the order of 8000t/h, is in the order of 20-40mm for simulations of approximately 20.0 to 30.0s in duration. On the overall physical scale of these systems, analysis of volumetric flow is the necessity and from a practical point of view, it is the assessment of flow or no flow that is typically required.

In order to investigate the influence of DEM parameters with an increase in particle size, slump tests were performed with Rocky for 30mm mono-size spherical particles, with the control volume of the slump test also scaled linearly. For the tests presented previously, the slump test control volume characterising length was based on a linear scale of 30:1 (linear dimension 300mm to particle size 10mm) and 60:1 (linear dimension of 300mm to particle size 5mm). The dimensions of the test apparatus volume for calibration with larger particles were selected to be 900mm cubed, maintaining a linear scale of characterising length to maximum particle size of 30:1. The graphs obtained from the analysis, including the influence of adhesion between the particles are presented in Figure 4-34 and Figure 4-35. Particle density was based on a median value of 2590kg/m$^3$. The adhesive parameters selected were based on $1/10^{th}$ particle diameter ($s_{adh}$) and coefficient of adhesive fraction of 0.5 ($\mu_{adh}$).
Results shown in the above figures are in good correlation with the served results presented in Section 4.3.2.2 and 4.3.2.3. This indicates that the parameters obtained in the smaller scale test simulations are also applicable when modelling larger particles, given that the control volume is of identical ratio to the particle size considered. As a real world validation of the coal calibrated in the above simulations, the image presented in Figure 4-36 below presents a stockpile observed at a coal handling terminal reflective of the angle of repose which would be expected for a large pile - if it could be simulated. The physical height of the pile shown is in the order of 10.0m.
4.3.3 **Summary and Specification of Calibration Parameters**

Referring to the results presented in Figure 4-22 to Figure 4-35, it is worthwhile to note that above a particle sliding friction $\mu_p$ of 0.5, the parameter of greatest influence on the simulated angles of repose, $\theta_R$, and slump plane angles, $\phi_i$, is particle rolling friction, $\mu_r$. The influence of particle density can be observed to have little effect as shown in Figure 4-25.

Implementing the calibration tests presented in Section 4.3.1 and 4.3.2, parameters selected for modelling the bulk solid materials described in Section 4.2 are presented in Table 4-3. It is obvious that more than one combination of parameters inevitably exists that would be adequate in describing the flow behaviour of each of these materials. However, the parameters outlined below are based on typical characterising parameters for:

1. Free flowing materials with low angle of internal friction and low angle of repose - riversand
2. General granular materials with moderate internal angle of friction and angle of repose - gravel and magnetite
3. Moderately difficult to handle cohesive materials with high internal angle of friction and angle of repose – coal and iron ore

The parameters specified for riversand were based on the low end of the range, for gravel and magnetite on the middle of the range, and for coal and iron ore the top end of the range simulated. Due to more than one set of characterising parameters existing for each of the materials calibrated, a sensitivity analysis will accompany all results presented in subsequent chapters.
Table 4-3 – Summary of Characterising DEM Parameters

<table>
<thead>
<tr>
<th>Material</th>
<th>( \mu_p )</th>
<th>( \mu_r )</th>
<th>( \mu_{adh} )</th>
<th>( s_{adh} ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>gravel</td>
<td>0.8</td>
<td>0.4</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>magnetite</td>
<td>0.8</td>
<td>0.4</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>riversand</td>
<td>0.5</td>
<td>0.1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>coal, iron ore (5mm)</td>
<td>0.9</td>
<td>0.9</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>coal, iron ore (10mm)</td>
<td>0.9</td>
<td>0.9</td>
<td>0.6</td>
<td>1</td>
</tr>
<tr>
<td>coal (30mm)</td>
<td>0.9</td>
<td>0.9</td>
<td>0.5</td>
<td>3</td>
</tr>
</tbody>
</table>

4.4 Inclining Wall Friction Simulation

From previous studies, it is maintained that the coefficient of sliding friction between the particles and the boundary (wall) is defined as the coefficient of sliding friction, \( \mu_s \), and may be used directly as obtained from flow property tests, (Katterfeld [55-59]). To illustrate this further, the inclining wall friction test plate was set up so as to replicate the physical test from the laboratory in Rocky. Following the loading of particles into a cell of dimensions equal to the Jenike cell, the plate was set to rotate about the centre of curvature as in the laboratory experiments.

Particles were allowed to settle under gravity for a period of 3.0s, following which the rotation of the inclining plate was commenced at a rotational speed of 0.1 rad/s. Over a period of 7.0s the average velocity of the entire mass of particles was recorded versus time. Simulations were performed for all of the bulk solid materials identified in Section 4.2 and modelled thus far. Results are shown in Figure 4-37 to Figure 4-45 below. Also shown is the influence of rolling friction.

![Figure 4-37 – Gravel Inclining Wall Friction Test Results](image)

(5mm Spheres - \( \mu_p = 0.8 \), \( \mu_r = 0.4 \), \( \mu_{adh} = 0.6 \), \( \rho_p = 2590 \text{kg/m}^3 \))
Figure 4-38 - Gravel Inclining Wall Friction Test Results

(10mm Spheres - $\mu_p = 0.8$, $\mu_r = 0.4$, $\mu_w = 0.6$, $\rho_p = 2590\text{kg/m}^3$)

Figure 4-39 - Magnetite Inclining Wall Friction Test Results

(10mm Spheres - $\mu_p = 0.8$, $\mu_r = 0.4$, $\mu_w = 0.6$, $\rho_p = 3665\text{kg/m}^3$)
Figure 4-40 – Riversand Inclining Wall Friction Test Results

(5mm Spheres - $\mu_p = 0.5$, $\mu_r = 0.1$, $\mu_w = 0.5$, $\rho_p = 2590$ kg/m$^3$)

Figure 4-41 – Coal Inclining Wall Friction Test Results

(5mm Spheres - $\mu_p = 0.9$, $\mu_r = 0.9$, $\mu_w = 0.7$, $\rho_p = 1350$ kg/m$^3$)
Figure 4-42 - Coal Inclining Wall Friction Test Results

(5mm Spheres - \( \mu_p = 0.9, \mu_r = 0.9, \mu_w = 0.7, \mu_{adh} = 1, s_{adh} = 0.5\text{mm}, \rho_p = 1350\text{kg/m}^3\))

Figure 4-43 - Coal Inclining Wall Friction Test Results

(10mm Spheres - \( \mu_p = 0.9, \mu_r = 0.9, \mu_w = 0.7, \mu_{adh} = 0.6, s_{adh} = 1.0\text{mm}, \rho_p = 1350\text{kg/m}^3\))
Figure 4-44 – Iron Ore Inclining Wall Friction Test Results

(5mm Spheres - $\mu_p = 0.9$, $\mu_r = 0.9$, $\mu_w = 0.7$, $\rho_p = 3753 \text{kg/m}^3$)

Figure 4-45 – Iron Ore Inclining Wall Friction Test Results

(5mm Spheres - $\mu_p = 0.9$, $\mu_r = 0.9$, $\mu_w = 0.7$, $\mu_{adh} = 1$, $s_{adh} = 0.5 \text{mm}$, $\rho_p = 3753 \text{kg/m}^3$)

Referring to results presented in Figure 4-37 to Figure 4-45, incipient sliding of the entire mass of particles without the influence of rolling friction is indicative of the friction coefficient, $\mu_w$, between the particles and the inclining wall. For each of the simulations performed the value of the coefficient of wall friction chosen correlates well to the coefficients of friction at which sliding was observed in the simulations. Results of simulations without adhesion between the particles show that when particle friction, $\mu_p$, is equal to the rolling friction, $\mu_r$, sliding of the entire mass of particles will occur at the inclination angle, for which the friction of the wall is equal to the coefficient of friction of the inclination angle; i.e. $\tan^{-1}(\theta_w)$. For rolling friction values lower than particle friction, the particles will not slide, but...
roll down the incline of the wall at inclinations lower than the angle of the wall. Similarly, for particles exhibiting adhesion, results show that rolling friction parameters lower than the particle friction will slide down the wall at much higher angles of inclination compared to particles without adhesion. This is explained by the fact that adhesion between the particles acts as means of countering the rolling motion of the particles.

### 4.4.1 Summary

Standard experimental tests for the determination of the flow properties of bulk solid materials have been presented. From these tests a set of macroscopic characterising properties were obtained and are summarized in Table 4-2. These properties were further investigated using additional calibration tests performed in the laboratory.

The calibration process consisted of a suite of tests which were also simulated using DEM simulations. Particle parameters were selected in an iterative manner in order to establish simulated material behaviour akin to that observed during laboratory tests. From results presented, it is evident that more than one set of parameters describing the physical characteristics of bulk solid materials is possible. However, for the purpose of modelling in this thesis, only one set of DEM parameters has been chosen to be representative of each of the materials tested. These parameters are summarized in Table 4-4 below.

<table>
<thead>
<tr>
<th>Material</th>
<th>µw</th>
<th>µp</th>
<th>µr</th>
<th>µadh</th>
<th>sadh (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>gravel</td>
<td>0.6</td>
<td>0.8</td>
<td>0.4</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>magnetite</td>
<td>0.6</td>
<td>0.8</td>
<td>0.4</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>riversand</td>
<td>0.5</td>
<td>0.5</td>
<td>0.1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>coal, iron ore (5mm)</td>
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<td>0.9</td>
<td>0.9</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>coal, iron ore (10mm)</td>
<td>0.7</td>
<td>0.9</td>
<td>0.9</td>
<td>0.6</td>
<td>1</td>
</tr>
<tr>
<td>coal (30mm)</td>
<td>0.7</td>
<td>0.9</td>
<td>0.9</td>
<td>0.5</td>
<td>3</td>
</tr>
</tbody>
</table>

Typical categories of bulk solid materials related to the set of parameters of each material selected can be summarized as follows:

1. Free flowing materials - riversand
2. General granular materials - gravel and magnetite
3. Moderately difficult to handle cohesive materials – coal and iron ore

For representative DEM parameters to be selected, the importance of understanding the nature of the physical properties of bulk solid material was demonstrated. The behaviour of bulk solid materials in physical tests allows for insight into behaviour that would be observed in practice. In view of the possibility of more than one set of characterising parameters, a sensitivity analysis will accompany DEM simulations and analysis presented in subsequent chapters.
Chapter 5 - Bulk Solid and Conveyor Belt Interaction: Laboratory Simulation Tests

An experimental laboratory testing facility was designed and built with the aim of simulating the opening and closing movement of the inclined sides of a troughed conveyor belt. The aim of the testing facility was to investigate the transverse behaviour of the bulk solid material due to the movement of the inclined sides.

A pressure pad measurement system (TekScan) was used to measure the loads exerted by bulk solid materials on the belt surface during the opening and closing cycle. Laboratory experiments were undertaken using a number of different bulk solid materials. Simulations using DEM were also performed and a comparison to experimental measurements is presented. A comparison is also made to the loads calculated implementing the existing theoretical approach by Krause and Hettler [64] outlined in Section 2.1.5.2.

Finally, a brief sensitivity analysis is performed on the DEM simulation parameters. The sensitivity analysis investigated the influence of particle properties, wall friction, oscillation frequency, troughing angle and the quantity of bulk solid material on the results obtained during opening and closing.

5.1 Laboratory Belt Simulation Test Facility

A testing facility was developed with the aim of simulating the motion of a belt conveyor and examining the movement of bulk solid materials. The facility consists of a fixed middle section and two pivoting sides. The fixed middle section is attached to the top of an oscillating plate and as the plate oscillates up and down the inclined sides pivot. Two transparent walls are fixed at either end allowing for the observation of cross-sectional behaviour of the bulk solid material as it is oscillated. A diagram of the testing facility is illustrated in Figure 5-1.

![Figure 5-1 - Experimental Testing Facility](image)
The dimensions and configuration of the testing facility were selected with the aim of comparing to an existing 600mm wide re-circulating belt conveyor system with a three-roll idler roll troughing profile of 35° located at TUNRA Bulk Solids laboratories. The length of the test facility is 500mm. The width of the middle horizontal section is 220mm, based on the idler roll length of the 600mm wide re-circulating belt conveyor system. The frequency and stroke during testing was nominally set to 2.5Hz and 12mm respectively. This is based on a conveyor belt velocity of 3m/s, idler spacing of 1.2m and 1.0% sag. The vertical displacement resulted in the inclined sides moving through an angular displacement of 3°, from an initial inclination equal to the idler roll inclination of the re-circulating belt conveyor of 35°.

5.2 Preliminary Testing and Analysis

Preliminary testing was performed on three bulk solid materials, namely riversand, gravel and magnetite, with material characteristics as detailed in Table 4-2.

5.2.1 Experimental Procedure

Each bulk solid material was poured into the middle of the troughed section of the testing facility illustrated in Figure 5-1 and shaped manually to conform to the load profile presented in Section 2.1.3. The contact perimeter and pile height was the same for all tests. A 100% cross section, calculated by Eqn. (2.1.4) was used resulting in a pile height of 120mm. This resulted in the length of contact between the bulk solid material and the horizontal section of the belt to be 220mm, and the length of contact with the inclined side to be 130mm. The experimental test facility was then oscillated at a frequency of 2.5Hz and a stroke of 12mm for a period of 100 seconds, during which data was recorded with the I-Scan software.

The I-Scan [92] matrix-based pressure measurement system consists of a thin (0.1mm) flexible tactile TekScan sensor. The sensor is connected to an interface electronic handle which is connected to the PC via a USB or PS2/PCI Card link. The TekScan sensor pad (model no. 5051, with a load rating of 20 Psi) used in the experiments consists of a matrix width and matrix height of 55mm x 55mm and 1936 sensels is shown in Figure 5-2.

![Figure 5-2 - TekScan Sensor Model 5051 [92]](image-url)
The TekScan sensor pad was calibrated by using two known masses (0.5kg and 2kg) in a two point loading procedure. Following calibration, the pad was placed on a flat surface and a known mass of granular material placed on it to verify the accuracy of calibration results.

The position of the TekScan sensor pad during testing is shown in Figure 5-3. Three tests were conducted on each bulk solid material with the sensor pad located in the top and bottom position as labeled. For each series of tests, the sensor pad was placed on the belt, followed by a 1.5mm thick layer of insertion rubber to preserve the friction of the belt surface and to also prevent damage to the sensor.

![Figure 5-3 - TekScan Pad Measuring Positions](image)

### 5.2.2 DEM Simulation with PFC3D

Discrete Element Modelling (DEM) simulations were performed using PFC3D, utilizing a user defined contact model as outlined in Section 3.1.1 and modeling each material as mono-sized spherical particles. Two different particle diameters were modelled, namely 5mm and 10mm. In addition, to investigate the influence of a particle size distribution, river sand was also modelled with particles in the range of 2-5mm ([Ilic et al.](#)). The simulation involved loading (refer Section 3.2.1) the spherical particles onto a cross section of the belt resembling that of the testing facility. The longitudinal length of the simulated cross section was 65mm compared to 500mm in laboratory experiments in order to minimize the number of particles. Measurement boundaries of 55mm x 55mm were set up in the top and bottom position on each side in the simulation akin to that illustrated in Figure 5-4. The inclined sides were set to oscillate at a frequency of 2.5Hz and an angular displacement of 3° (refer Section 3.2).

![Figure 5-4 –Modelling with PFC3D](image)
5.2.3 **Comparison of Results and Discussion**

Results from experimental testing and DEM simulations for gravel, magnetite and riversand are shown in Figure 5-5, Figure 5-6 and Figure 5-7 respectively. The TekScan results presented are typical of the entire 100s of testing but are only shown for the first 50s for clarity between the top and bottom pad.

![Figure 5-5 – Gravel: Experimental and DEM Simulation Results](image-url)
Figure 5-6 – Magnetite: Experimental and DEM Simulation Results

a) Measured TekScan results

b) DEM Simulation Results (10mm Particles)

c) DEM Simulation Results (5mm Particles)
Comparison of normal force data presented in Figure 5-5 to Figure 5-7 shows good overall correlation between measurements obtained using TekScan sensor pads and results obtained from DEM (PFC3D) simulation analysis. It is evident that the force on the bottom pad is of greater magnitude than the force on the top pad. The DEM (PFC3D) simulation results shown in Figure 5-5 to Figure 5-7 also indicate minimal influence of the particle size (within the range tested) on the overall force. This provides confidence in modelling granular materials with larger particle diameters to reduce computational time.

In addition to the DEM simulation for the purpose of comparing normal forces on the measured sensor pads, simulations were also undertaken to investigate the normal pressure profile of the bulk solid material acting on the inclined sides of the belt. Over a length of 10.0s of simulation time, normal forces were calculated for every occurrence of the belt in the closed and open position. Normal pressure was calculated and the results for each material are presented in Figure 5-8 to Figure 5-10.
Figure 5-8 – Gravel: DEM Average Load Profile on Inclined Side

Figure 5-9 – Magnetite: DEM Average Load Profile on Inclined Side
During the opening cycle, Figure 5-8 to Figure 5-10 show that the maximum pressure occurs towards the idler junction and the minimum stress occurs at the edge of the free top surface of bulk solid material. Similarly, during the closing cycle, the maximum pressure occurs at a distance of 0.04m-0.06m from the idler junction. That is, approximately 30% to 45% of the length of the bulk solid material in contact with the inclined side of the belt.

5.2.4 Conclusion

Preliminary testing and analysis indicated good overall correlation between the normal forces produced by the DEM simulations and the measured TekScan data for both the opening and closing of the inclined sides of the belt. Limitations based on the size of the TekScan sensor pads were evident and prevented the DEM results from being compared over the entire contact perimeter. To overcome this limitation a larger sensor pad was installed, with the results presented in the following section.

5.3 Modified Test Facility

The existing test facility illustrated in Figure 5-1 was once again used, however, during preliminary testing the two smaller TekScan sensor pads were replaced with a large pad that covered the entire belt cross section. Additionally, the test facility was modified to accept a 1050mm wide belt. For the 1050mm belt width, the horizontal section had a width of 440mm, and the corresponding length of the inclined side was 255mm. The frequency and angular displacement during testing was as described in Section 5.1. In both cases, DEM simulations were compared with the experimental data.

For the 600mm belt width, simulations were performed with DEM (PFC3D) and DEM (Rocky). For the 1050mm belt width, simulations were performed with DEM (Rocky) only.
5.3.1 Experimental Procedure

A larger TekScan sensor pad (model no. 5351, with a load rating of 5 Psi) was used in the modified experiments. The larger sensor consists of a matrix width and matrix height of 487.7mm x 426.7mm and 2016 sensels and is shown in Figure 5-11. The larger TekScan sensor was also slightly thicker (0.33mm) compared to the 5051 sensor used during preliminary tests and is shown as it fits on the 600mm wide belt in Figure 5-12.

![Figure 5-11 - TekScan Sensor 5351](image)

A thin (1.5mm) layer of insertion rubber was placed over the entire TekScan sensor, to preserve the frictional properties of the rubber, and to prevent damage to the surface of the sensor pad.

![Figure 5-12 – TekScan Sensor on 600mm Belt](image)

Four bulk solid materials were tested namely, coal, gravel, magnetite and riversand, with bulk solid material properties specified in Section 4.2.6. The experimental procedure involved pouring each bulk
solid material into the middle of the test facility and shaping manually to obtain 100% cross section, based on CEMA [6].

The 5351 TekScan sensor was calibrated in an air bladder type equilibration and calibration device (type PB100B) [92]. A photograph showing the calibration device with the TekScan sensor inserted is shown in Figure 5-13. The procedure involved applying a uniform pressure (air) over the entire sensing area and applying a two point loading calibration force. Following calibration, the pad was placed on a flat surface and known mass of material placed on it to verify accuracy of calibration results.

![Figure 5-13 - PB100B Calibration Device](image)

The two points generally chosen for calibration were at 20% and 80% of the expected load. This generally meant the application of either 2 - 5kPa or 3 - 7kPa loads for testing with the 600mm and 1050mm belt width, respectively.

For all experimental results reported, testing was performed for an oscillation time of 10.0s during which data was obtained for 100 frames. Typical results that were obtained from the I-Scan system are shown in Figure 5-14 and Figure 5-15. In both figures, the left image shows the pressure profile on the sensor when the sides are in the lowermost (open) and the right image shows the pressure profile when the sides are in the uppermost (closed) position.

![Figure 5-14 – Typical Data (3D) Shown in the I-Scan Software](image)
From a normal force versus frame plot for each test, the time and frame corresponding to the maximum and minimum normal force exerted by each bulk solid material on the TekScan pressure sensor can be determined. A typical plot obtained from the TekScan measurements is shown in Figure 5-16.

![Figure 5-15 – Typical Data (top view) Shown in the I-Scan Software](image)

The maximum and minimum values correspond to the when the inclined sides are in the lowermost (open) position and in the uppermost (closed) position respectively. Normal pressure data acting on the entire sensor (each sensel) is exported for each test, however, only the data bound by measurement boxes present in the middle of the pad as illustrated in Figure 5-15 was analysed.

![Figure 5-16 – Typical Normal Force vs. Frame Plot Obtained from I-Scan software (coal)](image)

During each test, all frames in the open position were averaged to obtain the “TekScan Open” pressure profile as a function of the location on the belt. Similarly, all frames corresponding to the closed position of the inclined sides were averaged to obtain the “TekScan Closed” pressure profile. Analysis of the data shows there are three distinct sections within the cross-section of the bulk solid material: a middle section which remained mostly stationary throughout the whole testing process and two “active” sections which ultimately moved relative to a slip surface.
5.3.2 Wall Loads on Inclined Sides

The theoretical approach by Krause and Hettler [64], presented in Section 2.1.5.2, assumes an active stress state is initiated during opening of the inclined sides, and a passive stress state is initiated during closing. While opening, the wedge of bulk solid material on the inclined side will be thrust downwards and outwards. The force acting on the inclined side will be a resultant of the internal friction force, $F_{ia}$, and the force due to the weight of the wedge, $F_{msa}$ (see Figure 2-10). Similarly during closing, the wedge of the bulk solid material on the inclined side will be compressed in an inward and upward direction. The force acting on the inclined side will be a resultant of the internal friction force, $F_{ip}$, this time acting in the opposite direction, and the force due to the weight of the wedge, $F_{msp}$ (see Figure 2-11). The force vector diagrams presented in Figure 2-12 and Figure 2-13 were used to determine the normal forces, $F_{nsa}$, and $F_{nsp}$, on the inclined sides of the belt due to opening and closing.

In the examination of the above theoretical approach it is important to note that for a 100% loaded belt, the conditions (for validity of Krause and Hettler analysis) noted in Section 2.1.5.2 are not satisfied. The conditions are once again presented below:

$$\theta_a \geq \theta_c, \quad \theta_p \geq \theta_c, \quad \lambda \leq \phi_i \quad \text{and} \quad \beta \geq \phi_w$$

Referring to the bulk solid material properties shown in Table 4-2 for the materials investigated in this study, the belt surcharge angle, $\lambda$, and active, $\theta_a$, and passive, $\theta_p$ stress state angles according to Eqn. (2.1.28) and Eqn. (2.1.30) are summarized in Table 5-1. Also shown is the measured angle from the junction of the inclined side and horizontal section to the top and centre of the bulk solid material height, $\theta_c$.

<table>
<thead>
<tr>
<th>Material</th>
<th>$\lambda$</th>
<th>$\theta_p$</th>
<th>$\theta_a$</th>
<th>$\theta_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>coal</td>
<td>13°-15°</td>
<td>31°</td>
<td>75°</td>
<td>48°-53°</td>
</tr>
<tr>
<td>gravel</td>
<td>13°-15°</td>
<td>40°</td>
<td>56°</td>
<td>48°-53°</td>
</tr>
<tr>
<td>magnetite</td>
<td>13°-15°</td>
<td>41°</td>
<td>59°</td>
<td>48°-53°</td>
</tr>
<tr>
<td>riversand</td>
<td>13°-15°</td>
<td>47°</td>
<td>47°</td>
<td>48°-53°</td>
</tr>
<tr>
<td>iron ore</td>
<td>13°-15°</td>
<td>30°</td>
<td>72°</td>
<td>48°-53°</td>
</tr>
</tbody>
</table>

Note: Belt surcharge angle $\lambda$ and angle to the centre of the pile $\theta_c$ were controlled by the height of material on each rig. For the 600mm rig the height of the pile was 120mm which resulted in $\lambda = 13°$ and $\theta_c = 48°$. For the 1050mm rig the height of the pile was 250mm which resulted in $\lambda = 15°$ and $\theta_c = 53°$.

Referring to the material properties presented in Table 5-1 it can be observed that the angle from the junction of the inclined side and horizontal section, to the top and centre of the bulk solid material height, $\theta_c$, is always greater than the calculated passive stress state slip plane angle, $\theta_p$. Similarly, it can be
observed that, the active stress state slip plane angle, $\theta_a$, is not greater than, $\theta_c$, for the riversand material. Importantly, from the passive force vector diagrams shown in Figure 2-13, it can be visualized that, for a constant side inclination angle, $\beta$ and wall friction angle, $\phi_w$, if the internal angle of friction is significantly high, there exists a value of $\theta_p + \phi_i$ at which the line, $F_{ip}$, will never cross the line, $F_{sp}$.

The theoretical approach outlined in Section 2.1.5.2, was applied to calculate the normal force acting on the inclined side of the belt during opening and closing. From the normal force calculation for $F_{nsa}$ and $F_{nsp}$, and knowing the cross sectional area of the inclined side along which the forces are acting, the corresponding normal pressures, $P_{nsa}$ and $P_{nsp}$ were calculated. Where the conditions described by relation (2.1.5.2*) were not satisfied, the passive stress state angle, $\theta_p$, was assumed equal to, $\theta_c$, during closing. Similarly, for the active stress state, where the conditions were not satisfied, the active stress state angle, $\theta_a$, was assumed equal to, $\theta_c$, during opening (riversand only). Table 5-2 and Table 5-3 below present a summary of calculated normal force, $F_{nsa}$ and, $F_{nsp}$, and corresponding normal pressure, $P_{nsa}$ and, $P_{nsp}$, for the 600mm wide belt. Also presented are the active and passive forces, $F_{sa}$ and, $F_{sp}$, determined from the Krause and Hettler theoretical approach.

### Table 5-2 - Calculated Wall Load Profiles for the 600mm Test Rig (Opening)

<table>
<thead>
<tr>
<th>Material</th>
<th>$F_{sa}$ (N/m)</th>
<th>$F_{nsa}$ (N)</th>
<th>$p_{nsa}$ (kPa)</th>
<th>$P_{nsa}$ (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>coal</td>
<td>42.7</td>
<td>2.0</td>
<td>0.3</td>
<td>0.6</td>
</tr>
<tr>
<td>gravel</td>
<td>112.9</td>
<td>5.3</td>
<td>0.7</td>
<td>1.5</td>
</tr>
<tr>
<td>magnetite</td>
<td>127.3</td>
<td>6.1</td>
<td>0.9</td>
<td>1.7</td>
</tr>
<tr>
<td>riversand</td>
<td>108.3</td>
<td>5.4</td>
<td>0.8</td>
<td>1.5</td>
</tr>
</tbody>
</table>

### Table 5-3 - Calculated Wall Load Profiles for the 600mm Test Rig (Closing)

<table>
<thead>
<tr>
<th>Material</th>
<th>$F_{sp}$ (N/m)</th>
<th>$F_{nsp}$ (N)</th>
<th>$p_{nsp}$ (kPa)</th>
<th>$P_{nsp}$ (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>coal</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>gravel</td>
<td>424.3</td>
<td>19.8</td>
<td>2.8</td>
<td>5.5</td>
</tr>
<tr>
<td>magnetite</td>
<td>495.1</td>
<td>23.6</td>
<td>3.3</td>
<td>6.6</td>
</tr>
<tr>
<td>riversand</td>
<td>231.6</td>
<td>11.5</td>
<td>1.6</td>
<td>3.2</td>
</tr>
</tbody>
</table>

**Note:**

$F_{sa}$ – Total Active Force,

$F_{sp}$ – Total Passive Force;

*Longitudinal length of material – 0.055m,

*Length of material on inclined side, $L_s$ – 0.13m;

*Total Normal Force ($F_{nsa}$, $F_{nsp}$) = Total Force $\times$ longitudinal length of material $\times$ cos$\theta_w$,

*Average Normal Pressure ($p_{nsa}$, $p_{nsp}$) = Total Normal Force/ (longitudinal length of material $\times L_s$)

*Total Normal Pressure ($P_{nsa}$, $P_{nsp}$) = 2 $\times$ Average Normal Pressure (assumed to act 1/3 of the length up the inclined side
In a similar manner, Table 5-4 and Table 5-5 present a summary of the calculated normal force, \( F_{nsa} \) and \( F_{nsp} \) and corresponding normal pressure, \( P_{nsa} \) and \( P_{nsp} \) for the 1050mm wide belt. Also presented are the active and passive forces, \( F_{sa} \) and \( F_{sp} \), determined from the theoretical approach of Krause and Hettler.

### Table 5-4 - Calculated Wall Load Profiles for the 1050mm Test Rig (Opening)

<table>
<thead>
<tr>
<th>Material</th>
<th>( F_{sa} ) (N/m)</th>
<th>( F_{nsa} ) (N)</th>
<th>( P_{nsa} ) (kPa)</th>
<th>( P_{nsp} ) (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>coal</td>
<td>175.4</td>
<td>7.4</td>
<td>0.6</td>
<td>1.2</td>
</tr>
<tr>
<td>gravel</td>
<td>474.7</td>
<td>20.1</td>
<td>1.6</td>
<td>3.2</td>
</tr>
<tr>
<td>magnetite</td>
<td>443.2</td>
<td>19.2</td>
<td>1.5</td>
<td>3.0</td>
</tr>
<tr>
<td>riversand</td>
<td>453.9</td>
<td>20.4</td>
<td>1.6</td>
<td>3.2</td>
</tr>
<tr>
<td>iron ore</td>
<td>482.6</td>
<td>20.0</td>
<td>1.6</td>
<td>3.1</td>
</tr>
</tbody>
</table>

### Table 5-5 - Calculated Wall Load Profiles for the 1050mm Test Rig (Closing)

<table>
<thead>
<tr>
<th>Material</th>
<th>( F_{sp} ) (N/m)</th>
<th>( F_{nsp} ) (N)</th>
<th>( P_{nsp} ) (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>coal</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>gravel</td>
<td>2057</td>
<td>87.2</td>
<td>6.8</td>
</tr>
<tr>
<td>magnetite</td>
<td>2345.9</td>
<td>101.6</td>
<td>8.0</td>
</tr>
<tr>
<td>riversand</td>
<td>1005.4</td>
<td>45.2</td>
<td>3.5</td>
</tr>
<tr>
<td>iron ore</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

**Note:**

- \( F_{sa} \) – Total Active Force,
- \( F_{sp} \) – Total Passive Force;
- **Longitudinal length of material** – 0.05m,
- **Length of material on inclined side**, \( L_s = 0.255m \)
- **Total Normal Force** (\( F_{nsa}, F_{nsp} \)) = Total Force \( \times \) Longitudinal length of material \( \times \) cos\( \theta \)
- **Average Normal Pressure** (\( p_{nsa}, p_{nsp} \)) = Total Normal Force/ (longitudinal length of material \( \times L_s \))
- **Total Normal Pressure** (\( P_{nsa}, P_{nsp} \)) = 2 \( \times \) Average Normal Pressure (assumed to act 1/3 of the length up the inclined side)

Referring to Table 5-2 to Table 5-5, for coal and iron ore during closing the force vector diagrams were not able to be implemented as the internal angle of friction, \( \phi_i \), was too large for equilibrium to occur. These two materials exhibited high cohesion and as such the theoretical approach was modified by assuming that the wedge of material located above the inclined side will fail internally at either the internal angle of friction, \( \phi_i \), or the active state angle, \( \theta_a \), given by Eqn. (2.1.28). In order to compare between the experimental and simulated values in figures presented in Section 5.2.3, the calculated normal pressures due to the weight of the wedge are also shown for the remaining materials tested. Where the conditions described by (2.1.5.2*) were not satisfied, the passive stress state angle, \( \theta_p \), was assumed equal to the angle, \( \theta_c \), during closing.
Similarly, for the active stress state, where the conditions described by (2.1.5.2*) were not satisfied, the active stress state angle, $\theta_a$, was assumed equal to, $\theta_c$, during opening (riversand only).

In all of the figures presented in Section 5.2.3, a comparison of the weight of the wedge in the absence of frictional forces between the bulk solid material and the belt during belt closing is also presented. The lower bound value (as shown in Table 5-6 and Table 5-7 below), $P_{ms}(\theta_a)$ and the upper bound value, $P_{ms}(\phi)$ or $P_{ms}(\theta_c)$ were calculated for both belt widths of 600mm and 1050mm.

<table>
<thead>
<tr>
<th>Material</th>
<th>$P_{ms}(\theta_a)$ or $P_{ms}(\theta_c)$ (kPa)</th>
<th>$P_{ms}(\phi)$ or $P_{ms}(\theta)$ (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>coal</td>
<td>0.7</td>
<td>0.8</td>
</tr>
<tr>
<td>gravel</td>
<td>1.8</td>
<td>2.2</td>
</tr>
<tr>
<td>magnetite</td>
<td>2.2</td>
<td>2.7</td>
</tr>
<tr>
<td>riversand</td>
<td>2.2</td>
<td>2.4</td>
</tr>
<tr>
<td>iron ore</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Table 5-7 - Calculated Wall Load Profiles Assuming Only Weight of Wedge Acting During Closing (1050mm Belt)

<table>
<thead>
<tr>
<th>Material</th>
<th>$P_{ms}(\theta_a)$ or $P_{ms}(\theta_c)$ (kPa)</th>
<th>$P_{ms}(\phi)$ or $P_{ms}(\theta)$ (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>coal</td>
<td>1.4</td>
<td>1.7</td>
</tr>
<tr>
<td>gravel</td>
<td>4.0</td>
<td>4.3</td>
</tr>
<tr>
<td>magnetite</td>
<td>5.2</td>
<td>5.4</td>
</tr>
<tr>
<td>riversand</td>
<td>4.1</td>
<td>4.1</td>
</tr>
<tr>
<td>iron ore</td>
<td>3.1</td>
<td>3.7</td>
</tr>
</tbody>
</table>

5.3.3  Discrete Element Modelling with PFC3D and Rocky

Discrete Element Modelling (DEM) simulations were performed on the 600mm wide belt with both PFC3D and Rocky software. Simulations were performed on the 1050mm wide belt using only the DEM (Rocky) software.

Figure 5-17 – Modelling with DEM (Rocky): 600mm wide belt
The simulation procedure with DEM (PFC3D) was identical to that described in Section 5.2.2 and illustrated in Figure 5-4. However, normal forces and pressures were only exported for simulation duration of 10.0s. Also, only the results for normal pressure (and not normal force) with respect to location on the belt are presented. Discrete Element Modelling (DEM) simulations with Rocky, utilizing the contact model as outlined in Section 3.1 were performed by modeling each material as mono-sized 5mm or 10mm spherical particles. Spherical particles were generated (refer Section 3.2.2) onto a cross section of each belt resembling that of the experimental facility for the full longitudinal length of 500mm. Measurement boundaries of 50mm in length were placed in the middle of the simulated test facility akin to that illustrated in the left image in Figure 5-17 (shown in red and white). For each belt width (600mm and 1050mm) simulated, the dimensions of the measurement boundaries are presented in Section 3.2.2. Similarly to the PFC3D simulations, the inclined sides were set to oscillate at a frequency of 2.5Hz and an angular displacement of 3° about the pivot point which is located at a distance of ±0.11m for the 600mm wide belt and at ±0.19m for the 1050mm wide belt.

For the Rocky simulations, the test rig was loaded with particles and profiled to the cross section observed during experimental testing over a time period of 3.0s, following which oscillation commenced. To investigate the modelled behaviour of each bulk solid material, the duration of each simulation was performed for a period of 30.0s. Normal force results were averaged (over a period of 10.0s only) corresponding to the open and closed position of the inclined sides of the simulated belt conveyor test facility, from which the normal pressures were calculated. As mentioned previously, only the normal pressures are presented in the following sections.

The results obtained from PFC3D and Rocky simulations are presented in Section 5.2.3 and a comparison is made to results obtained from the TekScan experimental measurements and the theoretical approach by Krause and Hettler.

5.3.4 Comparison of Experimental, Simulated and Theoretical Results

Three distinct sections within the transverse profile of each bulk solid material were once again visible during oscillation simulations: a middle section which remained mostly stationary throughout the whole testing process and two “active” sections which moved relative to a slip surface. Although this process is not truly representative of that observed in practice, it acts as a guide in determining the influence of the opening and closing of the sides of the belt on the bulk solid material. In practice, the middle section will not remain stationary due to the undulating movement of the bulk solid material on the conveyor belt as it travels between successive idler roll sets.

5.3.4.1 600mm Experimental Testing Facility

Figure 5-18 and Figure 5-19 below show shearing of the bulk solid material stream observed during experimental laboratory testing with coal following 30.0s of oscillation. This is due to the cohesive nature of the coal at the moisture content tested.
The cross section co-ordinates of the coal modelled in DEM (Rocky) are presented in Figure 5-20. Comparison of the results presented in Figure 5-19 and Figure 5-20 indicates good overall correlation between the transverse shape of the bulk solid material cross section observed during experimental testing and DEM (Rocky) simulations. However, from close inspection it can be observed that due to the particle size modelled, the DEM simulation is not able to predict the crack propagating the entire depth of the bulk solid material to the belt surface (as was observed during experimental testing).

A comparison of the normal pressure measured from experimental testing with the TekScan sensor pad and pressure calculated based on gravity is shown in Figure 5-21. The pressure due to gravity was determined based on the simple $\rho \cdot g \cdot h$ calculation, where $h$ was the measured height of the bulk solid.
material pile at the location considered. Results indicate that the loading on the inclined sides is very close to the gravitational load. This presents the notion that the passive stress state is not initiated.

Figure 5-21 – Coal: TekScan Results

Comparison of results obtained experimentally with the TekScan sensor, and results obtained from PFC3D and Rocky DEM simulations is presented in Figure 5-22. Also presented are the calculated theoretical values from Table 5-2, Table 5-3 and Table 5-6 based on the method of Krause and Hettler. DEM (Rocky) results presented in Figure 5-22 are for 5mm mono-sized spheres with parameters of $\mu_p = 0.9$, $\mu_r = 0.9$, $\mu_w = 0.7$ with and without adhesion between the particles.

Figure 5-22 – Coal: Open and Closed Load Profile Summary

(5mm Spheres - $\mu_p = 0.9$, $\mu_r = 0.9$, $\mu_w = 0.7$, $\rho_p = 1350$kg/m$^3$)

Results presented in Figure 5-22 indicate that the adhesive parameters chosen for the simulation do not have a significant influence on the pressure exerted by the bulk solid material on the belt surface. For the opening cycle, good correlation between the experimental, DEM simulations and the theoretical approach
by Krause and Hettler can be observed. For the closed cycle, DEM simulations predict a higher load on the inclined sides compared to the measured data.

From comparative observation of Figure 5-19 and Figure 5-20, it is evident that the DEM simulations do not predict the transverse cracking propagating the entire material burden depth most likely due to the particle size modelled. This results in an over prediction in the normal pressure profile on the inclined side given by the DEM simulations as the effect of shearing and separation of the wedge from the remainder of the material stream is not entirely captured. This is further emphasized from the comparison of the theoretical calculation to the experimental results, which indicates that the total pressure on the inclined side of the belt correlates well with the weight of the wedge only.

The observed transverse profile for gravel during experimental laboratory testing and the co-ordinates obtained from DEM (Rocky) simulations following 30.0s of oscillation are shown in Figure 5-23 and Figure 5-24 respectively. Good comparison between the experimental results and DEM simulation is evident.

From the comparison of experimental data and gravitational loads presented in Figure 5-25, the loading on the inclined sides in the open cycle is in good agreement. The loading evident in the closed cycle indicates an increase above the gravitational component.
Experimental results obtained from the TekScan sensor and a comparison to DEM simulations is presented in Figure 5-25 and Figure 5-26. Also presented are the calculated theoretical values based on the approach by Krause and Hettler. DEM results presented in Figure 5-26 are for 10mm mono-sized spheres with parameters of $\mu_p = 0.8$, $\mu_r = 0.4$ and $\mu_w = 0.6$.

Comparison of results in Figure 5-26 indicates that the pressure acting on the inclined side of the belt, as calculated by the theory of Krause and Hettler, is over predicted. This indicates that the passive stress state is not initiated during the closing cycle. A better correlation is evident when comparing the pressure acting on the inclined sides simply due to the weight of the wedge only.
Good correlation between the observed experimental and simulated transverse profile for the magnetite material is also shown in Figure 5-27 and Figure 5-28. A comparison of the normal pressure measured from experimental testing with the TekScan sensor and pressure calculated based on gravity is shown in Figure 5-29. The measured pressure profile in the closed cycle acting on the inclined sides is shown to be greater than gravity. This indicates resistance of the material to the contraction of the cross section and is a function of the internal friction of the bulk solid material.

Figure 5-27 – Magnetite: Cross Section Following 30.0s of Oscillation - Experimental

Figure 5-28 – Magnetite: Cross Section Co-ordinates Following 30.0s of Oscillation - DEM (Rocky)

(10mm Spheres - $\mu_r = 0.8$, $\mu_v = 0.4$, $\mu_a = 0.6$, $\rho_p = 3665\text{kg/m}^3$)

Figure 5-29 – Magnetite: TekScan Results
Results obtained from experimental testing with the TekScan sensor, results from DEM simulations and theoretical calculations are presented in Figure 5-30. DEM results presented in these figures are for 10mm mono-sized spheres with parameters of $\mu_p = 0.8$, $\mu_r = 0.4$ and $\mu_w = 0.6$. Similarly to the gravel material, these results indicate that applying the theory of Krause and Hettler, the pressure on the inclined sides for the magnetite material is over predicted. This again demonstrates that the passive stress state is not initiated as the theory assumes. A better correlation is evident when comparing the pressure acting on the inclined sides simply due to the weight of the wedge only. It is believed that the representative pressure distribution will be somewhere in between these two values.

![Figure 5-30 – Magnetite: Open and Closed Load Profile Summary](image)

The transverse profile observed for riversand during experimental laboratory testing is shown in Figure 5-31 following 30.0s of oscillation.
Similarly, the cross section co-ordinates of the riversand modelled with DEM (Rocky) are presented in Figure 5-32 at the corresponding time of oscillation.

Comparison of the transverse profile results presented indicates good correlation between the shape of the bulk solid material cross section observed during experimental testing and DEM simulation. The corresponding comparison of the normal pressure measured from experimental testing with the TekScan sensor and pressure calculated based on gravity is shown in Figure 5-33.
Comparison between the experimental, simulated and the theoretical approach are presented in Figure 5-34. Simulations were performed with 5mm mono-sized spheres with parameters of $\mu_p = 0.5$, $\mu_r = 0.1$ and $\mu_w = 0.5$.

Results regarding the open cycle indicate good correlation between the experimental, simulated and theoretical calculation. In the closed cycle, the results obtained from the DEM simulations over predict the loads on the inclined sides of the belt compared to the measured values. The consistently lower values measured from the experimental tests are believed to be due to the relatively low loads measured when compared to the pressure sensitivity of the TekScan sensor pad. Comparison between simulated and theoretical results shows good correlation, indicating that the passive stress is initiated. This correlation is in line with the notion that the theoretical approach by Krause and Hettler is based on free flowing materials, such as riversand.

Experimental testing and DEM simulation results for the 600mm wide belt presented in Figure 5-21 to Figure 5-34 indicate that the material with highest particle and loose poured density (magnetite) exhibited the greatest normal pressure on the belt surface during opening and closing. The material with the lowest particle and loose poured density (coal) exhibited the lowest normal pressure. Overall, the results obtained from both the experimental tests and DEM simulations for the opening of the sides show good correlation to the values calculated from the theoretical approach of Krause and Hettler. This indicates that the active stress state is being initiated with the relatively small (i.e. $3^\circ$) angular displacement of the inclined sides.

For the closing cycle, the experimental and DEM simulation results both show lower values than those calculated from the theoretical approach of Krause and Hettler for all materials investigated, with the exception of riversand. Furthermore, the gravel and magnetite results indicate the pressure on the inclined side of the belt is nearer to only the weight of the wedge of bulk solid material.
The investigation presented in Section 5.3.4.1, has thus far focused on the loads exerted by bulk solid materials on the inclined sides of the belt. To further illustrate the correlation observed for the open cycle and closed cycles, a comparison of the pressure profiles, including the resultant load on the middle section of the belt, $\text{P}_m$, is presented in Figure 5-35. The results shown for gravel are typical to what would be observed for the range of materials presented.

![Figure 5-35 – Gravel: Pressure Profile including Loading on the Middle Portion of the Belt](image)

Note: The pressure acting on the middle section of the belt was determined based on the vertical components of the pressure calculated for the sides subtracted from the total gravitational load acting.

From the above results it is evident that good correlation between the measured, simulated and theoretical results based on the approach of Krause and Hettler is attained for the open cycle. For the closed cycle, due to the over prediction of the load acting on the inclined sides of the belt by the theory, the load calculated on the middle portion of the belt is significantly lower compared to the measured and simulated values.

This indicates two important conclusions from this investigation:

1. The theoretical approach of Krause and Hettler is only valid for free flowing bulk solid materials, with low internal angles of friction and low friction between the bulk solid material and the conveyor belt, and;

2. The passive stress state is not initiated during the closing cycle for the angular displacement investigated.

In general, there was good agreement between results obtained from simulations with PFC3D and Rocky DEM codes across the range of parameters simulated. This provides confidence that the values obtained from Rocky simulations alone would be sufficient in quantifying the mechanisms of particle interactions.
for this particular application. Subsequently, DEM simulations with Rocky were only performed on the larger belt width of 1050mm.

The transverse profile of all bulk solid materials modelled indicated good correlation with the transverse bulk solid material profile observed during experimental testing. This indicates that the transverse bulk solid material behaviour modelled using DEM simulations as mono-sized spherical particles with parameters obtained via calibration testing presented in Chapter 4 is reflective of what would be observed in reality.

### 5.3.4.2 1050mm Test Facility

In view of experimental findings from the 600mm belt test facility, namely discrepancies between the DEM and TekScan results, the dimensions of the belt were enlarged in order to increase the measurement resolution. All simulations were performed using Rocky, with parameters identical to those presented for the 600mm wide belt simulations in Figure 5-22 to Figure 5-34. Due to symmetry observed in results presented in Section 5.3.4.1, results are only presented for one half of the 1050mm belt. In Figure 5-36 to Figure 5-39, the location on the belt corresponding to “0” is the centre of the belt trough. The junction between the horizontal and inclined sides of the belt is located at ±0.19m from the centre of the belt. The edges of the material cross section are located at ±0.445m from the centre ($L_s = 0.255m$).

Iron ore was chosen as a substitute material for the coal in the DEM simulations due to its cohesive nature and higher bulk density. DEM parameters for iron ore were based on $\mu_p = 0.9$ and $\mu_r = 0.9$ with and without adhesion between the particles. Iron ore and riversand were modeled as 5mm mono-sized spheres. Gravel and magnetite were modelled as 10mm mono-sized spheres.

Figure 5-36 to Figure 5-39 show a comparison of results obtained from experimental testing and DEM simulations for gravel, magnetite, riversand and iron ore respectively. Also presented is a comparison to the theoretical approach by Krause and Hettler, with calculated values from Table 5-4, Table 5-5 and Table 5-7.
Figure 5-36 – Gravel: Open and Closed Load Profile Summary - 1050mm Belt

(10mm Spheres - $\mu_p = 0.8$, $\mu_r = 0.4$, $\mu_w = 0.6$, $\rho_p = 2590\text{kg/m}^3$)

Figure 5-37 – Magnetite: Open and Closed Load Profile Summary - 1050mm Belt

(10mm Spheres - $\mu_p = 0.8$, $\mu_r = 0.4$, $\mu_w = 0.6$, $\rho_p = 3665\text{kg/m}^3$)
Results presented in Figure 5-36 to Figure 5-39 indicate that values calculated based on the existing theoretical approach by Krause and Hettler during opening are in good agreement to experimental and simulated results for all materials tested. Similarly to the 600mm wide belt analysis presented in Section 5.3.4.1, the best correlation between experimental and simulation results is for the riversand (most free flowing) material with the lowest frictional characteristics with the belt surface.

For the closing of the belt cycle, results show some discrepancy between the calculated values and indicate the existing theory over predicts the forces/pressure during closing of the belt. These results once again show that it is highly likely that the passive stress state will not form during closing of the troughed shape.
Comparison of experimental and simulated results indicates that the pressure present on the inclined side of the belt during closing is likely to be somewhere between the weight of the wedge of the material located above the inclined side of the belt, and the passive value calculated from theory, assuming that $\theta_p = \theta_c$.

In addition to the results presented above the loading present on the middle section of the belt, $P_{m}$, was also calculated. Similarly to results presented for the 600mm test facility, results presented in Figure 5-40 below for gravel are typical of what would be expected for all of the materials simulated.

![Figure 5-40 – Gravel: Profile including Loading on the Middle Portion of the Belt](image)

Note: The pressure acting on the middle section of the belt was calculated based on the vertical components of the pressure for the sides subtracted from the total gravitational load acting.

From the above results, good correlation can be observed between the experimental, simulated and theoretical results for the open cycle. As seen in the results shown for the 600mm test facility, for the closed cycle, the calculated load on the middle portion of the belt is significantly lower compared to the measured and simulated values.

The cohesive shearing/separation of the material cross sections in the DEM simulations were found to be close to that observed in the experimental tests. However, similarly to the results presented for coal previously in Figure 5-20, the extent of the crack propagation was once again found to not penetrate the entire depth of the material. Figure 5-41 below shows the particle co-ordinates of the modelled iron ore obtained from the Rocky simulation following 30.0s of oscillation.
Despite qualitative agreement, the expected loads on the belt do not correlate with those measured via experimental testing. It is believed that the particle size modelled may be too large to exhibit loads that are in better correlation with the experimental measurements. This may be explained by the analogy of compressing fine moist sand to compressing large marbles.
It is evident that over prediction of the loads by DEM on the inclined sides is due to the failure to model the entire separation of the wedge from the remainder of the material burden. In the experimental tests cracking was observed to propagate through the entire burden depth, however, cracking observed in DEM simulations was only present for approximately one half of the depth. Similarly to the coal results presented earlier, this is further emphasized in the comparison of experimental results and theoretical calculation, assuming that only the weight of the wedge is acting on the inclined side.

The investigation of bulk solid and conveyor belt interactions presented above leads to the affirmation that the passive stress state is not fully developed within the bulk solid material during belt closing.

5.4 Influence of DEM Parameters

In addition to the analysis presented in Section 5.2 and Section 5.3 a parametric sensitivity analysis was conducted on the 600mm wide test facility with both PFC3D and Rocky DEM codes. Simulations were performed with 5 and 10mm mono-sized spherical particles.

5.4.1 Influence of Particle Properties

Figure 5-44 to Figure 5-47 below show a sensitivity analysis of the particle properties simulated and their influence on the pressure profile exerted by the modelled bulk solid material on the surface of the belt.

![Figure 5-44 – Gravel: DEM (PFC3D) - Open and Closed Loading Profile](image)

Influence of Particle Parameters
Figure 5-45 – Gravel: DEM (Rocky) – Open and Closed Loading Profile

Influence of Particle Parameters

Figure 5-46 – Magnetite: DEM (PFC3D) – Open and Closed Loading Profile

Influence of Particle Parameters
In general, Figure 5-44 to Figure 5-47 show the normal pressure on the belt during closing increases with an increase in both the particle friction and rolling friction. This provides correlation to the calibration parameters shown and discussed in Chapter 4. Furthermore, values obtained with Rocky indicate slightly higher pressures than those obtained from PFC3D.

Analysis of bulk solid materials simulated indicate that lower particle and rolling friction results in lower angles of repose and angles of internal friction (or slump plane angles). Similarly, higher particle and rolling friction results in larger simulated angles of repose and internal friction. Therefore it can be concluded that higher angles of internal friction and angle of repose would produce greater loads on the inclined sides of the belt during the closing cycle.

For the opening cycle of the inclined sides, no notable change was observed due to the variation in modelling parameters. This further indicates that the active stress state is formed and the pressure acting on the belt surface is essentially due to the weight of the mass only.

5.4.2 Influence of Wall Friction

Figure 5-48 to Figure 5-49 present the influence of wall friction on the pressures exerted by the bulk solid material on the belt during opening and closing of the inclined sides. In general, an increase in the friction between the particles and belt indicates an increase in resultant normal pressure during closing and an insignificant change during opening.
Figure 5-48 – Coal: DEM (PFC3D) – Open and Closed Loading Profile

Influence of Wall Friction

\((5\text{mm Spheres} - \mu_p = 0.9, \mu_r = 0.9, \rho_p = 1350\text{kg/m}^3)\)

Figure 5-49 – Coal: DEM (Rocky) Open and Closed Loading Profile

Influence of Wall Friction

\((5\text{mm Spheres} - \mu_p = 0.9, \mu_r = 0.9, \rho_p = 1350\text{kg/m}^3)\)

5.4.3 Influence of Particle Adhesion

Figure 5-50 to Figure 5-51 below show the DEM simulation results for coal, with and without adhesion present between the particles using Rocky. Results are identical to those observed in Section 5.3, and indicate that the adhesion between the particles does not show a significant variation in the pressure exerted by the bulk solid material on the belt during opening or closing.
5.4.4 Influence of Trough Angle

Figure 5-52 to Figure 5-54 present the influence of a greater side inclination angle (troughing angle) on the pressures exerted by the bulk solid material on the belt. A $45^\circ$ troughing angle was compared to the existing $35^\circ$ troughing results for coal, gravel and magnetite. Results presented indicate an increase in the normal pressure on the belt with an increase in the troughing angle.
Figure 5-52 – Coal: DEM (Rocky) – Open and Closed Loading Profile

Influence of Troughing Angle

(5mm Spheres - $\mu_p = 0.9$, $\mu_s = 0.9$, $\mu_w = 0.7$, $\rho_p = 1350$kg/m$^3$)

Figure 5-53 – Gravel: DEM (Rocky) – Open and Closed Loading Profile

Influence of Troughing Angle

(10mm Spheres - $\mu_p = 0.8$, $\mu_s = 0.4$, $\mu_w = 0.6$, $\rho_p = 2590$kg/m$^3$)
5.4.5 Influence of Oscillating Frequency

Figure 5-55 to Figure 5-57 below present the investigation of oscillating frequency on the normal pressure exerted by the bulk solid material on the belt during opening and closing. Results show an insignificant variation in pressure profiles across the range of frequencies, materials and properties considered.
Figure 5-56 – Gravel: DEM (Rocky) – Open and Closed Loading Profiles

Influence of Oscillating Frequency

(10mm Spheres - $\mu_p = 0.8, \mu_r = 0.4, \mu_w = 0.6, \rho_p = 2590$kg/m$^3$)

Figure 5-57 – Magnetite: DEM (Rocky) – Open and Closed Loading Profiles

Influence of Oscillating Frequency

(10mm Spheres - $\mu_p = 0.8, \mu_r = 0.4, \mu_w = 0.6, \rho_p = 3665$kg/m$^3$)
Chapter 6 - Bulk Solid and Conveyor Belt Interaction: Site Tests

A re-circulating belt conveyor testing facility was employed to investigate bulk solid material cross sectional shape and loading on the belt during conveying. The test facility consists of two parallel belt conveyors each 65m long and feeding onto each other via storage bins and feeders located at each end. The system comprises of a 600mm wide fabric belt of class PN300/2, with a 35° offset idler troughing profile. A photograph of the testing facility is shown in Figure 6-1.

Conveyor surcharge angle testing was performed and compared to belt surcharge angles calculated based on theoretical approaches predicted by CEMA [6] and Colijn [17]. Experimental testing also involved the measurement of the pressure exerted by the bulk solid material on the surface of the conveyor belt. These pressures were measured using TekScan sensor pads during the conveying of a number of bulk solid materials.

The conveying process was modelled with DEM (Rocky) based on a 35° idler roll troughing profile obtained via the application of the finite difference model of Wheeler [106]. The finite difference approach was also implemented to model the same width belt at an idler roll inclination profile of 45° and both sets of results are compared to experimental measurements. Subsequently, a 3D laser scan of a section of the re-circulating belt conveyor testing facility was undertaken from which the belt profile was developed. The profile obtained was then modelled with DEM (Rocky) and results were compared to the experimental tests. A brief parametric sensitivity analysis regarding the influence of DEM simulation parameters on the normal force exerted on the belt was also performed.

In order to investigate the influence of sag on the cross sectional shape of the belt, two additional 3D laser scans were performed at greater and lower belt sag. The cross sectional profile of the greater sag was then also modelled with DEM and a comparison of results to the initial modelling is also presented. Finally, a comparison of experimental, DEM simulation and theoretical results based on the work of Spaans [90] and Mulani [69] was also undertaken and results are presented.
6.1 Belt Surcharge Angle Testing and Analysis

Four bulk solid materials were tested, namely gravel, coal, magnetite and river sand. The procedure involved loosely pouring each material onto the belt along the length of two idler pitches, maintaining the natural angle of repose. The conveyor was then operated in forward motion, stopped and reversed to the original position at which the cross sectional profile was assessed. Tests were repeated three times for each material at a speed of 2.0m/s and photographs taken at each end of the cross section prior to and following each test. Photographs of the observed profile typical of each bulk solid material tested before and after testing are presented in Figure 6-2. Belt surcharge angles were measured in a CAD package from which a summary is presented in Table 6-1.

![Gravel Before and After](image1)

Before | After
---|---
a) Gravel

![Coal Before and After](image2)

Before | After
---|---
b) Coal

![Riversand Before and After](image3)

Before | After
---|---
c) Riversand

![Magnetite Before and After](image4)

Before | After
---|---
d) Magnetite

Figure 6-2 – Conveyor Surcharge Angle Tests
The conveyor surcharge angles measured on the re-circulating belt conveyor facility are presented in Table 6-1, compared with those calculated by the CEMA [6] and Colijn [17] approaches. For each material, the angle of repose value used in the calculations was taken from the range of values presented in Table 4-2.

<table>
<thead>
<tr>
<th>Material</th>
<th>Conveyor Surcharge Angle (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CEMA</td>
</tr>
<tr>
<td>gravel</td>
<td>20°-25°</td>
</tr>
<tr>
<td>coal</td>
<td>25°-30°</td>
</tr>
<tr>
<td>magnetite</td>
<td>20°-25°</td>
</tr>
<tr>
<td>riversand</td>
<td>15°-20°</td>
</tr>
</tbody>
</table>

As can be seen from Table 6-1 above, measured conveyor surcharge angle results are within the range of values indicated by CEMA. The values calculated by Colijn are shown to be lower compared to the measured values and correlate with the lower end of the range specified by CEMA. It is worthy to note that the length of time and the distance of transportation will also affect the measured results i.e. 3 runs (up and back) is not a significant time for settlement of material to occur. Consequently, this may be reason that the surcharge angles measured on the re-circulating facility are higher than those predicted by Colijn.

6.2 3-D Laser Scan of Loaded Belt

A 3D laser scan of a section of the test facility was undertaken using a FARO® Laser Scanner Focus 3D S [112] in order to measure the belt profile during tests. The laser scan measurements involved loading the belt with gravel (2 idler pitch lengths) and profiling the bulk solid material manually, maintaining a head height of 120mm and length of contact with the inclined side of the belt of 130mm. The experimental set-up of the FARO® 3D laser scan is shown in Figure 6-3.
The belt profile obtained from the 3D laser scan used to determine the belt cross section geometry modelled in Section 6.5.2 is shown in Figure 6-4 for belt sag of approximately 24mm and is also typical of the scan output from which belt cross section geometry presented in Section 6.5.2.3 was obtained.

![Figure 6-4 - FARO 3D Scan: Conveyor Belt Geometry Profile](image)

### 6.3 TekScan Testing Procedure

Load measurements exerted by each bulk solid material on the belt during conveying were obtained using the TekScan sensor pad (as described previously in 5.3.1) on the belt surface. A thin insertion rubber, 1.5mm thick covered the sensor, and two bulk bags were placed at each end in order to prevent the bulk solid material from dispersing in the direction of conveyor belt travel. The TekScan testing set-up is shown in Figure 6-5. Following loading, the belt was conveyed at a velocity of 1.5m/s, during which 10 seconds of data were recorded.

![Empty Belt](image) ![Loaded Belt](image)

![Figure 6-5 – TekScan: Test Set-Up](image)

A side view obtained from the 3D laser scan of the 1.2m length of belt in between two consecutive idler roll sets for belt sag of 24mm is shown in Figure 6-6. Also shown is the corresponding loading profile typical to that observed in the TekScan tests.
Figure 6-6 – Typical TekScan Results

The 100mm sections depicted in Figure 6-6 are locations along the length of the belt related to the following:

1. Section A – Immediately prior to the idler roll set
2. Section B – At the horizontal centre idler roll
3. Section C – At the side or wing idler rolls
4. Section D – Immediately following the idler roll set
5. Section E – Half of the idler roll set pitch

Three tests were conducted for each bulk solid material from which an average transverse pressure profile (across the belt) for each region depicted in Figure 6-6 is presented in the following section.

6.4 TekScan Results

The transverse pressure profile for each bulk solid material was averaged from the TekScan tests at each of the five sections A to E and results are presented in Figure 6-7 to Figure 6-10. Also shown in each plot is the calculated hydrostatic pressure (gravitational load) profile, with the normal component calculated for the inclined sides.
Figure 6-7 – Coal: TekScan Transverse Pressure Profile

Figure 6-8 – Gravel: TekScan Transverse Pressure Profile
From the results presented in Figure 6-7 to Figure 6-10 previously, similar trends can be observed for all materials tested. Namely, the transverse pressure profile is almost identical at Section A and Section D (before and after the idler roll set) and for these two sections a close correlation to the pressure profile due to gravity alone can also be observed.

From Section A to Section B, an increase in the pressure over the centre section of the belt is observed, with this value exceeding the gravitational component. The pressure across the centre of the belt then reduces as the loaded belt travels between Section B to Section C, back towards the gravitational component. The peak pressure then shifts towards the position on the belt located at the junction of the horizontal and wing idler rolls.
The pressure present at the centre section of the belt then decreases to lower than the gravitational component, while the peak pressure is maintained at a location on the belt near to the junction of the idler rolls. This is shown by the pressure profile measured at Section E.

For each material, the summation of the normal force acting over an entire length of the idler pitch of 1.2m was obtained from the experimental results. These data are presented in Figure 6-11. The results show that the peak normal force exhibited on the belt is located at the junction of the idler rolls. For all materials tested, a decrease in the normal force in the centre of the belt is also evident.

**Figure 6-11 – TekScan: Measured Sum of Normal Forces Transverse Profile**

**Idler Pitch of 1.2m**
From the above summation of normal forces, the normal force on the inclined side of the belt and the normal force acting on the centre of the belt were calculated and are presented in Table 6-2.

<table>
<thead>
<tr>
<th>Material</th>
<th>TekScan Normal Force on Each Side (N)</th>
<th>TekScan Normal Force on Centre (N)</th>
<th>TekScan Total Normal Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>coal</td>
<td>62</td>
<td>155</td>
<td>279</td>
</tr>
<tr>
<td>gravel</td>
<td>86</td>
<td>250</td>
<td>422</td>
</tr>
<tr>
<td>magnetite</td>
<td>171</td>
<td>440</td>
<td>782</td>
</tr>
<tr>
<td>riversand</td>
<td>133</td>
<td>340</td>
<td>606</td>
</tr>
</tbody>
</table>

Based on the values presented in Table 6-2 a relationship between the normal force exerted on the inclined side and on the centre of the belt as a percentage of the total normal force acting is presented in Figure 6-12 as a function of the internal angle of friction. The internal angle of friction corresponds to the angles of each bulk solid material tested previously and presented in Table 4-2.

Results presented in Figure 6-12 show that the ratio of the normal force acting on the inclined side compared to the normal force acting on the centre of the belt remains constant across the range of internal angles of friction tested. This result indicates that the percentage of the load exerted onto the inclined sides of the belt is approximately 45% and the percentage of the load supported by the centre section of the belt is 55%.

From the TekScan sensor image shown in Figure 6-6 it can be observed that in the location between the two idler roll sets, the pad was less inclined to pick up loading towards the outer edges of the belt. Subsequently, the sum of normal force data presented in Figure 6-11 is likely to be lower than that
expected in reality. Due to this potential inaccuracy, an additional (extrapolated) sum of normal forces profiles was also investigated. For this analysis each of the loading profiles shown as Section A to D, in Figure 6-7 to Figure 6-10, were assumed to act over 1/3 of the idler spacing. The loading profile shown as Section E and located in the middle of the two idler roll sets was assumed to act over the remaining 2/3 of the spacing. This assumption was based on the typical results obtained from the TekScan software, which indicate that the loading profile shown as Section E, dominates the loading profile on the belt. This can be visualized from the typical TekScan result presented in Figure 6-6. For the materials tested, the results of the extrapolated analysis are presented in Figure 6-13.

![Figure 6-13 – TekScan: Extrapolated Sum of Normal Forces Transverse Profile](image)

In general, an increase in the force exerted on the belt can be observed when compared to results presented in Figure 6-11, however an almost identical trend is evident. Similarly to the measured results,
the peak normal force on the belt was observed to occur at the location governed by the inclined idler roll junction. A drop in normal pressure on the centre section of the belt is also observed. As conducted previously, the alternate results presented in Figure 6-13 were used to calculate the normal force acting on the inclined side and the centre of the belt, with the results presented in Table 6-3.

<table>
<thead>
<tr>
<th>Material</th>
<th>TekScan Normal Force on Each Side (N)</th>
<th>TekScan Normal Force on Centre (N)</th>
<th>TekScan Total Normal Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>coal</td>
<td>73</td>
<td>286</td>
<td>432</td>
</tr>
<tr>
<td>gravel</td>
<td>120</td>
<td>498</td>
<td>738</td>
</tr>
<tr>
<td>magnetite</td>
<td>191</td>
<td>693</td>
<td>1075</td>
</tr>
<tr>
<td>riversand</td>
<td>126</td>
<td>507</td>
<td>759</td>
</tr>
</tbody>
</table>

A relationship between the force exerted by the bulk solid material on the inclined side of the belt and on the centre of the belt as a percentage of the total force acting is presented in Figure 6-14.

For results presented in Figure 6-13 and calculated values presented in Table 6-3 and Figure 6-14, it is assumed that the transverse pressure profile corresponding to Section E acts along 2/3 of the idler pitch. The remaining 1/3 of the idler pitch consists of four equal components which comprise of the transverse pressure profiles given by Section A – immediately prior to the idler set, Section B – at the horizontal centre idler roll, Section C – at the wing idler roll and Section D – immediately following the idler roll set. Each of these components is assumed to act along 1/12 of the idler pitch.

Based on these assumptions, the results presented in Figure 6-14 show a similar trend to the results presented in Figure 6-12, with the internal angle of friction not of noticeable influence on the ratio of
forces acting on the middle of the belt and on the inclined sides. The percentage of the total load exerted by each bulk solid material on the inclined sides of the belt is approximately 35% and the percentage of the total load supported by the centre section of the belt is in the order of 65%.

In summary, the results from the TekScan measurements shown in Figure 6-6 to Figure 6-10 show that 5 different sections (A-E) with varying transverse pressure profile were observed. From the results presented in Figure 6-11 and Figure 6-13 for the sum of normal forces over an idler spacing of 1.2m, the measured and extrapolated results show that the peak loading force on the belt occurs at the location of the idler roll junction. Both sets of results also show a decrease in normal force across the centre of the belt.

Furthermore, Figure 6-12 and Figure 6-14 show that the percentage of the total normal force exerted by the bulk solid materials on the inclined sides of the belt is in the 35-45% range. Thus the percentage of the total normal force exerted on the centre section of the belt is in the range of 55-65%. These ratios were observed to occur across the entire range of internal angles of friction for each bulk solid material tested. Due to potential inaccuracies regarding the summation of forces obtained directly from the measured TekScan results, the extrapolated TekScan results are used for direct comparison to subsequent DEM analysis presented in Section 6.5.

### 6.5 DEM Simulation of Belt Deflection

In order to analyse the above experimental results in further depth, two different belt deflection profiles were modelled using DEM (Rocky). The first belt profile was based on a finite difference approach developed by Wheeler [106]. The second belt profile was directly obtained from a 3D laser scan of a portion of the re-circulating belt conveying test facility.

As previously mentioned in Section 3.2.2 only one half of the belt was modelled. The modelled belt comprises of 28 individual analysis boundaries, each assigned translational and rotational motion parameters (in the x-y plane). With every boundary moving individually, their collective profile moves from a troughed shape at the idler roll set, to a curved shape at one half of the idler roll set pitch. This is illustrated in Figure 6-15 with the nominated x and y direction being positive.
The period of motion between each successive troughed and curved profile was set to a time of 0.4s. This corresponds to a belt velocity of 1.5m/s and idler roll pitch of 1.2m. The length of the section of the conveyor belt modelled was 50mm and the width of the analysis boundaries of the 35° idler roll trough profile are summarized in Table 6-4 below.

<table>
<thead>
<tr>
<th>Boundary</th>
<th>Width (mm)</th>
<th>Boundary</th>
<th>Width (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.2</td>
<td>15</td>
<td>12.2</td>
</tr>
<tr>
<td>2</td>
<td>9.2</td>
<td>16</td>
<td>12.2</td>
</tr>
<tr>
<td>3</td>
<td>9.2</td>
<td>17</td>
<td>12.2</td>
</tr>
<tr>
<td>4</td>
<td>9.2</td>
<td>18</td>
<td>12.2</td>
</tr>
<tr>
<td>5</td>
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</tr>
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<td>6</td>
<td>9.2</td>
<td>20</td>
<td>12.2</td>
</tr>
<tr>
<td>7</td>
<td>9.2</td>
<td>21</td>
<td>12.2</td>
</tr>
<tr>
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<td>9.2</td>
<td>22</td>
<td>12.2</td>
</tr>
<tr>
<td>9</td>
<td>9.2</td>
<td>23</td>
<td>12.2</td>
</tr>
<tr>
<td>10</td>
<td>9.2</td>
<td>24</td>
<td>12.2</td>
</tr>
<tr>
<td>11</td>
<td>9.2</td>
<td>25</td>
<td>12.2</td>
</tr>
<tr>
<td>12</td>
<td>9.5</td>
<td>26</td>
<td>12.2</td>
</tr>
<tr>
<td>13</td>
<td>10.6</td>
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<td>12.2</td>
</tr>
<tr>
<td>14</td>
<td>12.2</td>
<td>28</td>
<td>12.2</td>
</tr>
</tbody>
</table>

The above dimensions were used for modelling and analysis in Section 6.5.1.1 and Section 6.5.2. Slightly different dimensions were used in the modelling presented in Section 6.5.1.2.

Following manual shaping of the particles on the belt as described in Section 3.2.2, opening and closing of the belt commenced. The motion of each individual boundary governed the geometry of the belt cross section as it moves from closed (at the idler roll trough) to open (at one half of the idler pitch). The translation and rotation of each analysis boundary was only applied in the x-y plane.

The sum of normal forces exerted by the bulk solid material on each of the boundary elements was recorded over an idler pitch of 1.2m and averaged over a simulation time of 8.0s for all 4 materials tested. Referring to Figure 6-15, for all simulation results presented, the centre section of the belt refers to boundary elements 1-12 and the inclined side of the belt refers to boundary elements 13 and above.

### 6.5.1 Finite Difference Belt Loading Profile

During conveying, the transverse profile of the belt changes from a confined troughed shape at the idler roll set, to a semi-confined curved profile between the idler roll sets. In this process, the exact geometrical profile of the belt is dependent on the belt stiffness, idler roll pitch, belt sag, belt speed, bulk solid material properties and is therefore difficult to predict.
Following the analysis and results presented in Chapter 5, a different mode of deformation of the belt compared to simply pivoting the inclined sides about a point was also investigated. Output from a belt deflection program previously developed by Wheeler [106] was implemented to obtain an initial cross sectional profile of a 600mm wide belt.

Simulations were performed using DEM (Rocky) and the belt loading profile due to opening and closing of the above cross sectional geometry. These results are presented in Section 6.5.1.1. Also, presented in 6.5.1.2 is the implementation of the belt geometry profile predicted by the finite difference approach to investigate the influence of a greater idler inclination angle.

### 6.5.1.1 35° Idler Roll Trough Loading Profile

For a troughed idler roll inclination of 35°, modelled using the belt cross sectional profile shown in Figure 6-16. Specification of the analysis boundary motion is summarized in Table 6-5.

#### Table 6-5 - 35° Idler Roll Trough: Finite Difference Geometry - Boundary Motion

<table>
<thead>
<tr>
<th>Boundary</th>
<th>Vₓ (m/s)</th>
<th>Vᵧ (m/s)</th>
<th>ω (rad/s)</th>
<th>Boundary</th>
<th>Vₓ (m/s)</th>
<th>Vᵧ (m/s)</th>
<th>ω (rad/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>±7.1e⁻²</td>
<td>0</td>
<td>15</td>
<td>±3.0e⁻²</td>
<td>±5.0e⁻²</td>
<td>±1.5e⁻¹</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>±7.1e⁻²</td>
<td>±5.5e⁻³</td>
<td>16</td>
<td>±1.1e⁻²</td>
<td>±5.2e⁻²</td>
<td>±1.4e⁻¹</td>
</tr>
<tr>
<td>3</td>
<td>±4.6e⁻⁴</td>
<td>±7.1e⁻²</td>
<td>±2.2e⁻²</td>
<td>17</td>
<td>±1.1e⁻²</td>
<td>±5.4e⁻²</td>
<td>±3.9e⁻¹</td>
</tr>
<tr>
<td>4</td>
<td>±9.1e⁻⁴</td>
<td>±7.1e⁻²</td>
<td>±4.6e⁻²</td>
<td>18</td>
<td>±1.1e⁻²</td>
<td>±5.4e⁻²</td>
<td>±1.4e⁻¹</td>
</tr>
<tr>
<td>5</td>
<td>±3.9e⁻⁷</td>
<td>±7.0e⁻²</td>
<td>±8.7e⁻²</td>
<td>19</td>
<td>±1.1e⁻²</td>
<td>±5.5e⁻²</td>
<td>±2.6e⁻²</td>
</tr>
<tr>
<td>6</td>
<td>±1.8e⁻³</td>
<td>±7.0e⁻²</td>
<td>±1.5e⁻¹</td>
<td>20</td>
<td>±1.1e⁻²</td>
<td>±5.4e⁻²</td>
<td>±3.9e⁻²</td>
</tr>
<tr>
<td>7</td>
<td>±5.7e⁻⁷</td>
<td>±6.8e⁻²</td>
<td>±2.2e⁻¹</td>
<td>21</td>
<td>±1.1e⁻²</td>
<td>±5.4e⁻²</td>
<td>±4.8e⁻²</td>
</tr>
<tr>
<td>8</td>
<td>±1.5e⁻⁴</td>
<td>±6.6e⁻²</td>
<td>±3.3e⁻¹</td>
<td>22</td>
<td>±1.1e⁻²</td>
<td>±5.3e⁻²</td>
<td>±4.7e⁻²</td>
</tr>
<tr>
<td>9</td>
<td>±3.5e⁻⁴</td>
<td>±6.3e⁻²</td>
<td>±4.4e⁻¹</td>
<td>23</td>
<td>±1.1e⁻²</td>
<td>±5.2e⁻²</td>
<td>±4.1e⁻²</td>
</tr>
<tr>
<td>10</td>
<td>±7.2e⁻²</td>
<td>±5.9e⁻²</td>
<td>±5.8e⁻¹</td>
<td>24</td>
<td>±1.1e⁻²</td>
<td>±5.1e⁻²</td>
<td>±3.0e⁻²</td>
</tr>
<tr>
<td>11</td>
<td>±1.4e⁻⁴</td>
<td>±5.4e⁻²</td>
<td>±7.1e⁻¹</td>
<td>25</td>
<td>±1.1e⁻²</td>
<td>±5.1e⁻²</td>
<td>±2.0e⁻²</td>
</tr>
<tr>
<td>12</td>
<td>±2.4e⁻⁴</td>
<td>±4.7e⁻²</td>
<td>±2.0e⁻¹</td>
<td>26</td>
<td>±1.1e⁻²</td>
<td>±5.1e⁻²</td>
<td>±8.9e⁻³</td>
</tr>
<tr>
<td>13</td>
<td>±3.0e⁻³</td>
<td>±4.5e⁻²</td>
<td>±5.6e⁻²</td>
<td>27</td>
<td>±3.9e⁻³</td>
<td>±5.1e⁻²</td>
<td>±2.3e⁻³</td>
</tr>
<tr>
<td>14</td>
<td>±2.7e⁻³</td>
<td>±4.6e⁻²</td>
<td>±3.0e⁻¹</td>
<td>28</td>
<td>±6.2e⁻³</td>
<td>±5.1e⁻²</td>
<td>±6.8e⁻³</td>
</tr>
</tbody>
</table>
In Table 6-5, the $\mp$ symbol indicates that the initial translational velocities, $V_x$ and $V_y$, of the corresponding analysis boundary will be negative. Contrary to this, $\pm$ indicates positive initial translation. Similarly for the angular rotation, $\omega$, of each analysis boundary, $\pm$ indicates initial rotational movement is clockwise and $\mp$ indicates initial rotational movement is anti-clockwise. With a set motion time period of 0.4s between the troughed and opened belt geometry profile, each analysis boundary will cycle between positive and negative x and y translational and rotational velocity as shown in Table 6-5.

For each analysis boundary, the sum of the normal forces acting over an equivalent belt length of 1.2m was obtained and results are presented in Figure 6-17 to Figure 6-20. Also shown on each graph is a comparison to the calculated gravitational component of each bulk solid material and the extrapolated TekScan results re-plotted from Figure 6-13. Coal simulation results are only shown for parameters with adhesion present between the particles, as the transverse normal force profile for parameters without adhesion were almost identical. However, both sets of results are summarized in Table 6-6. The same procedure was also adopted for all subsequent simulation results.

Results show good overall correlation between the experimentally measured and modelled loading profiles using belt geometry approximated with the finite difference model of Wheeler [106]. Also, less scatter and smoother loading profiles were observed for DEM simulations with smaller diameter particles.

![Figure 6-17 – 35° Idler Roll Trough: Finite Difference Geometry - Coal](image)

**Sum of Normal Forces Transverse Profile - Idler Pitch of 1.2m**

(5mm Spheres - $\mu_p = 0.9$, $\mu_c = 0.9$, $\mu_{adh} = 1$, $s_{adh} = 0.5$mm, $\mu_w = 0.7$, $\rho_p = 1350$kg/m$^3$)
Figure 6-18 – 35° Idler Roll Trough: Finite Difference Geometry - Gravel

Sum of Normal Forces Transverse Profile - Idler Pitch of 1.2m

(10mm Spheres - $\mu_p = 0.8$, $\mu_r = 0.4$, $\mu_w = 0.6$, $\rho_p = 2590$kg/m$^3$)

Figure 6-19 – 35° Idler Roll Trough: Finite Difference Geometry - Magnetite

Sum of Normal Forces Transverse Profile - Idler Pitch of 1.2m

(10mm Spheres - $\mu_p = 0.8$, $\mu_r = 0.4$, $\mu_w = 0.6$, $\rho_p = 3665$kg/m$^3$)
Figure 6-20 – 35° Idler Roll Trough: Finite Difference Geometry - Riversand

Sum of Normal Forces Transverse Profile - Idler Pitch of 1.2m

(5mm Spheres - \( \mu_p = 0.5, \mu_r = 0.1, \mu_w = 0.5, \rho_p = 2590\text{kg/m}^3 \))

The sum of normal forces across the transverse profile for the range of bulk materials presented in Figure 6-17 to Figure 6-20 allows the normal force on the inclined side of the belt and the normal force acting on the centre of the belt to be calculated. These values are presented in Table 6-6. The force values presented for coal with \( \phi_i = 50^\circ \) are equivalent to DEM parameters indicated in Figure 6-17, however, without adhesion (i.e. 5mm Spheres - \( \mu_p = 0.9, \mu_r = 0.9, \mu_w = 0.7, \rho_p = 1350\text{kg/m}^3 \)).

Table 6-6 - 35° Idler Roll Trough: Finite Difference Geometry Normal Force on Belt

<table>
<thead>
<tr>
<th>Idler Pitch of 1.2m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
</tr>
<tr>
<td>coal ( (\phi_i = 50^\circ) )</td>
</tr>
<tr>
<td>coal ( (\phi_i = 65^\circ) )</td>
</tr>
<tr>
<td>gravel</td>
</tr>
<tr>
<td>magnetite</td>
</tr>
<tr>
<td>riversand</td>
</tr>
</tbody>
</table>

From Table 6-6, the normal force exerted on the inclined side and on the centre of the belt as a percentage of the total normal force acting is presented in Figure 6-21 as a function of the internal angle of friction.
Figure 6-21 shows that the percentage of the load exerted onto the inclined sides of the belt is in the order of 40-48% and the percentage of the load supported by the centre section of the belt is 52-60%. The results presented in Table 6-6 and are within the range of values obtained from the measured and extrapolated TekScan results.

6.5.1.2 45° Idler Roll Troughing Profile

The potential influence of a larger idler roll troughing inclination angle of 45° was also investigated, with the belt geometry used in the simulations shown in Figure 6-22. Belt geometry was based on the finite difference belt geometry presented previously, but with a greater angle of the inclined sides. Similarly to the 35° idler roll troughing simulations, only one half of the belt was modelled.

Figure 6-22 – Belt Cross Sectional Profile: Finite Difference Geometry

45° Idler Roll Troughing Profile

For the 45° trough profile, 31 analysis boundaries were modelled, with their dimensions outlined in Table 6-7. The motion specification corresponding to each boundary is summarized in Table 6-8.
Table 6-7 - 45° Idler Roll Trough: Finite Difference Profile - Boundary Dimensions

<table>
<thead>
<tr>
<th>Boundary</th>
<th>Width (mm)</th>
<th>Boundary</th>
<th>Width (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>9.9</td>
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<td>9.9</td>
</tr>
<tr>
<td>16</td>
<td>9.9</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 6-8 - 45° Idler Roll Trough: Finite Difference Geometry - Boundary Motion

<table>
<thead>
<tr>
<th>Boundary</th>
<th>( V_x ) (m/s)</th>
<th>( V_y ) (m/s)</th>
<th>( \omega ) (rad/s)</th>
<th>Boundary</th>
<th>( V_x ) (m/s)</th>
<th>( V_y ) (m/s)</th>
<th>( \omega ) (rad/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>( \pm 7.1e^{-2} )</td>
<td>0</td>
<td>17</td>
<td>( \pm 3.3e^{-1} )</td>
<td>( \pm 5.3e^{-2} )</td>
<td>( \pm 4.9e^{-1} )</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>( \pm 7.1e^{-2} )</td>
<td>( \pm 5.5e^{-3} )</td>
<td>18</td>
<td>( \pm 3.6e^{-1} )</td>
<td>( \pm 5.3e^{-2} )</td>
<td>( \pm 4.0e^{-1} )</td>
</tr>
<tr>
<td>3</td>
<td>( \pm 4.6e^{-3} )</td>
<td>( \pm 7.1e^{-2} )</td>
<td>( \pm 2.2e^{-1} )</td>
<td>19</td>
<td>( \pm 3.6e^{-1} )</td>
<td>( \pm 5.3e^{-2} )</td>
<td>( \pm 4.6e^{-1} )</td>
</tr>
<tr>
<td>4</td>
<td>( \pm 9.1e^{-1} )</td>
<td>( \pm 7.1e^{-2} )</td>
<td>( \pm 4.6e^{-2} )</td>
<td>20</td>
<td>( \pm 3.3e^{-1} )</td>
<td>( \pm 5.3e^{-2} )</td>
<td>( \pm 6.5e^{-2} )</td>
</tr>
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<td>5</td>
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<td>( \pm 7.0e^{-2} )</td>
<td>( \pm 8.7e^{-1} )</td>
<td>21</td>
<td>( \pm 2.8e^{-1} )</td>
<td>( \pm 5.2e^{-2} )</td>
<td>( \pm 7.9e^{-1} )</td>
</tr>
<tr>
<td>6</td>
<td>( \pm 1.8e^{-5} )</td>
<td>( \pm 7.0e^{-2} )</td>
<td>( \pm 1.5e^{-1} )</td>
<td>22</td>
<td>( \pm 2.3e^{-1} )</td>
<td>( \pm 5.2e^{-2} )</td>
<td>( \pm 7.6e^{-1} )</td>
</tr>
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<td>( \pm 5.7e^{-5} )</td>
<td>( \pm 6.8e^{-2} )</td>
<td>( \pm 3.6e^{-1} )</td>
<td>23</td>
<td>( \pm 1.7e^{-1} )</td>
<td>( \pm 5.1e^{-2} )</td>
<td>( \pm 6.5e^{-1} )</td>
</tr>
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<td>8</td>
<td>( \pm 1.5e^{-5} )</td>
<td>( \pm 6.6e^{-2} )</td>
<td>( \pm 3.6e^{-1} )</td>
<td>24</td>
<td>( \pm 1.3e^{-1} )</td>
<td>( \pm 5.1e^{-2} )</td>
<td>( \pm 4.8e^{-1} )</td>
</tr>
<tr>
<td>9</td>
<td>( \pm 3.5e^{-6} )</td>
<td>( \pm 6.3e^{-2} )</td>
<td>( \pm 4.5e^{-1} )</td>
<td>25</td>
<td>( \pm 9.2e^{-1} )</td>
<td>( \pm 5.0e^{-2} )</td>
<td>( \pm 3.1e^{-1} )</td>
</tr>
<tr>
<td>10</td>
<td>( \pm 7.2e^{-5} )</td>
<td>( \pm 5.9e^{-2} )</td>
<td>( \pm 5.9e^{-1} )</td>
<td>26</td>
<td>( \pm 7.1e^{-1} )</td>
<td>( \pm 5.0e^{-2} )</td>
<td>( \pm 1.3e^{-1} )</td>
</tr>
<tr>
<td>11</td>
<td>( \pm 1.4e^{-4} )</td>
<td>( \pm 5.4e^{-2} )</td>
<td>( \pm 7.4e^{-1} )</td>
<td>27</td>
<td>( \pm 6.1e^{-1} )</td>
<td>( \pm 5.0e^{-2} )</td>
<td>( \pm 3.2e^{-1} )</td>
</tr>
<tr>
<td>12</td>
<td>( \pm 2.4e^{-4} )</td>
<td>( \pm 4.7e^{-2} )</td>
<td>( \pm 2.2e^{-1} )</td>
<td>28</td>
<td>( \pm 5.9e^{-1} )</td>
<td>( \pm 5.0e^{-2} )</td>
<td>( \pm 1.1e^{-1} )</td>
</tr>
<tr>
<td>13</td>
<td>( \pm 3.0e^{-4} )</td>
<td>( \pm 4.5e^{-2} )</td>
<td>( \pm 1.1e^{-1} )</td>
<td>29</td>
<td>( \pm 6.7e^{-1} )</td>
<td>( \pm 5.0e^{-2} )</td>
<td>( \pm 1.6e^{-1} )</td>
</tr>
<tr>
<td>14</td>
<td>( \pm 2.6e^{-4} )</td>
<td>( \pm 4.6e^{-2} )</td>
<td>( \pm 4.6e^{-1} )</td>
<td>30</td>
<td>( \pm 7.8e^{-1} )</td>
<td>( \pm 5.0e^{-2} )</td>
<td>( \pm 8.7e^{-1} )</td>
</tr>
<tr>
<td>15</td>
<td>( \pm 3.6e^{-4} )</td>
<td>( \pm 4.9e^{-2} )</td>
<td>( \pm 2.3e^{-1} )</td>
<td>31</td>
<td>( \pm 8.4e^{-1} )</td>
<td>( \pm 5.0e^{-2} )</td>
<td>( \pm 1.8e^{-1} )</td>
</tr>
<tr>
<td>16</td>
<td>( \pm 1.9e^{-3} )</td>
<td>( \pm 5.1e^{-2} )</td>
<td>( \pm 2.1e^{-1} )</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

For each bulk solid material modelled, the general trend of the transverse sum of normal force profiles were found to be very similar to those of the results modelled with DEM (Rocky) shown in Figure 6-17 to Figure 6-20. No experimental results were available for comparison to be made. The total volume of bulk solid material modelled was maintained identical to the simulations presented in Section 6.5.1.1. The
force on the inclined side of the belt and the force acting on the centre of the belt were calculated and results are presented in Table 6-9.

<table>
<thead>
<tr>
<th>Material</th>
<th>Normal Force on Each Side (N)</th>
<th>Normal Force on Centre (N)</th>
<th>Total Normal Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>coal ($\phi_i = 50^\circ$)</td>
<td>73</td>
<td>206</td>
<td>352</td>
</tr>
<tr>
<td>coal ($\phi_i = 65^\circ$)</td>
<td>72</td>
<td>205</td>
<td>349</td>
</tr>
<tr>
<td>gravel</td>
<td>133</td>
<td>404</td>
<td>670</td>
</tr>
<tr>
<td>magnetite</td>
<td>189</td>
<td>566</td>
<td>944</td>
</tr>
<tr>
<td>riversand</td>
<td>135</td>
<td>448</td>
<td>718</td>
</tr>
</tbody>
</table>

A comparison of results presented in Table 6-9 with the results for the 35° idler roll trough inclination presented in Table 6-6 shows an increase in the total normal force on the belt. The force on the centre section of the belt was observed to increase, however, the force acting on the inclined side of the belt was observed to decrease. The normal force exerted on the inclined side and on the centre section of the belt as a percentage of the total normal force acting is shown in Figure 6-23 as a function of the internal angle of friction.

The results presented in Figure 6-23 show that the percentage of the total load exerted onto the inclined sides and the middle section of the belt is approximately constant at 40% and 60% respectively. This relationship was observed to occur across the range of internal angles of friction modelled and is of similar nature when compared to results presented for the 35° idler roll troughing profile presented in Figure 6-21. The trend observed in Table 6-9 is further illustrated in Figure 6-24 below which shows the
cross section modelled for the 35° and 45° trough with the total volume of the material maintained constant.

![Figure 6-24 – Modelled Cross Sections](image)

From Figure 6-24 it can be visualized that the total cross section of the bulk solid material located above the centre section of the belt is greater at greater troughing angle. However, the cross sectional area located above the inclined side is slightly lower.

**6.5.2 Belt Loading Profile Based on 3D Laser Scan**

Following good overall correlation between experimental and modelled results obtained using the finite difference approach presented in Section 6.5.1, additional belt geometry was modelled using results obtained from the 3D laser scan. The scanned cross sectional profile obtained from the 3D laser scan of the conveyor belt test facility at the idler roll set, and at a distance of one half of the idler pitch, is shown in Figure 6-25. The geometry presented in Figure 6-25 was taken directly from the 3D laser scan image shown in Figure 6-4. Belt sag (vertical displacement) was observed to be approximately 24mm.

![Figure 6-25 – Belt Cross Sectional Profile: 3D Laser Scan Belt Geometry - 24mm Sag](image)

Data obtained from the laser scan was used to obtain x-y co-ordinates of the transverse belt geometry with the results presented in Figure 6-26.

![Figure 6-26 – Belt Cross Sectional Profile: 3D Laser Scan Belt Geometry - 24mm Sag](image)
The x-y co-ordinates of the belt geometry were then used to develop analysis boundaries for the DEM (Rocky) model, with the motion specification of each of the boundaries presented in Table 6-10.

Table 6-10 – 3D Laser Scan Belt Geometry: 24mm Sag – Boundary Motion

<table>
<thead>
<tr>
<th>Boundary</th>
<th>V_x (m/s)</th>
<th>V_y (m/s)</th>
<th>ω (rad/s)</th>
<th>Boundary</th>
<th>V_x (m/s)</th>
<th>V_y (m/s)</th>
<th>ω (rad/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>±6.0e-2</td>
<td>±3.2e-2</td>
<td>15</td>
<td>±3.1e-1</td>
<td>±5.3e-2</td>
<td>±5.1e-1</td>
</tr>
<tr>
<td>2</td>
<td>±8.3e-6</td>
<td>±6.0e-2</td>
<td>±3.1e-2</td>
<td>16</td>
<td>±6.1e-1</td>
<td>±5.8e-2</td>
<td>±3.2e-1</td>
</tr>
<tr>
<td>3</td>
<td>±1.7e-6</td>
<td>±5.9e-2</td>
<td>±3.1e-2</td>
<td>17</td>
<td>±8.2e-1</td>
<td>±6.2e-2</td>
<td>±1.7e-1</td>
</tr>
<tr>
<td>4</td>
<td>±3.5e-6</td>
<td>±5.9e-2</td>
<td>±3.1e-2</td>
<td>18</td>
<td>±9.3e-1</td>
<td>±6.3e-2</td>
<td>±1.2e-1</td>
</tr>
<tr>
<td>5</td>
<td>±1.4e-3</td>
<td>±5.9e-2</td>
<td>±3.1e-2</td>
<td>19</td>
<td>±1.0e-2</td>
<td>±6.5e-2</td>
<td>±4.6e-2</td>
</tr>
<tr>
<td>6</td>
<td>±7.1e-6</td>
<td>±5.9e-2</td>
<td>±5.6e-2</td>
<td>20</td>
<td>±1.0e-1</td>
<td>±6.5e-2</td>
<td>±7.3e-1</td>
</tr>
<tr>
<td>7</td>
<td>±1.3e-5</td>
<td>±5.8e-2</td>
<td>±7.2e-2</td>
<td>21</td>
<td>±9.9e-1</td>
<td>±6.4e-2</td>
<td>±2.2e-1</td>
</tr>
<tr>
<td>8</td>
<td>±3.1e-5</td>
<td>±5.7e-2</td>
<td>±7.2e-2</td>
<td>22</td>
<td>±8.3e-1</td>
<td>±6.2e-2</td>
<td>±3.7e-1</td>
</tr>
<tr>
<td>9</td>
<td>±3.2e-3</td>
<td>±5.7e-2</td>
<td>±3.4e-1</td>
<td>23</td>
<td>±5.5e-1</td>
<td>±5.9e-2</td>
<td>±4.1e-1</td>
</tr>
<tr>
<td>10</td>
<td>±2.4e-4</td>
<td>±5.4e-2</td>
<td>±4.6e-1</td>
<td>24</td>
<td>±2.3e-1</td>
<td>±5.5e-2</td>
<td>±3.8e-1</td>
</tr>
<tr>
<td>11</td>
<td>±6.4e-4</td>
<td>±4.9e-2</td>
<td>±5.7e-1</td>
<td>25</td>
<td>±6.6e-1</td>
<td>±5.1e-2</td>
<td>±4.6e-1</td>
</tr>
<tr>
<td>12</td>
<td>±1.2e-3</td>
<td>±4.4e-2</td>
<td>±9.1e-3</td>
<td>26</td>
<td>±4.3e-1</td>
<td>±4.7e-2</td>
<td>±5.9e-1</td>
</tr>
<tr>
<td>13</td>
<td>±1.3e-3</td>
<td>±4.4e-2</td>
<td>±2.9e-1</td>
<td>27</td>
<td>±9.1e-3</td>
<td>±4.2e-2</td>
<td>±8.4e-1</td>
</tr>
<tr>
<td>14</td>
<td>±1.2e-4</td>
<td>±4.7e-2</td>
<td>±5.5e-1</td>
<td>28</td>
<td>±1.6e-2</td>
<td>±3.5e-2</td>
<td>±8.9e-1</td>
</tr>
</tbody>
</table>

The transverse loading profile for all materials is presented in Figure 6-27 to Figure 6-30.

Figure 6-27 – 3D Laser Scan Belt Geometry: 24mm Sag - Coal

Sum of Normal Forces Transverse Profile - Idler Pitch of 1.2m

(5mm Spheres - μ_p = 0.9, μ_r = 0.9, μ_adh = 1, s_adh = 0.5mm, μ_n = 0.7, ρ_p = 1350kg/m^3)
Figure 6-28 – 3D Laser Scan Belt Geometry: 24mm Sag - Gravel

Sum of Normal Forces Transverse Profile - Idler Pitch of 1.2m

\[ (10\text{mm Spheres} - \mu_p = 0.8, \mu_c = 0.4, \mu_w = 0.6, \rho_p = 2590\text{kg/m}^3) \]

Figure 6-29 – 3D Laser Scan Belt Geometry: 24mm Sag - Magnetite

Sum of Normal Forces Transverse Profile - Idler Pitch of 1.2m

\[ (10\text{mm Spheres} - \mu_p = 0.8, \mu_c = 0.4, \mu_w = 0.6, \rho_p = 3665\text{kg/m}^3) \]
As mentioned previously in Section 6.5.1, simulation results for coal presented in Figure 6-27 are only shown for parameters with adhesion present between the particles. Good correlation between the transverse loading profiles obtained with DEM (Rocky) and those obtained from extrapolated TekScan results can be observed. Reduced scatter was once again observed for simulations involving smaller diameter particles. From the sum of the transverse normal force profiles presented in Figure 6-27 to Figure 6-30 the normal force exerted on the inclined side and on the centre section of the belt were calculated and results are presented in Table 6-11. The forces presented for the coal are with and without adhesion between the particles. Good overall correlation can be observed when comparing results from Table 6-11 and results obtained using the finite difference geometry presented in Table 6-6.

Figure 6-31 shows the forces on the inclined side and centre section of the belt as a percentage of the total normal force and as a function of the internal angle of friction.
From the results presented, it can be observed that the percentage of the load exerted onto the inclined sides of the belt and on the centre section of the belt is approximately 40% and 60% respectively. This was observed across the range of internal angles of friction simulated. The trend indicated in Figure 6-31 and the results presented in Table 6-11 are within the range of values obtained from measured and extrapolated TekScan results. Good overall correlation can also be observed when compared to results obtained from the modelling of the 35° troughing angle presented in Section 6.5.1.

Further to the summation of normal forces on the belt acting over one idler pitch presented in Table 6-11 and Figure 6-31, loads during the trough opening cycle and the trough closing cycle were also investigated separately. Referring to Figure 6-6, the trough opening cycle may be visualized as the motion of the belt travelling from the idler roll set (Section B/Section C) to one half of the idler pitch (Section E). Similarly, the belt trough closing cycle may be visualized as travelling from one half of the idler pitch to the idler roll set. These results are summarized in Table 6-12. The total difference stated is the normal force on the belt during the closing cycle subtracted from the normal force during the opening cycle. A negative value indicates that the normal force on the belt during the closing cycle exceeds the normal force during the opening cycle.
From the results presented in Table 6-12 it can be observed that, for all materials modelled, the normal force acting on the belt is greater during the trough closing cycle. This indicates that the belt would consume energy during closing of the troughed shape.

### 6.5.2.1 Influence of Particle Parameters

In order to investigate the influence of material parameters modelled with DEM (Rocky), a brief sensitivity analysis was conducted with the sum of normal force results summarized in Table 6-13 to Table 6-20. Referring to results presented in Table 6-13, Table 6-15, Table 6-17 and Table 6-19, it can be observed that the total normal force exerted by each bulk solid material on the belt over one idler pitch, decreases with an increase in the modelled internal angle of friction. To investigate this in greater detail, analysis of the sum of normal forces exerted on the belt during the trough opening and trough closing cycles separately are also presented in Table 6-14, Table 6-16, Table 6-18 and Table 6-20, for each of the materials modelled.

### Table 6-12 - 3D Laser Scan Belt Geometry: 24mm Sag – Normal Force on Belt

**Trough Opening and Closing, Idler Pitch of 1.2m**

<table>
<thead>
<tr>
<th>Material</th>
<th>Trough Opening (N)</th>
<th>Trough Closing (N)</th>
<th>Total Difference (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>coal ($\phi_i = 50^\circ$)</td>
<td>161</td>
<td>182</td>
<td>-21</td>
</tr>
<tr>
<td>coal ($\phi_i = 65^\circ$)</td>
<td>156</td>
<td>180</td>
<td>-24</td>
</tr>
<tr>
<td>gravel</td>
<td>297</td>
<td>338</td>
<td>-41</td>
</tr>
<tr>
<td>magnetite</td>
<td>431</td>
<td>483</td>
<td>-52</td>
</tr>
<tr>
<td>riversand</td>
<td>332</td>
<td>360</td>
<td>-28</td>
</tr>
</tbody>
</table>

### Table 6-13 – Coal (5mm spheres - $\rho_p = 1350\text{kg/m}^3$): Normal Force on Belt

| Idler Pitch of 1.2m |
|---------------------|-------------------|-------------------|-------------------|
| Material            | Normal Force on Each Side (N) | Normal Force on Centre (N) | Total Normal Force (N) |
| $\mu_p = 0.3, \mu_s = 0.1 (\phi_i = 31^\circ)$ | 72                | 222               | 366               |
| $\mu_p = 0.8, \mu_s = 0.4 (\phi_i = 45^\circ)$ | 68                | 216               | 352               |
| $\mu_p = 0.9, \mu_s = 0.9 (\phi_i = 50^\circ)$ | 66                | 212               | 344               |
| $\mu_p = 0.9, \mu_s = 0.9, \mu_{adh} = 1, s_{adh} = 0.5\text{mm} (\phi_i = 65^\circ)$ | 65                | 206               | 336               |
Table 6-14 – Coal (5mm spheres - \( \rho_p = 1350\text{kg/m}^3 \)): Normal Force on Belt Trough Opening and Closing, Idler Pitch of 1.2m

<table>
<thead>
<tr>
<th>Material</th>
<th>Trough Opening (N)</th>
<th>Trough Closing (N)</th>
<th>Total Difference (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_p = 0.3, \mu_r = 0.1 (\phi_i = 31^\circ) )</td>
<td>175</td>
<td>191</td>
<td>-16</td>
</tr>
<tr>
<td>( \mu_p = 0.8, \mu_r = 0.4 (\phi_i = 45^\circ) )</td>
<td>168</td>
<td>184</td>
<td>-16</td>
</tr>
<tr>
<td>( \mu_p = 0.9, \mu_r = 0.9 (\phi_i = 50^\circ) )</td>
<td>161</td>
<td>183</td>
<td>-22</td>
</tr>
<tr>
<td>( \mu_p = 0.9, \mu_r = 0.9, \mu_{adh} = 1, s_{adh} = 0.5\text{mm} (\phi_i = 65^\circ) )</td>
<td>156</td>
<td>180</td>
<td>-24</td>
</tr>
</tbody>
</table>

Table 6-15 – Gravel (10mm Spheres - \( \rho_p = 2590\text{kg/m}^3 \)): Normal Force on Belt Idler Pitch of 1.2m

<table>
<thead>
<tr>
<th>Material</th>
<th>Normal Force on Each Side (N)</th>
<th>Normal Force on Centre (N)</th>
<th>Total Normal Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_p = 0.3, \mu_r = 0.1 (\phi_i = 31^\circ) )</td>
<td>145</td>
<td>370</td>
<td>658</td>
</tr>
<tr>
<td>( \mu_p = 0.8, \mu_r = 0.4 (\phi_i = 45^\circ) )</td>
<td>118</td>
<td>397</td>
<td>633</td>
</tr>
<tr>
<td>( \mu_p = 0.9, \mu_r = 0.9 (\phi_i = 50^\circ) )</td>
<td>130</td>
<td>368</td>
<td>628</td>
</tr>
</tbody>
</table>

Table 6-16 – Gravel (10mm Spheres - \( \rho_p = 2590\text{kg/m}^3 \)): Normal Force on Belt Trough Opening and Closing, Idler Pitch of 1.2m

<table>
<thead>
<tr>
<th>Material</th>
<th>Trough Opening (N)</th>
<th>Trough Closing (N)</th>
<th>Total Difference (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_p = 0.3, \mu_r = 0.1 (\phi_i = 31^\circ) )</td>
<td>309</td>
<td>349</td>
<td>-40</td>
</tr>
<tr>
<td>( \mu_p = 0.8, \mu_r = 0.4 (\phi_i = 45^\circ) )</td>
<td>296</td>
<td>337</td>
<td>-41</td>
</tr>
<tr>
<td>( \mu_p = 0.9, \mu_r = 0.9 (\phi_i = 50^\circ) )</td>
<td>286</td>
<td>341</td>
<td>-55</td>
</tr>
</tbody>
</table>

Table 6-17 – Magnetite (10mm Spheres - \( \rho_p = 3665\text{kg/m}^3 \)): Normal Force on Belt Idler Pitch of 1.2m

<table>
<thead>
<tr>
<th>Material</th>
<th>Normal Force on Each Side (N)</th>
<th>Normal Force on Centre (N)</th>
<th>Total Normal Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_p = 0.3, \mu_r = 0.1 (\phi_i = 31^\circ) )</td>
<td>199</td>
<td>566</td>
<td>964</td>
</tr>
<tr>
<td>( \mu_p = 0.8, \mu_r = 0.4 (\phi_i = 45^\circ) )</td>
<td>190</td>
<td>534</td>
<td>914</td>
</tr>
<tr>
<td>( \mu_p = 0.9, \mu_r = 0.9 (\phi_i = 50^\circ) )</td>
<td>175</td>
<td>544</td>
<td>894</td>
</tr>
</tbody>
</table>
Table 6-18 – Magnetite (10mm Spheres - $\rho_p = 3665$kg/m$^3$): Normal Force on Belt

Trough Opening and Closing, Idler Pitch of 1.2m

<table>
<thead>
<tr>
<th>Material</th>
<th>Trough Opening (N)</th>
<th>Trough Closing (N)</th>
<th>Total Difference (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_p = 0.3, \mu_r = 0.1 (\phi_i = 31^\circ)$</td>
<td>459</td>
<td>505</td>
<td>-46</td>
</tr>
<tr>
<td>$\mu_p = 0.8, \mu_r = 0.4 (\phi_i = 45^\circ)$</td>
<td>431</td>
<td>483</td>
<td>-52</td>
</tr>
<tr>
<td>$\mu_p = 0.9, \mu_r = 0.9 (\phi_i = 50^\circ)$</td>
<td>416</td>
<td>480</td>
<td>-64</td>
</tr>
</tbody>
</table>

Table 6-19 – Riversand (5mm Spheres - $\rho_p = 2590$kg/m$^3$): Normal Force on Belt

Idler Pitch of 1.2m

<table>
<thead>
<tr>
<th>Material</th>
<th>Normal Force on Each Side (N)</th>
<th>Normal Force on Centre (N)</th>
<th>Total Normal Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_p = 0.3, \mu_r = 0.1 (\phi_i = 31^\circ)$</td>
<td>135</td>
<td>432</td>
<td>702</td>
</tr>
<tr>
<td>$\mu_p = 0.8, \mu_r = 0.4 (\phi_i = 45^\circ)$</td>
<td>134</td>
<td>398</td>
<td>666</td>
</tr>
<tr>
<td>$\mu_p = 0.9, \mu_r = 0.9 (\phi_i = 50^\circ)$</td>
<td>132</td>
<td>392</td>
<td>656</td>
</tr>
</tbody>
</table>

Table 6-20 – Riversand (5mm Spheres - $\rho_p = 2590$kg/m$^3$): Normal Force on Belt

Trough Opening and Closing, Idler Pitch of 1.2m

<table>
<thead>
<tr>
<th>Material</th>
<th>Trough Opening (N)</th>
<th>Trough Closing (N)</th>
<th>Total Difference (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_p = 0.3, \mu_r = 0.1 (\phi_i = 31^\circ)$</td>
<td>338</td>
<td>364</td>
<td>-26</td>
</tr>
<tr>
<td>$\mu_p = 0.8, \mu_r = 0.4 (\phi_i = 45^\circ)$</td>
<td>318</td>
<td>348</td>
<td>-30</td>
</tr>
<tr>
<td>$\mu_p = 0.9, \mu_r = 0.9 (\phi_i = 50^\circ)$</td>
<td>311</td>
<td>345</td>
<td>-34</td>
</tr>
</tbody>
</table>

Importantly, results presented in Table 6-14, Table 6-16, Table 6-18 and Table 6-20 show that the total difference in the normal force acting on the belt between the trough opening and closing cycles increases with an increase in the modelled internal angle of friction.

Therefore, for constant particle density and friction between the bulk solid material and the belt during transportation, DEM analysis indicates that the sum of normal forces acting on the belt over an idler pitch of 1.2m is likely to be higher for materials exhibiting lower internal angles of friction. Or in other words, materials exhibiting a greater level of cohesion (or adhesion between the particles) are likely to exert on average, a lower total normal force on the belt during transportation. Furthermore, investigation of the sum of normal forces acting on the belt during the trough opening and closing cycles separately, shows that the total difference between the normal forces exerted by bulk solid materials on the belt increases
with an increase in the modelled internal angle of friction. Assuming that the deflection of the belt remains identical during the opening and closing of the troughed shape, the trend in the analysis shows that the energy loss attributed to the closing of the troughed shape of the belt will increase with an increase in the internal angle of friction.

This outcome of analysis is analogous to prior research work performed by Krause and Hettler [64], Spaans [90], Mulani [69], CEMA [6] and Wheeler [106], that state greater energy loss is attributed to bulk solid material characterized by larger angles of internal friction.

### 6.5.2.2 Influence of Wall Friction

The influence of wall friction was also investigated and typical results obtained for riversand and coal only are presented in Table 6-21 and Table 6-22 below.

**Table 6-21 – Riversand: Wall Friction Variation: Normal Force on Belt - Idler Pitch of 1.2m**

(5mm Spheres - \( \mu_p = 0.5, \mu_r = 0.1, \rho_p = 2590\text{kg/m}^3 \))

<table>
<thead>
<tr>
<th>Wall Friction Coefficient, ( \mu_w )</th>
<th>Normal Force on Each Side (N)</th>
<th>Normal Force on Centre (N)</th>
<th>Total Normal Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>131</td>
<td>428</td>
<td>690</td>
</tr>
<tr>
<td>0.7</td>
<td>132</td>
<td>426</td>
<td>690</td>
</tr>
</tbody>
</table>

**Table 6-22 – Coal: Wall Friction Variation: Normal Force on Belt - Idler Pitch of 1.2m**

(5mm Spheres - \( \mu_p = 0.9, \mu_r = 0.9, \rho_p = 1350\text{kg/m}^3 \))

<table>
<thead>
<tr>
<th>Wall Friction Coefficient, ( \mu_w )</th>
<th>Normal Force on Each Side (N)</th>
<th>Normal Force on Centre (N)</th>
<th>Total Normal Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>67</td>
<td>209</td>
<td>343</td>
</tr>
<tr>
<td>0.7</td>
<td>66</td>
<td>212</td>
<td>344</td>
</tr>
</tbody>
</table>

Results indicate that for the given range of values simulated, wall friction variation does not play a significant role in the overall sum of normal forces acting. This observation is inevitably related to the postulation of Mulani [69] that wall friction is not of great relevance during the steady state conveying of bulk solid materials, as it acts to nullify the presence of net shear forces acting along the belt.

### 6.5.2.3 Influence of Belt Sag

To investigate the belt deflection profile due to other belt sag values, additional 3D laser scans were performed at 1.3% and 2.7% sag (16mm and 32mm respectively) for the same portion of the recirculating belt conveyor test facility shown in Section 6.2. Additional scanned transverse belt profiles are
shown in Figure 6-32 and Figure 6-34. The initial 24mm sag profile is once again shown in Figure 6-33 for comparison purposes.

When comparing the deflection geometry of the belt from the troughed shape at the idler set to the curved profile at one half of the idler pitch, an important observation which can be made is that the belt sides are not shown to pivot about the idler junction. This is clearly shown in Figure 6-32 and Figure 6-34. The progression from 16mm sag to 32mm sag in the images presented also shows that the sides of the belt are more likely to pivot about the edges of the belt. This motion is contrary to the existing theoretical analysis proposed by Krause and Hettler [64], applied by Spaans [90] and investigated in Chapter 5 which assumes that the inclined side pivots around the idler junction.

![Figure 6-32 – Belt Cross Sectional Profile: 3D Laser Scan Belt Geometry - 16mm Sag](image1)

![Figure 6-33 – Belt Cross Sectional Profile: 3D Laser Scan Belt Geometry - 24mm Sag](image2)

![Figure 6-34 – Belt Cross Sectional Profile: 3D Laser Scan Belt Geometry - 32mm Sag](image3)

Data obtained from the laser scan of the belt with 32mm sag presented in Figure 6-34 was used to obtain x-y co-ordinates of the transverse belt geometry with the results presented in Figure 6-35.
Analysis boundaries were developed from the x-y co-ordinates of the belt geometry presented in Figure 6-35 and the motion specification of each is presented in Table 6-23. In a similar manner to that described previously, the process of opening and closing of the belt, modelled as 28 analysis boundaries was then modelled with DEM (Rocky). Results presented in Table 6-23 show that for identical conveying speed (time between troughed and opened cross section) to simulations performed with the 24mm belt sag geometry, the vertical velocity, \( V_y \), of each boundary is typically of greater magnitude.

Table 6-23 –3D Laser Scan Belt Geometry: 32mm Sag – Boundary Motion

<table>
<thead>
<tr>
<th>Boundary</th>
<th>( V_x ) (m/s)</th>
<th>( V_y ) (m/s)</th>
<th>( \omega ) (rad/s)</th>
<th>Boundary</th>
<th>( V_x ) (m/s)</th>
<th>( V_y ) (m/s)</th>
<th>( \omega ) (rad/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>( \pm 8.2e^{-2} )</td>
<td>( \pm 3.2e^{-2} )</td>
<td>15</td>
<td>( \pm 3.1e^{-3} )</td>
<td>( \pm 7.5e^{-2} )</td>
<td>( \pm 5.1e^{-1} )</td>
</tr>
<tr>
<td>2</td>
<td>( \pm 8.3e^{-6} )</td>
<td>( \pm 8.2e^{-2} )</td>
<td>( \pm 3.1e^{-2} )</td>
<td>16</td>
<td>( \pm 6.1e^{-3} )</td>
<td>( \pm 8.0e^{-2} )</td>
<td>( \pm 3.2e^{-1} )</td>
</tr>
<tr>
<td>3</td>
<td>( \pm 1.7e^{-6} )</td>
<td>( \pm 8.1e^{-2} )</td>
<td>( \pm 3.1e^{-2} )</td>
<td>17</td>
<td>( \pm 8.2e^{-1} )</td>
<td>( \pm 8.4e^{-2} )</td>
<td>( \pm 1.7e^{-1} )</td>
</tr>
<tr>
<td>4</td>
<td>( \pm 3.5e^{-6} )</td>
<td>( \pm 8.1e^{-2} )</td>
<td>( \pm 3.1e^{-2} )</td>
<td>18</td>
<td>( \pm 9.3e^{-3} )</td>
<td>( \pm 8.5e^{-2} )</td>
<td>( \pm 1.2e^{-1} )</td>
</tr>
<tr>
<td>5</td>
<td>( \pm 1.4e^{-3} )</td>
<td>( \pm 8.1e^{-2} )</td>
<td>( \pm 3.1e^{-2} )</td>
<td>19</td>
<td>( \pm 1.0e^{-2} )</td>
<td>( \pm 8.7e^{-2} )</td>
<td>( \pm 4.6e^{-2} )</td>
</tr>
<tr>
<td>6</td>
<td>( \pm 7.1e^{-6} )</td>
<td>( \pm 8.0e^{-2} )</td>
<td>( \pm 5.6e^{-2} )</td>
<td>20</td>
<td>( \pm 1.0e^{-2} )</td>
<td>( \pm 8.7e^{-2} )</td>
<td>( \pm 7.3e^{-2} )</td>
</tr>
<tr>
<td>7</td>
<td>( \pm 1.3e^{-5} )</td>
<td>( \pm 8.0e^{-2} )</td>
<td>( \pm 7.2e^{-2} )</td>
<td>21</td>
<td>( \pm 9.9e^{-1} )</td>
<td>( \pm 8.6e^{-2} )</td>
<td>( \pm 2.2e^{-1} )</td>
</tr>
<tr>
<td>8</td>
<td>( \pm 3.1e^{-3} )</td>
<td>( \pm 7.9e^{-2} )</td>
<td>( \pm 7.2e^{-2} )</td>
<td>22</td>
<td>( \pm 8.3e^{-3} )</td>
<td>( \pm 8.4e^{-2} )</td>
<td>( \pm 3.7e^{-1} )</td>
</tr>
<tr>
<td>9</td>
<td>( \pm 3.2e^{-3} )</td>
<td>( \pm 7.9e^{-2} )</td>
<td>( \pm 3.4e^{-1} )</td>
<td>23</td>
<td>( \pm 5.5e^{-3} )</td>
<td>( \pm 8.1e^{-2} )</td>
<td>( \pm 4.1e^{-1} )</td>
</tr>
<tr>
<td>10</td>
<td>( \pm 2.4e^{-3} )</td>
<td>( \pm 7.5e^{-2} )</td>
<td>( \pm 4.6e^{-1} )</td>
<td>24</td>
<td>( \pm 2.3e^{-3} )</td>
<td>( \pm 7.7e^{-2} )</td>
<td>( \pm 3.8e^{-1} )</td>
</tr>
<tr>
<td>11</td>
<td>( \pm 6.4e^{-4} )</td>
<td>( \pm 7.1e^{-2} )</td>
<td>( \pm 5.7e^{-1} )</td>
<td>25</td>
<td>( \pm 6.6e^{-4} )</td>
<td>( \pm 7.3e^{-2} )</td>
<td>( \pm 4.6e^{-1} )</td>
</tr>
<tr>
<td>12</td>
<td>( \pm 1.2e^{-3} )</td>
<td>( \pm 6.6e^{-2} )</td>
<td>( \pm 9.1e^{-1} )</td>
<td>26</td>
<td>( \pm 4.3e^{-3} )</td>
<td>( \pm 6.9e^{-2} )</td>
<td>( \pm 5.9e^{-1} )</td>
</tr>
<tr>
<td>13</td>
<td>( \pm 1.3e^{-3} )</td>
<td>( \pm 6.6e^{-2} )</td>
<td>( \pm 2.9e^{-1} )</td>
<td>27</td>
<td>( \pm 9.1e^{-3} )</td>
<td>( \pm 6.4e^{-2} )</td>
<td>( \pm 8.4e^{-1} )</td>
</tr>
<tr>
<td>14</td>
<td>( \pm 1.2e^{-3} )</td>
<td>( \pm 6.9e^{-2} )</td>
<td>( \pm 5.5e^{-1} )</td>
<td>28</td>
<td>( \pm 1.6e^{-2} )</td>
<td>( \pm 5.6e^{-2} )</td>
<td>( \pm 8.9e^{-1} )</td>
</tr>
</tbody>
</table>

The general trends of the transverse loading profiles observed were very similar to results shown in Figure 6-27 to Figure 6-30 as the total volume of bulk solid material modelled was maintained identical to the simulations of the 24mm belt sag geometry. Results presented in Table 6-24 show the normal force on the inclined side and on the centre of the belt.
Table 6-24 – 3D Laser Scan Belt Geometry: 32mm Sag – Normal Force on Belt

Idler Pitch of 1.2m

<table>
<thead>
<tr>
<th>Material</th>
<th>Normal Force on Each Side (N)</th>
<th>Normal Force on Centre (N)</th>
<th>Total Normal Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>coal ($\phi_i = 50^\circ$)</td>
<td>66</td>
<td>210</td>
<td>342</td>
</tr>
<tr>
<td>coal ($\phi_i = 65^\circ$)</td>
<td>66</td>
<td>204</td>
<td>336</td>
</tr>
<tr>
<td>gravel</td>
<td>119</td>
<td>406</td>
<td>644</td>
</tr>
<tr>
<td>magnetite</td>
<td>194</td>
<td>533</td>
<td>921</td>
</tr>
<tr>
<td>riversand</td>
<td>140</td>
<td>414</td>
<td>694</td>
</tr>
</tbody>
</table>

In direct comparison to the 24mm belt sag geometry results presented in Table 6-11 previously, no significant variation in the normal force exerted on the belt can be observed. Similarly, Figure 6-36 below shows very close correlation to the results presented in Figure 6-31 regarding the percentage of the total normal force supported by the inclined side (40%) and centre section of the belt (60%).

![Figure 6-36 – 3D Laser Scan Belt Geometry: 32mm Sag](image)

**Force Ratio vs. Internal Angle of Friction**

To investigate this further, a comparison was also undertaken regarding the normal force simulation data from the DEM (Rocky) model between the 24mm and 32mm belt sag. Coal and Magnetite materials were chosen due to their relative particle densities being at the uppermost and lowermost of the range considered. Figure 6-37 and Figure 6-38 show the variation in normal force on the inclined side for the duration of the simulation.
From results presented above, it is evident that the amplitude of oscillation between the open and closed belt geometry is greater for the increased belt sag of 32mm. The maximum and minimum values summarized from the above results are presented in Table 6-25 and Table 6-26. Also presented are the average force values for the duration of the simulation.
Table 6-25 – 3D Laser Scan Belt Geometry: 32mm Sag

Loading Comparison on Inclined Side Summary

<table>
<thead>
<tr>
<th>Material</th>
<th>Minimum Normal Force (N)</th>
<th>Maximum Normal Force (N)</th>
<th>Average Normal Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>coal (24mm)</td>
<td>1</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>coal (32mm)</td>
<td>0</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>magnetite (24mm)</td>
<td>3</td>
<td>13</td>
<td>8</td>
</tr>
<tr>
<td>magnetite (32mm)</td>
<td>1</td>
<td>12</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 6-26 – 3D Laser Scan Belt Geometry: 32mm Sag

Loading Comparison on Centre Summary

<table>
<thead>
<tr>
<th>Material</th>
<th>Minimum Normal Force (N)</th>
<th>Maximum Normal Force (N)</th>
<th>Average Normal Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>coal (24mm Sag)</td>
<td>3</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>coal (32mm Sag)</td>
<td>0</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>magnetite (24mm Sag)</td>
<td>6</td>
<td>15</td>
<td>11</td>
</tr>
<tr>
<td>magnetite (32mm Sag)</td>
<td>2</td>
<td>17</td>
<td>11</td>
</tr>
</tbody>
</table>

Results presented in Table 6-25 and Table 6-26 emphasize that the total amplitude of the normal force acting on both the inclined side and centre section of the belt is higher at greater belt sag. It can also be observed that the average force remains similar. During the opening and closing of the troughed shape over an identical motion period of 0.4s, the vertical displacement of each of the boundary elements will be greater at greater belt sag. Consequently, both the inclined sides and the centre of the belt will cycle through greater amplitude of force. This implies a greater amount of work would be done by the belt; therefore more energy will be consumed. This is in accordance with the findings of Mustoe and Bin [70], Nordell [72], Behrens [5], Spaans [90], Mulani [69], CEMA [6] and Wheeler [106].

6.6 Calculated Belt Loading Profile

The measured and modelled belt loading profiles obtained from the experimental testing and DEM simulations presented in Section 6.4 and Section 6.5 are compared to the calculated gravitational loads and also those based on the existing theory presented in Chapter 2. The inclination of the belt sides used in the analysis was governed by the idler roll troughing inclination angle of 35°. The conveyor surcharge angle was 13° and wall friction angles were in the range of $26^\circ < \phi_w < 33^\circ$ across the range of bulk solid materials investigated.
6.6.1 *Comparison to Gravity*

For each bulk solid material tested, the force due to gravity was calculated based on a simple $\rho g h$ calculation across the transverse section of the 600mm wide belt considered, with the normal component calculated for the inclined sides. These values are summarized in Table 6-27.

**Table 6-27 – Gravitational Only Normal Force on Belt – Idler Pitch of 1.2m**

<table>
<thead>
<tr>
<th>Material</th>
<th>Normal Force on Each Side (N)</th>
<th>Normal Force on Centre (N)</th>
<th>Total Normal Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>coal</td>
<td>40</td>
<td>225</td>
<td>305</td>
</tr>
<tr>
<td>gravel</td>
<td>82</td>
<td>461</td>
<td>625</td>
</tr>
<tr>
<td>magnetite</td>
<td>103</td>
<td>581</td>
<td>787</td>
</tr>
<tr>
<td>riversand</td>
<td>79</td>
<td>447</td>
<td>605</td>
</tr>
</tbody>
</table>

The force on the inclined side and on the centre section of the belt as a percentage of the total normal force acting is shown in Figure 6-39.

![Figure 6-39 – Gravitational Force vs. Internal Angle of Friction](image)

Results from Figure 6-39 show the total load exerted by the bulk solid material on the belt comprises of a 26% to 74% relationship between the normal force on the inclined side and the centre section of the belt respectively. Referring to results presented in Section 6.4 and Section 6.5, during transportation, the percentage of the total normal force will increase on the inclined side of the belt and decrease on the centre section of the belt. The total normal force exerted on the belt is also shown to be greater than gravity during transportation. Referring to the extrapolated TekScan results presented in Table 6-3, the normal forces on the inclined sides of the belt are in the order of 1.1 to 1.9 times greater compared to
gravity. The gravitational forces for the centre section of the belt presented in Table 6-27 fall within the range of the measured and extrapolated TekScan measurements presented in Table 6-2 and Table 6-3.

Referring to forces exhibited on the 24mm belt sag geometry presented Table 6-11, it can be observed that the normal force on the inclined side of the belt is in the order of 1.5 to 1.8 times compared to gravitational forces presented in Table 6-27. Forces exerted by the bulk solid material on the centre section of the belt, as given by the DEM simulation results presented in Table 6-11, are slightly lower compared to the gravitational loads presented in Table 6-27. However, the forces calculated using DEM simulations are within 86-96% of the calculated gravitational force.

### 6.6.2 Comparison to Spaans

The theoretical approach of Spaans [90] outlined in Section 2.1.5.4 was applied to calculate the loading on the inclined sides and the centre section of the belt with the results summarized in Table 6-28. The forces shown are based on the assumption that the passive stress state is initiated and is given by the passive pressure factor, $\lambda_p$, from equation (2.1.31).

<table>
<thead>
<tr>
<th>Material</th>
<th>Normal Force on Each Side, $F_Z$ (N)</th>
<th>Normal Force on Centre, $F_m$ (N)</th>
<th>Total Normal Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>coal ($\phi_i = 50^\circ$)</td>
<td>165</td>
<td>173</td>
<td>503</td>
</tr>
<tr>
<td>coal ($\phi_i = 65^\circ$)</td>
<td>283</td>
<td>92</td>
<td>658</td>
</tr>
<tr>
<td>gravel</td>
<td>290</td>
<td>380</td>
<td>960</td>
</tr>
<tr>
<td>magnetite</td>
<td>349</td>
<td>484</td>
<td>1182</td>
</tr>
<tr>
<td>riversand</td>
<td>200</td>
<td>403</td>
<td>803</td>
</tr>
</tbody>
</table>

Results from Table 6-28 are applied to plot the percentage of the total force exerted on the inclined side and on the centre of the belt in Figure 6-40 as a function of the internal angle of friction.
Figure 6-40 shows that the composition of the total force is very much in disagreement with both the measured and extrapolated TekScan results presented in Section 6.4. The results according to Spaans also diverge from DEM (Rocky) simulation results presented in Section 6.5. According to Spaans, the ratio between the percentage of the total normal force acting on the inclined side and the centre of the belt is shown to vary significantly with an increase in the angle of internal friction. In general, a comparison of results from Table 6-28 with results presented in Table 6-2, Table 6-3 and Table 6-11 indicates an over prediction of the normal force exerted on the inclined side and an under prediction of the normal force on the centre section of the belt.

Furthermore, as the laser belt scan geometries from Figure 6-32 to Figure 6-34 show, the mode of deflection of the belt is more likely to be governed by a pivot point located towards the outer edges of the belt rather than at the idler roll junction. As such, the above results imply the passive stress state may actually propagate from the outer edges of the belt. Similar to the findings of Chapter 5, results presented reaffirm the notion that only a fraction of the passive stress state is initiated.

Based on these findings, it is proposed that a multiplying coefficient be applied to the passive pressure factor, $\lambda_p$, in order to transmit only a fraction of the passive stress state. In a sequential manner, multiplying coefficients were investigated with the aim of reproducing a relationship comprising the normal force on the inclined side and centre section of the belt, similar in nature to that observed in Section 6.4 and Section 6.5. The normal forces presented in Table 6-29 are based on the assumption that the multiplying coefficient is equal to 0.25.
Table 6-29 – Calculated Normal Force on Belt: Spaans - 0.25 $\lambda_p$

Idler Spacing of 1.2m

<table>
<thead>
<tr>
<th>Material</th>
<th>Normal Force on Each Side, $F_Z$ (N)</th>
<th>Normal Force on Centre, $F_m$ (N)</th>
<th>Total Normal Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>coal ($\phi_i = 50^\circ$)</td>
<td>62</td>
<td>249</td>
<td>373</td>
</tr>
<tr>
<td>coal ($\phi_i = 65^\circ$)</td>
<td>87</td>
<td>238</td>
<td>412</td>
</tr>
<tr>
<td>gravel</td>
<td>117</td>
<td>515</td>
<td>749</td>
</tr>
<tr>
<td>magnetite</td>
<td>138</td>
<td>662</td>
<td>938</td>
</tr>
<tr>
<td>riversand</td>
<td>96</td>
<td>504</td>
<td>696</td>
</tr>
</tbody>
</table>

The percentages of the total normal force acting on the inclined side and the centre of the belt as a function of the internal angle of friction are plotted in Figure 6-41.

![Graph showing force ratio vs. internal angle of friction](image)

Figure 6-41 – Force Ratio vs. Internal Angle of Friction: Spaans - 0.25 $\lambda_p$

Results presented in Figure 6-41 above indicate a slightly improved correlation to results of TekScan testing and DEM simulations. As such, further investigation was conducted to specify a multiplying coefficient in closer agreement with results presented in Section 6.4 and Section 6.5 respectively.

Consequently, Table 6-30 shows the normal forces on the belt calculated using the approach of Spaans and based on the assumption that the multiplying coefficient, for free flowing to moderate to handle materials up to an internal angle of friction of 45°, is 0.4. For bulk solid materials with an internal angle of friction over 45° (coal), the multiplying coefficient was selected to be 0.2.
Table 6-30 – Calculated Normal Force on Belt: Spaans - 0.4 · \( \lambda_p \) (0.2 · \( \lambda_p \) for \( \phi_i > 45^\circ \))

<table>
<thead>
<tr>
<th>Material</th>
<th>Normal Force on Each Side, ( F_Z ) (N)</th>
<th>Normal Force on Centre, ( F_m ) (N)</th>
<th>Total Normal Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>coal (( \phi_i = 50^\circ ))</td>
<td>56</td>
<td>226</td>
<td>366</td>
</tr>
<tr>
<td>coal (( \phi_i = 65^\circ ))</td>
<td>74</td>
<td>248</td>
<td>396</td>
</tr>
<tr>
<td>gravel</td>
<td>152</td>
<td>488</td>
<td>792</td>
</tr>
<tr>
<td>magnetite</td>
<td>181</td>
<td>626</td>
<td>988</td>
</tr>
<tr>
<td>riversand</td>
<td>117</td>
<td>484</td>
<td>718</td>
</tr>
</tbody>
</table>

The percentages of the total normal force acting on the inclined side and the centre of the belt as a function of the internal angle of friction are shown in Figure 6-42.

Figure 6-42 - Force Ratio vs. Internal Angle of Friction: Spaans - 0.4 · \( \lambda_p \) (0.2 · \( \lambda_p \) for \( \phi_i > 45^\circ \))

It can be observed that the results plotted in Figure 6-42 are in much closer agreement with the extrapolated TekScan results presented in Figure 6-14. Closer agreement with the DEM simulation results based on 3D laser scan geometry presented in Figure 6-31 is also evident.

Evaluation of Spaans’ approach to experimental and simulation results of Section 6.4 and Section 6.5 shows an over prediction of normal forces on the inclined sides of the belt and an under prediction on the centre section of the belt. An over prediction of the total normal force exerted by the bulk solid material on the belt is also observed. Ultimately, this indicates that the passive stress state is not initiated.

Analysis presented shows that the these discrepancies may be counteracted with the application of a multiplying coefficient in the order of 0.4 for easy to moderate to handle bulk solid materials. The internal angles of friction of these materials are in the range of \( 30^\circ < \phi_i < 45^\circ \). For bulk solid materials
characterized by internal angles of friction above 45°, better correlation to experimental and simulated results was exhibited applying a multiplying coefficient value of 0.2.

6.6.3 Comparison to Mulani

In a similar manner to Section 6.6.2, the theoretical approach by Mulani [69] presented in Section 2.1.5.1 was applied to calculate the normal forces acting on the inclined sides and centre section of the belt. Typically, Mulani assumes the passive stress state is not entirely developed during conveying and as such Equation (2.1.26) assumes that a multiplying coefficient of 0.85 is applied to the passive (Rankine) pressure factor, $K_{1p}$. The calculated forces are summarized in Table 6-31.

<table>
<thead>
<tr>
<th>Material</th>
<th>Normal Force on Each Side, $F_{sn}$ (N)</th>
<th>Normal Force on Centre, $F_{cm}$ (N)</th>
<th>Total Normal Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>coal ($\phi_i = 50^\circ$)</td>
<td>76</td>
<td>143</td>
<td>295</td>
</tr>
<tr>
<td>coal ($\phi_i = 65^\circ$)</td>
<td>146</td>
<td>29</td>
<td>321</td>
</tr>
<tr>
<td>gravel</td>
<td>138</td>
<td>324</td>
<td>600</td>
</tr>
<tr>
<td>magnetite</td>
<td>174</td>
<td>409</td>
<td>757</td>
</tr>
<tr>
<td>riversand</td>
<td>108</td>
<td>356</td>
<td>572</td>
</tr>
</tbody>
</table>

Components of the total normal force acting on the inclined side and on the centre section of the belt are shown as a function of the internal angle of friction in Figure 6-43.

Similarly to results shown in Table 6-28 and Figure 6-40, it is evident that results according to Mulani presented in Table 6-31 and Figure 6-43 are also in disagreement with the TekScan and DEM simulation.
results presented previously. Figure 6-43 shows that the constituents of the total normal force acting on the inclined side and the centre section of the belt vary significantly with the internal angle of friction. Also, in comparison to outcomes of Section 6.4 and Section 6.5, an under prediction of the normal force exerted on the centre section of the belt and the total normal force acting on the belt can be observed.

Subsequently, a multiplying coefficient of 0.4 for internal angle of friction up to 50° and a multiplying coefficient of 0.2 for internal angle of friction of 65° were assumed and additional analysis undertaken. The procedure was identical to the analysis previously proposed in Section 6.6.2 and the normal forces exerted by the bulk solid material on the belt are presented in Table 6-32 and plotted in Figure 6-44.

<table>
<thead>
<tr>
<th>Material</th>
<th>Normal Force on Each Side, $F_{sn}$ (N)</th>
<th>Normal Force on Centre, $F_{m}$ (N)</th>
<th>Total Normal Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>coal ($\phi_i = 50^\circ$)</td>
<td>45</td>
<td>195</td>
<td>285</td>
</tr>
<tr>
<td>coal ($\phi_i = 65^\circ$)</td>
<td>60</td>
<td>170</td>
<td>290</td>
</tr>
<tr>
<td>gravel</td>
<td>104</td>
<td>380</td>
<td>588</td>
</tr>
<tr>
<td>magnetite</td>
<td>131</td>
<td>480</td>
<td>742</td>
</tr>
<tr>
<td>river sand</td>
<td>91</td>
<td>384</td>
<td>566</td>
</tr>
</tbody>
</table>

From the results presented in Table 6-32 and Figure 6-44, an improved correlation to the extrapolated TekScan and DEM simulation results of Section 6.4 and Section 6.5 can be observed.
In view of results presented in Section 6.4 and Section 6.5, and those calculated using the approach of Mulani and Spaans with the derived multiplying coefficients, it is evident that a closer correlation to experimental and DEM simulation results is obtained using the approach of Spaans, presented in Section 6.6.2. For both theoretical approaches, further analysis shows that for an internal angle of friction of 65°, a multiplication coefficient lower than 0.2 is required for the percentage of total normal force acting on the inclined side to decrease to within the 30-40% range.

6.7 Conclusion

Conveyor surcharge angle measurements conducted on a re-circulating belt conveyor testing facility were observed to be within the range indicated by CEMA [6]. Values calculated using the approach of Colijn [17] were found to be lower compared to the measured values and correlate with the bottom end of the CEMA range.

From TekScan measurements conducted on the same test facility, results show that up to five different regions with distinct transverse pressure profiles were typically observed for each bulk solid material tested. Pressure profiles immediately prior to, and immediately following, the idler roll set were observed to be similar, and in close correlation to the pressure profile due to gravity acting alone. As the belt approaches the horizontal roll of the idler set, an increase in the pressure on the centre section of the belt was observed to occur to a value exceeding the gravitational component. The pressure present in the middle section of the belt then decreases back towards the gravitational component as the belt travels from the horizontal roll to the inclined wing idler rolls. This change is due to the offset configuration of the idler rolls. Also, during this process, peak pressure shifts towards the location on the belt situated at the junction of the centre and wing idler rolls. As the belt travels away from the idler roll set, pressure within the centre section of the belt then decreases to lower than the gravitational component, while peak pressure is maintained at a location on the belt related to the junction of the centre and wing idler rolls. During travel from one idler set to the next, experimental analysis indicates that the pressure profile exerted by the bulk solid material on the belt at one half of the idler pitch will dominate the pressure profile. Consequent analysis of the sum of normal forces acting over one idler pitch for both the measured and extrapolated TekScan results show that the peak loading force on the belt occurs at the location of the belt governed by the junction of the centre and wing idler rolls.

Experimental results also show that during transportation, the percentage of the total normal force exerted by the bulk solid material on the inclined sides of the belt is in the 35-45% range. Thus, the normal force exerted by the bulk solid on the centre section of the belt is in the 55-65% range of the total normal force acting. These proportions of the total normal force were observed to occur across the entire range of internal angles of friction related to each bulk solid material tested.

DEM simulation of belt geometry obtained from the finite difference method of Wheeler [106] shows that the normal force exerted on the inclined sides of the belt is in the order of 40-48% and the normal force
supported by the centre section of the belt is 52-60% of the total normal force. Similarly, DEM simulation of geometry obtained using 3D laser scanning shows the normal force exerted on the inclined sides of the belt and on the centre section of the belt to be 40% and 60% of the total normal force respectively.

Results attained using DEM simulations were found to be within the range of values obtained from experimental TekScan results. An almost constant relationship between the normal force exerted on the inclined side of the belt and on the centre of the belt was observed to occur across the range of bulk solid materials modelled and their respective internal angles of friction. The abovementioned observations from experimental and DEM simulation results show reasonable overall correlation to the work of Grabner et al [29], and presented in Figure 2-2, from which a 30% to 70% relationship was obtained for a 30° idler roll trough. However, that work involved measuring forces due to both the bulk solid material and the belt.

An investigation of the increase in idler roll troughing angle shows an increase in the total normal force exerted by the bulk solid material on the belt. The force on the centre section of the belt was also observed to increase, however, the normal force exerted on the inclined side of the belt was found to decrease.

A brief sensitivity analysis of DEM simulation parameters indicates an increase in the total normal force exerted on the belt with a decrease in the simulated internal angle of friction. Importantly, during the opening and closing cycle of the troughed shape of the belt, DEM analysis shows that the total difference between the normal forces exerted on belt will actually increase with an increase in the modelled internal angle of friction. For a constant belt deflection during the trough opening and closing cycles, this indicates that the energy loss attributed to the closing of the belt increases with an increase in the internal angle of friction. This outcome of analysis is analogous to research by Krause and Hettler [64], Spaans [90], Mulani [69], CEMA [6] and Wheeler [106], who state that greater energy loss is attributed to bulk solid materials characterized by larger angles of internal friction. Analysis also showed that wall friction variation was not of great influence on the normal force exerted on the belt within the range of values investigated. The influence of wall friction is related to Mulani’s assertion that during the steady state conveying of bulk solid materials, wall friction is not of great relevance, as it acts to nullify the presence of net shear forces acting along the belt.

Assessment of conveyor belt 3D laser scan geometry indicates that during the opening and closing of the troughed cross section, inclined sides of the belt are likely to pivot about a point located towards the outer edges. This motion is contrary to the existing theoretical analysis proposed by Krause and Hettler [64], applied by Spaans [90] and investigated in Chapter 5, which assumes that the inclined sides pivot around a point located at the idler roll junction.

Investigation regarding the influence of belt sag showed that at greater sag, the inclined sides and centre section of the belt cycle through a normal force of greater amplitude. However, the average normal force
was observed to remain relatively unchanged. When comparing to a belt with smaller sag, at identical conveying speed, the vertical displacement and subsequent velocity of the belt in the vertical direction is higher at greater sag. As such, a greater amount of work would be performed by the belt at greater belt sag resulting in higher energy losses of the system. This finding is in accordance with the findings of Mustoe and Bin [70], Nordell [72], Behrens [5], Spaans [90], Mulani [69] and Wheeler [106].

Experimental results of Section 6.4 and DEM simulation results presented in Section 6.5.2 show that the force on the inclined sides of the belt are in the order of 1.1 to 1.9 times greater compared to gravity acting alone. These results are in good overall correlation with results observed by Behrens [5] who indicates 1.2, for a 30° idler troughing inclination angle and force 1.9 times greater at an idler inclination of 45°. Similarly, investigation of forces located on centre section of the belt shows experimental measurements and DEM simulation results are in reasonable overall correlation to observations of Behrens [5], who states that the normal force acting on the centre idler roll is given by the weight force of the volume of the material located directly above it. It is important to note however, that Behrens measured the forces due to both the bulk solid material and the belt on the idler rolls, while the results presented herein are presented for the loading due to the bulk solid material only.

Evaluation of the existing theoretical approach of Spaans [90] to experimental and DEM simulation results of Section 6.4 and Section 6.5 shows an over prediction of forces on the inclined sides of the belt and an under prediction on the centre section of the belt. An over prediction of the total normal force exerted by the bulk solid material on the belt is also observed. From a similar assessment, results using the approach of Mulani [69] show an under prediction of the normal force exerted on the centre section of the belt and the total normal force acting on the belt.

Both sets of results, according to Spaans and Mulani, show that the constituents of the total normal force acting on the inclined side and the centre section of the belt vary significantly with the internal angle of friction. This was observed to be in disagreement with experimental and DEM simulation results.

In view of the findings presented in Chapter 5, discrepancies between theoretically calculated results and those obtained using experimental testing and DEM simulations further goes to indicate that the passive stress state is not fully initiated during the conveying of bulk solid materials. Based on the assumption that only a fraction of the passive stress state is initiated, additional analysis shows that application of a multiplying coefficient using the theoretical approaches of Spaans and Mulani provides results closer to those obtained from experimental testing and DEM simulations. For bulk solid materials which can be classified as easy to moderate to handle, analysis indicates that the multiplying coefficient is in the order of 0.4. The internal angles of friction of these materials are in the range of $30^\circ < \phi_i < 45^\circ$. Assuming that the relation between the normal force on the inclined side and on the centre of the belt is within the range of 30/40% to 70/60% of the total normal force, analysis indicates the multiplying coefficient will decrease above $\phi_i = 45^\circ$. For bulk solid materials characterized by internal angles of
friction exceeding 45°, good correlation with experimental and simulation results was exhibited by applying a multiplying coefficient of 0.2.

Although the multiplying coefficients derived above exhibited good correlation to results of experimental testing and DEM simulations, further research into the multiplying coefficients across a vast range of materials exhibiting different internal angles of friction would be warranted. A review of both fabric and steel cord belts and variable belt widths and troughing profiles should also be undertaken.
Chapter 7 - Bulk Solid Interactions: Transition Zone and Discharge

The geometric shape of the bulk solid material cross section prior to discharge changes due to the variation in the profile of the conveyor belt - from a trapezoidal shape governed by the geometry of the last fully troughed idler set, to a flat profile at the head pulley. How the bulk solid material behaves in this zone is a function of the material properties of the bulk solid and the transition time, which in turn depends on the transition length and belt speed. An accurate determination of the discharge profile is essential for effective transfer chute design, specifically the geometry and position of the re-directing deflector or impact plate.

In the transition zone, bulk solid materials will have a tendency to shear and spread laterally as the belt troughing angle decreases from the last fully troughed idler set to a flat profile at discharge. This motion is similar to the theory used to analyse active stress states associated with a retaining wall pivoting about its base as described in soil mechanics by Terzaghi and Peck [93], Terzaghi [94] and Reimbert [77,79], Nedderman [71], and as found in the application to belt conveying by Krause and Hettler [64] presented in Chapter 2.

7.1 Transition Zone Analysis and DEM Simulation

The laboratory testing facility described in Chapter 5 was modified in order to investigate lowering the sides at a drop time slower than gravity. This was achieved with the use of a pneumatic cylinder as shown in Figure 7-1 below.

![Figure 7-1 - Modified Laboratory Testing Facility](image-url)
The experimental procedure involved pouring each bulk solid material into the middle of the testing facility and then shaping manually in order to produce a parabolic profile representative of that observed on the re-circulating testing facility described and shown in Section 6.1.

The sides were then lowered over a time interval of 0.5s. All tests were captured with a high-speed video camera from which active state shear planes were observed. The experimental results were compared to existing theory and to results obtained from DEM (Rocky) simulations. Investigations were made and compared to those based on Rankine’s calculation for the active slip plane angle, $\theta_a$, given by Eqn. (2.1.14), Reimbert’s active state slip plane given by Eqn. (2.1.17) and Krause and Hettler’s calculation given by Eqn. (2.1.28) utilizing the full range of material properties from Chapter 4, Table 4-2.

The results for the calculation of the active state slip plane angle, $\theta_a$, are summarized in Table 7-1 below.

<table>
<thead>
<tr>
<th>Material</th>
<th>Rankine Active State $\theta_a$ Eqn. (2.1.14)</th>
<th>Reimbert and Reimbert $\theta_a$ Eqn. (2.1.17)</th>
<th>Krause and Hettler $\theta_a$ Eqn. (2.1.28)</th>
</tr>
</thead>
<tbody>
<tr>
<td>gravel</td>
<td>68°</td>
<td>56°</td>
<td>56°</td>
</tr>
<tr>
<td>magnetite</td>
<td>68°</td>
<td>55°</td>
<td>59°</td>
</tr>
<tr>
<td>riversand</td>
<td>61°</td>
<td>55°</td>
<td>47°</td>
</tr>
<tr>
<td>coal</td>
<td>78°</td>
<td>58°</td>
<td>75°</td>
</tr>
</tbody>
</table>

For the coarse, free flowing materials, Figure 7-2, Figure 7-6 and Figure 7-10 show superimposed images from high-speed video footage typically observed at the start and also at the instant the sides are in the fully open position. Directly below the experimental images, shown in Figure 7-3, Figure 7-7 and Figure 7-11 are images from the drop tests simulated with DEM, showing the cross section at the instant the sides reach the fully open position. The final cross section from the experimental tests is again shown in Figure 7-4, Figure 7-8 and Figure 7-12 for comparison to the particle co-ordinates from the DEM simulations that are shown in Figure 7-5, Figure 7-9 and Figure 7-13.
Figure 7-2 – Gravel: Experimental Results Showing Overlaid Still Images

Figure 7-3 – Gravel: DEM (Rocky) Results

(10mm Spheres - $\mu_p = 0.8$, $\mu_r = 0.4$, $\mu_w = 0.6$, $\rho_p = 2590\text{kg/m}^3$)

Figure 7-4 – Gravel: Experimental Results – Fully Open

Figure 7-5 – Gravel: DEM (Rocky) Results - Particle Co-ordinates

(10mm Spheres - $\mu_p = 0.8$, $\mu_r = 0.4$, $\mu_w = 0.6$, $\rho_p = 2590\text{kg/m}^3$)
Figure 7-6 – Magnetite: Experimental Results Showing Overlaid Still Images

Figure 7-7 – Magnetite: DEM (Rocky) Results

(10mm Spheres - $\mu_p = 0.8$, $\mu_v = 0.4$, $\mu_w = 0.6$, $\rho_p = 3665\text{kg/m}^3$)

Figure 7-8 – Magnetite: Experimental Results – Fully Open

Figure 7-9 – Magnetite: DEM (Rocky) Results - Particle Co-ordinates

(10mm Spheres - $\mu_p = 0.8$, $\mu_v = 0.4$, $\mu_w = 0.6$, $\rho_p = 3665\text{kg/m}^3$)
Referring to Figure 7-2 to Figure 7-13, it can be observed that the middle section remains stationary during the test, while the bulk solid material supported by the inclined sides moves. The process clearly identifies moving and stationary bulk solid material and therefore the presence of slip planes.
For gravel, riversand and magnetite, it was observed that a suitable approximation of the range of active state shear plane angles could be obtained by Eqn. (2.1.14) - Rankine and Eqn. (2.1.17) – Reimbert. However, values given by Eqn. (2.1.28) – Krause and Hettler, provided the most accurate and representative evaluation, and was therefore deemed the most suitable for this particular application.

For coal, due to its cohesive nature, the bulk solid material fails internally, shearing and breaking up in the cross section. Results observed from experimental tests are shown in Figure 7-14 and Figure 7-16, while results obtained from DEM simulations are presented in Figure 7-15 and Figure 7-16. They indicate three separate sections of the bulk solid material cross section, one present above the middle section of the belt, and two sections corresponding to the inclined sides of the belt after they have been fully opened.

![Figure 7-14 – Coal: Experimental Results – Fully Open (Front View)](image1.png)

![Figure 7-15 – Coal: DEM (Rocky) Results - Particle Co-ordinates (Front View)](image2.png)

\[(5\text{mm Spheres} - \mu_p = 0.9, \mu_r = 0.9, \mu_a = 0.7, \mu_{adh} = 1, s_{adh} = 0.5\text{mm}, \rho_p = 1350\text{kg/m}^3)\]

Particle co-ordinates from the DEM simulation are shown in Figure 7-15 and Figure 7-18.
From observation of the results presented in Figure 7-14 to Figure 7-18, when the sides have been fully opened the results show good correlation to the active slip plane angle, $\theta_a$, towards centre of the bulk solid material pile and the internal angle of friction, $\phi_i$, towards the outer edges of the belt (from the side pivot junction). The active state slip plane angle, $\theta_a$, calculated by Krause and Hettler (Eqn. 2.1.28) and the internal angle of friction of $\phi_i = 65^\circ$ is plotted in Figure 7-17 and Figure 7-18.
Figure 7-19 to Figure 7-21 show a selected number of particles present on the top surface of each bulk solid material analysed and a corresponding curve fit that best describes the profile shape.

Figure 7-19 – Gravel: DEM (Rocky) Results - 6th Order Polynomial Fit of Top Surface Co-ordinates

\[ y = Ax^6 + Bx^5 + Cx^4 + Dx^3 + Ex^2 + Fx + G \]  \hspace{1cm} (6.1.1)

The constants for Eqn. (6.1.1) are summarized in Table 7-2 below.

<table>
<thead>
<tr>
<th>Material</th>
<th>( A )</th>
<th>( B )</th>
<th>( C )</th>
<th>( D )</th>
<th>( E )</th>
<th>( F )</th>
<th>( G )</th>
</tr>
</thead>
<tbody>
<tr>
<td>gravel</td>
<td>-1302.1</td>
<td>-84.2</td>
<td>91.2</td>
<td>3.8</td>
<td>-3.1</td>
<td>-28.3e-1</td>
<td>0.1</td>
</tr>
<tr>
<td>magnetite</td>
<td>-1914.6</td>
<td>-27.5</td>
<td>130.9</td>
<td>1.4</td>
<td>-3.7</td>
<td>-4.5e-1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Results presented in Figure 7-19 and Figure 7-20 indicate that the top surface of the gravel and magnetite can be approximated by a 6th order polynomial given by the equation:

\[ y = Ax^6 + Bx^5 + Cx^4 + Dx^3 + Ex^2 + Fx + G \]  \hspace{1cm} (6.1.1)
Results presented in Figure 7-21 show that the cross section of the riversand can be approximated by a parabola given by the equation:

\[ y = Jx^2 + Kx + L \]  \hspace{1cm} (6.1.2)

The constants for Eqn. (6.1.2) are summarized in Table 7-3 below.

Table 7-3 - Riversand Profile Shape Curve Fit Constants

<table>
<thead>
<tr>
<th>( J )</th>
<th>( K )</th>
<th>( L )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.8</td>
<td>1.3e^-3</td>
<td>0.11</td>
</tr>
</tbody>
</table>

For coal, due to cohesive shearing/breaking of the burden, it is not practical to fit any curve to the surface profile.

In considering the above findings a comparison to CEMA [6] and the parabolic approximation based on Roberts [86], is shown in Figure 7-22 to Figure 7-25 assuming that the head height of the material burden bed depth, H, is equal to 0.12m.
In the foregoing section it has been assumed that, during the opening of the bulk solid material cross section from a troughed profile to a flat profile, the active stress state is initiated. With this assumption results presented indicate that, for free flowing materials a representative value of the active slip plane angle, $\theta_a$, can be predicted by Krause and Hettler’s Eqn. (2.1.28)

For cohesive materials, such as coal, the results show that the cross section will tend to break or shear into three separate sections - one section present above the centre idler roll and two separate symmetrical sections on the outer edges bound by the contact length between the bulk solid material and the inclined sides of the belt. The middle section is once again bounded by the active slip plane angle, $\theta_a$, and can be
approximated by Krause and Hettler Eqn. (2.1.28). The outer sections are bounded by the internal angle of friction, $\phi_i$, of the bulk solid material and the inclined sides of the belt.

Results presented in Figure 7-19 to Figure 7-21 indicate that the geometric shape of the top surface of the bulk solid material varies considerably. However assessment of existing cross sectional geometric shape profiles at discharge given by CEMA [6] and Roberts [86] and shown in Figure 7-22 to Figure 7-25 indicates that these methods are useful and adequate in obtaining an approximation of the general geometric shape of the bulk solid material stream. In some cases, these methods may under predict the transverse spread for free flowing materials as shown in Figure 7-24 and the height of cohesive bulk solid material burden towards the outer sides of the conveyor belt as observed in Figure 7-25.

The results presented in this chapter are commonly observed in practice, with Figure 7-26 clearly showing the experimental and theoretical observations of the 6th order polynomial fit to the material profile at discharge. Ultimately the exact geometrical profile of the bulk solid material cross section at discharge will be governed by the properties of the bulk solid material at the moisture content handled.

![Figure 7-26 - Observed Shearing/Breaking of Bulk Solid Material Cross Section at Discharge (Coal)](image)

7.2 Application to Discharge Trajectories

As stated in Chapter 2, a number of different models for the determination of discharge trajectories have been developed, including those presented by CEMA [6], Dunlop Conveyor Manual [25], Korzen [63], Golka [28] and Booth [9], amongst others.

Experimental validation, assessment of accuracy, implementation of combinations of these models and differentiation between them have previously been studied in detail by Arnold and Hill [2], Huque and McLean [40,41] and Hastie [37]. Each of the trajectory methods is limited to two dimensions i.e. the vertical plane in the direction of belt travel. The work of Golka [27,28] describes a co-efficient of divergence of the flow in the plane of the conveyor belt travel and in the vertical direction perpendicular
to the belt travel direction, commencing at the head pulley centreline. Design guidelines for the determination of these coefficients are based on the characteristics of bulk solid materials and conveyor parameters and include; particle size, dustiness, environmental factors and belt speed.

In view of results presented in Section 7.1, namely the propensity of cohesive bulk solid materials to shear/break in the cross section during conveying, and also free flowing materials to spread laterally, two observations from practice are presented to highlight the influence of these findings. The images in Figure 7-27 and Figure 7-28 below show typical discharge trajectories observed for materials exhibiting cohesive properties typical of difficult to handle bulk solid materials. It can be observed that the breaking up of the cross section is not limited to material on the belt only, but also the cross section discharges as three separate streams, a middle section, and two outer wedge sections. Of further significance is the “clumping” of the bulk material shown in Figure 7-28.

![Figure 7-27 – Discharge of a Wet and Sticky Ore](image1)

![Figure 7-28 – Discharge of Bauxite Showing the Clumping Nature of a Cohesive Material](image2)
To investigate the influence of bulk solid material properties and the transition geometry on the cross sectional profile of the discharging material stream, a case study is presented. The case study will investigate the propensity of free flowing bulk solid materials to spread laterally and cohesive bulk solid materials to shear/break in the transverse direction.

### 7.2.1 Influence of Transition Geometry and Material Properties

Typical head end transition geometry for a conveyor belt in the plane of travel is shown in Figure 7-29.

![Transition Geometry in the Plane of Belt Travel](image)

In Figure 7-29, \( \alpha \) is the inclination of the conveyor, \( \epsilon \) is the inclination of the transition section for half trough depth, \( h_b \), \( R \) is the radius of the head pulley, \( L_t \) is the transition length and \( \epsilon_1 \) is the effective inclination angle of the outermost edge of the belt in the plane of the conveyor belt travel.

The manner in which the belt wraps around the head pulley gives rise to an inclination of the belt towards the outer edges that differs to the inclination of the middle section. This is illustrated in Figure 7-29, with the middle section of the belt inclined to the horizontal by the angle, \( \epsilon \). The outer edges of the belt will gradually reduce from this value to a final inclination angle to the horizontal at the outermost edges of the belt indicated by the angle \( \epsilon_1 \). Typical transition geometry of a conveyor belt in plan view is illustrated in Figure 7-30.

![Transition Geometry in Plan View](image)
From Figure 7-30, it can be visualized that the angle to the vertical of the transition of the belt will increase from the middle section towards the outer edges of the belt, and the outermost edges of the belt will have the greatest divergence. The divergence in plan view of the conveyor belt transition geometry is labeled, $\varepsilon_2$.

Consider a 2.0m wide conveyor belt, consisting of a 35° idler inclination, 3-roll troughing profile, handling coal at 8000t/h with a velocity of 5.2m/s. This particular study is based on a feed conveyor at a local coal handling facility. The nominal inclination of the conveyor is 15°, with a transition length of 6.35m, which provides an additional transition inclination of 1.6° (total inclination at discharge, $\alpha + \varepsilon = 16.6°$). In addition to the specified belt configuration (as present on site), two different transition lengths were also simulated and modeled, 1.5m and 3.175m, while maintaining identical inclination of the belt troughed lead-in and inclination of the middle section of the transition. The discharge was modeled by both the discharge trajectory approach by Roberts presented in Section 2.2.2, and DEM (Rocky) using 30mm spherical particles, with parameters of a cohesive coal identical to those described in Section 4.4.1. Additionally, the influence of material properties, on the discharging stream was also investigated by modelling a free flowing bulk solid material.

DEM (Rocky) parameters selected for the simulations of the cohesive coal were, particle friction, $\mu_p = 0.9$, rolling friction, $\mu_r = 0.9$, adhesive fraction, $\mu_{adh} = 0.5$ and particle adhesive distance $s_{adh} = 3\text{mm}$ (based on $1/10^{th}$ of the particle diameter). These values correspond to a difficult to handle coal with a slump test angle (internal angle of friction, $\phi_i$) of 65°.

Parameters selected for the free flowing coal simulations are particle friction, $\mu_p = 0.5$, and rolling friction, $\mu_r = 0.2$, corresponding to a simulated material which can be described as free flowing with a slump test angle (internal angle of friction, $\phi_i$) of 32°. For this free flowing material the simulated angle of repose was found to be identical to the internal angle of friction.

The cross section of the bulk solid material stream on the belt at the last idler set was calculated based on the approach by Roberts [86] and the belt surcharge angle $\lambda$, calculated using the method of Colijn [16] as outlined in Section 2.1.3.1. The geometry of the cross section for the cohesive bulk solid material at the last idler set before transition and the final cross section at discharge are shown in Figure 7-31 and Figure 7-32. These figures show both the DEM (Rocky) simulation results in addition to the continuum approach.
Similarly, for the free flowing material, the corresponding cross sections obtained from DEM (Rocky) overlaid with the cross section calculated from the continuum model, are shown in Figure 7-33 and Figure 7-34.
In the following analysis the material contact perimeter (B+2C) and head height, H, of the bulk solid material stream at discharge is assumed to be identical to the contact perimeter and head height at the last idler roll set before discharge. For the cohesive material, the bulk solid material cross section was assumed to shear/break in a manner described in Section 7.1, with the outer shear angle approximated by the internal angle of friction, \( \phi_i = 65^\circ \) and the inner shear plane angle approximated by the active state shear plane angle, \( \theta_a = 75^\circ \). The cross section at discharge is presented in Figure 7-31 and Figure 7-32.

When considering the free flowing material, the length of the inclined sides, C, was divided into four equal components of \( C_1, C_2, C_3 \) and \( C_4 \) as shown in Figure 7-33 and Figure 7-34.

For simplicity in the 3D CAD model, the top surface of each bulk solid material has been presented as a circular profile both at the last idler roll set prior to discharge and at discharge, however, a parabolic cross sectional profile was assumed in the analysis. Referring to Figure 7-31 and Figure 7-32, cross sectional parameters for the cohesive bulk solid material used in the continuum analysis are shown in Table 7-4 below.

<table>
<thead>
<tr>
<th>H (mm)</th>
<th>B (mm)</th>
<th>C (mm)</th>
<th>( \theta_R ) (°)</th>
<th>( \lambda ) (°)</th>
<th>( \phi ) (°)</th>
<th>( \theta_a ) (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>470</td>
<td>740</td>
<td>500</td>
<td>40</td>
<td>23</td>
<td>65</td>
<td>75</td>
</tr>
</tbody>
</table>

Similarly, referring to Figure 7-33 and Figure 7-34, cross sectional parameters for the free flowing bulk solid material used in the continuum analysis are shown in Table 7-5 below.

<table>
<thead>
<tr>
<th>H (mm)</th>
<th>B (mm)</th>
<th>C (mm)</th>
<th>( \theta_R ) (°)</th>
<th>( \lambda ) (°)</th>
<th>( \phi ) (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>430</td>
<td>740</td>
<td>580</td>
<td>32</td>
<td>14</td>
<td>32</td>
</tr>
</tbody>
</table>

Note: \( C_1 = C_2 = C_3 = C_4 = 145\text{mm} \)

### 7.2.2 6.35m Transition Length (existing)

#### 7.2.2.1 Free Flowing Material

In plan view, the transverse divergence of the material stream is bound by the outermost divergence angle, \( \varepsilon_2 = 1.0^\circ \) and zero divergence in the direction of the conveyor belt. In the side view, the inclination angle of the discharging material stream is reduced from 16.6° in middle of the trough (horizontal section) to 13.6° towards the outer edges of the belt.
Dividing the cross section into four equal components 145mm apart, and following the centreline of each component from the last idler roll set to discharge, the divergence and inclination of the trajectory may be determined for each component. This procedure is illustrated in Figure 7-35 and Figure 7-36.

![Figure 7-35 – Continuum Model: 6.35m Transition - Free Flowing Material](image)

The inclination angle at the point of discharge of the bulk solid material in contact with the inclined sides of the belt will progressively decrease from the inclination of the transition i.e. \((\alpha + \varepsilon) = 16.6^\circ\) in the middle of the belt to 13.6\(^\circ\) towards the outer edges of the belt, as illustrated in Figure 7-29 and Figure 7-30. Likewise in plan view, the divergence angle of the centre of each segment from last idler set to discharge will increase. The corresponding divergence angles and inclinations of the trajectory at discharge of each segment are summarized in Table 7-6 below.

![Figure 7-36 – Continuum Model (Side View): 6.35m Transition - Free Flowing Material](image)

<table>
<thead>
<tr>
<th>Segment</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Divergence angle (°)</td>
<td>0.1</td>
<td>0.4</td>
<td>0.6</td>
<td>1.0</td>
</tr>
<tr>
<td>Discharge angle (°)</td>
<td>16.2</td>
<td>15.5</td>
<td>14.7</td>
<td>13.6</td>
</tr>
</tbody>
</table>

Figure 7-37 and Figure 7-38 show an overlay of the discharge trajectory model and results obtained from DEM simulations for the free flowing material with the 6.35m transition length. The velocity colour scheme shown in Figure 7-38 is applicable to all DEM analysis images presented in this section.
When considering the cohesive material, the transverse divergence of the bulk solid material stream in plan view is bound by the outermost divergence angle of $\varepsilon_2 = 0.8^\circ$ towards the outer edges of the belt, and zero divergence in the direction of the conveyor belt. In the side view, the inclination angle of the discharging material stream is reduced from $16.6^\circ$ in the middle of the trough to $14.0^\circ$ towards the outer edges of the belt.
Figure 7-39 and Figure 7-40 below show an overlay of the continuum discharge trajectory model and results obtained from the DEM simulation for the cohesive material with a 6.35m transition length.

Figure 7-39 - 6.35m Transition: Discharge Trajectory Comparison (Side View)

(30mm Spheres - $\mu_p = 0.9$, $\mu_r = 0.9$, $\mu_w = 0.7$, $\mu_{adh} = 0.5$, $s_{adh} = 3.0\text{mm}$, $\rho_p = 1350\text{kg/m}^3$)

Results correlate well between the two theoretical approaches, with the divergence angle of the predicted continuum material stream matching the results obtained from DEM. Furthermore, the DEM results show the clumping of the bulk solid material stream during discharge that is evident in practice and clearly shown in Figure 7-28.

Figure 7-40 - 6.35m Transition: Discharge Trajectory Overlay Comparison

(30mm Spheres - $\mu_p = 0.9$, $\mu_r = 0.9$, $\mu_w = 0.7$, $\mu_{adh} = 0.5$, $s_{adh} = 3.0\text{mm}$, $\rho_p = 1350\text{kg/m}^3$)
7.2.3 3.175m Transition Length

7.2.3.1 Free Flowing Material

The transverse divergence of the bulk solid material stream is bound by outermost divergence angle of $e_2 = 1.7^\circ$ towards the outer edges of the belt in plan view and zero divergence in the direction of the conveyor belt. In the side view, the inclination angle of the discharging bulk solid material stream is reduced from $16.6^\circ$ in middle of the trough to $11.4^\circ$ towards the outer edges of the belt.

Following the centreline of each of the four segments from the last idler roll set to discharge and assuming an identical contact perimeter and height at discharge, the divergence and inclination of the trajectory may be determined. This process is illustrated in Figure 7-41 and Figure 7-42 below.

![Figure 7-41 – Continuum Model: 3.175m Transition - Free Flowing Material](image)

The inclination at discharge of the bulk solid material in contact with the inclined sides of the belt will progressively decrease, from an inclination of $16.6^\circ$ in the middle of the belt to $11.4^\circ$ towards the outer edges of the belt. In plan view, the divergence angle of the centre of each of the four segments from the last idler roll set to discharge will increase from $0.3^\circ$ of segment 1, to $1.7^\circ$ for segment 4. The angles of divergence in plan view and trajectory inclination at the onset of discharge for each segment are summarized in Table 7-7 below.

![Figure 7-42 – Continuum Model: 3.175m Transition - Free Flowing Material (Side View)](image)

<table>
<thead>
<tr>
<th>Segment</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Divergence angle (°)</td>
<td>0.3</td>
<td>0.8</td>
<td>1.2</td>
<td>1.7</td>
</tr>
<tr>
<td>Discharge angle (°)</td>
<td>15.9</td>
<td>14.4</td>
<td>12.9</td>
<td>11.4</td>
</tr>
</tbody>
</table>
Figure 7-43 and Figure 7-44 show an overlay of the continuum discharge trajectory model and results obtained from the DEM simulation for the free flowing material with a transition length of 3.175m.

**Figure 7-43 - 3.175m Transition: Discharge Trajectory Comparison (Side View)**

(30mm Spheres - $\mu_p = 0.5$, $\mu_r = 0.2$, $\mu_w = 0.7$, $\rho_p = 1350$kg/m³)

**Figure 7-44 - 3.175m Transition: Discharge Trajectory Overlay Comparison**

(30mm Spheres - $\mu_p = 0.5$, $\mu_r = 0.2$, $\mu_w = 0.7$, $\rho_p = 1350$kg/m³)

7.2.3.2 **Cohesive Material**

The transverse divergence of the bulk solid material stream in plan view is bound by the outermost divergence angle of $\theta_2 = 1.8^\circ$ towards the outer edges of the belt and zero divergence in the direction of
conveyor belt travel. In the side view, the inclination angle of the discharging bulk solid material stream is reduced from 16.6° in the middle of the trough to 11.2° towards the outer edges of the belt.

Figure 7-45 and Figure 7-46 below show combined results of the continuum discharge trajectory models and results obtained from the DEM simulation for the cohesive material with a transition length of 3.175m.

![Figure 7-45 - 3.175m Transition: Discharge Trajectory Comparison (Side View)](image)

(30mm Spheres - $\mu_p = 0.9$, $\mu_r = 0.9$, $\mu_w = 0.7$, $\mu_{adh} = 0.5$, $s_{adh} = 3.0\text{mm}$, $\rho_p = 1350\text{kg/m}^3$)

![Figure 7-46 - 3.175m Transition: Discharge Trajectory Overlay Comparison](image)

(30mm Spheres - $\mu_p = 0.9$, $\mu_r = 0.9$, $\mu_w = 0.7$, $\mu_{adh} = 0.5$, $s_{adh} = 3.0\text{mm}$, $\rho_p = 1350\text{kg/m}^3$)
Comparing the results for the transition lengths of 6.35m and 3.175m, for both the free flowing and cohesive material properties, it can be observed that the increase in divergence of the bulk solid material stream is only slight. Comparison of results between the cohesive and free flowing materials for the above transition lengths indicates that the free flowing material exhibits marginally greater divergence. Good correlation between the DEM and continuum results can be observed for both sets of analyses presented.

7.2.4 1.5m Transition Length

7.2.4.1 Free Flowing Material

For this relatively short transition distance the bulk solid material stream is bounded by an outermost divergence angle of $\varepsilon_2 = 3.7^\circ$. Similarly, the inclination of the discharging bulk solid material stream is reduced from $16.6^\circ$ in middle of the trough to $4.5^\circ$ towards the outer edges of the belt. Such a short transition length would seldom be used in practice; however, it has been included in this analysis to demonstrate the influence of the transition length on the divergence and trajectory of the discharging bulk solid material stream.

Dividing the cross section into four equal components, the cross section assumed from the last idler roll set to discharge is shown in Figure 7-47 and a side view at discharge is shown in Figure 7-48.

![Figure 7-47 – Continuum Model: 1.5m Transition: Free Flowing Material](image1)

![Figure 7-48 – Continuum Model: 1.5m Transition - Free Flowing Material (Side View)](image2)

The angles of divergence in plan view and the trajectory inclination angle at the onset of discharge for each segment is summarized in Table 7-8 below.
Table 7-8 – 1.5m Transition: Divergence and Inclination Angles at Discharge

<table>
<thead>
<tr>
<th>Segment</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Divergence angle (°)</td>
<td>0.5</td>
<td>1.5</td>
<td>2.4</td>
<td>3.7</td>
</tr>
<tr>
<td>Discharge angle (°)</td>
<td>15.0</td>
<td>11.9</td>
<td>8.9</td>
<td>6.0</td>
</tr>
</tbody>
</table>

Figure 7-49 and Figure 7-50 below show an overlay comparison of the continuum discharge trajectory model and the results obtained from the DEM simulation for the free flowing material for a transition length of 1.5m.

Figure 7-49 - 1.5m Transition: Discharge Trajectory Comparison (Side View)

(30mm Spheres - \( \mu_p = 0.5, \mu_s = 0.2, \mu_w = 0.7, \rho_p = 1350\text{kg/m}^3 \))

Figure 7-50 - 1.5m Transition: Discharge Trajectory Overlay Comparison

(30mm Spheres - \( \mu_p = 0.5, \mu_s = 0.2, \mu_w = 0.7, \rho_p = 1350\text{kg/m}^3 \))
The images above indicate that free flowing materials will have a tendency to continue to spread or diverge in the transverse direction following discharge, and for a constant set of material properties, the transverse divergence is influenced by the transition geometry.

From the comparison of analysis for the free flowing material presented in Figure 7-49 and Figure 7-50, results show that the transverse divergence of the material stream predicted by DEM exceeds the continuum model. This indicates that discharge may not be solely influenced by the transition geometry alone and factors such as belt speed and modelled bulk solid material parameters may also be of influence. Further practical validation and research into the contribution of these factors on the overall influence of the transverse divergence would be warranted.

7.2.4.2 Cohesive Material

For the cohesive case transverse divergence of the bulk solid material stream is bound by the outermost divergence angle of $\epsilon_2 = 3.2^\circ$ zero divergence in the direction of the conveyor belt travel. Similarly, the inclination angle of the discharging material stream is reduced from 16.6$^\circ$ in middle of the trough to 6.1$^\circ$ towards the outer edges of the belt.

Figure 7-51 and Figure 7-52 below show an overlay comparison of the continuum discharge trajectory model and results from the DEM simulation for the cohesive material with the 1.5m transition length.

![Figure 7-51 - 1.5m Transition: Discharge Trajectory Comparison (Side View)](image)

(30mm Spheres - $\mu_p = 0.9$, $\mu_r = 0.9$, $\mu_w = 0.7$, $\mu_{adh} = 0.5$, $s_{adh} = 3.0\text{mm}$, $\rho_p = 1350\text{kg/m}^3$)
Figure 7-52 - 1.5m Transition: Discharge Trajectory Overlay Comparison

(30mm Spheres - $\mu_p = 0.9$, $\mu_r = 0.9$, $\mu_w = 0.7$, $\mu_{adh} = 0.5$, $s_{adh} = 3.0\text{mm}$, $\rho_p = 1350\text{kg/m}^3$)

For the cohesive material modelled, results demonstrate the influence of the transition length on the transverse divergence of the discharging bulk solid material stream. Comparison of the continuum discharge trajectory model and results obtained from the DEM simulation show good correlation with the assumptions regarding the shearing effects of the cross section at discharge.

Comparison of the results obtained for the transition lengths of 6.35m, 3.175m and 1.5m, for both the free flowing and cohesive material properties show that decreasing the transition length leads to an increase in the transverse divergence of the discharging bulk solid material stream. A direct comparison between the cohesive and free flowing materials for each transition length indicates that the free flowing material exhibits greater divergence compared to the cohesive material.

7.2.5 Influence of Belt Speed

To gain an understanding of the validity of the transverse divergence predictions presented thus far, the bulk solid material discharge trajectory was also modelled at a lower belt speed of 2.6m/s. This analysis was only conducted for a transition length of 3.175m and assumed identical inclination and divergence parameters as stated previously for this transition length.
7.2.5.1 Free Flowing Material

The angles of divergence in plan view and inclinations of discharge trajectory for each segment are summarized in Table 7-7. Figure 7-53 and Figure 7-54 show an overlay comparison of the continuum discharge trajectory model and results obtained from the DEM simulation for the free flowing material at a transition length of 3.175m and a discharge velocity of 2.6m/s.

Figure 7-53 – 3.175m Transition: Discharge Trajectory Comparison – 2.6m/s (Side View)

(30mm Spheres - $\mu_p = 0.5, \mu_r = 0.2, \mu_w = 0.7, \rho_p = 1350\text{kg/m}^3$)

Figure 7-54 – 3.175m Transition: Discharge Trajectory Overlay Comparison – 2.6m/s

(30mm Spheres - $\mu_p = 0.5, \mu_r = 0.2, \mu_w = 0.7, \rho_p = 1350\text{kg/m}^3$)
7.2.5.2 Cohesive Material

For the cohesive material, the angle of transverse divergence in plan view and inclinations of the trajectory at the onset of discharge were as previously presented in Section 7.2.3. Figure 7-55 and Figure 7-56 show an overlay comparison of the continuum discharge trajectory models and results obtained from DEM simulation for the cohesive material at a transition length of 3.175m and a discharge velocity of 2.6m/s.

![Figure 7-55 – 3.175m Transition Discharge Trajectory Overlay Comparison – 2.6m/s (Side View)](image1)
(30mm Spheres - $\mu_p = 0.9$, $\mu_r = 0.9$, $\mu_u = 0.7$, $\mu_{adh} = 0.5$, $s_{adh} = 3.0$mm, $\rho_p = 1350$kg/m$^3$)

![Figure 7-56 – 3.175m Transition Discharge Trajectory Overlay Comparison – 2.6m/s](image2)
(30mm Spheres - $\mu_p = 0.9$, $\mu_r = 0.9$, $\mu_u = 0.7$, $\mu_{adh} = 0.5$, $s_{adh} = 3.0$mm, $\rho_p = 1350$kg/m$^3$)
Results presented in Figure 7-53 to Figure 7-56 show good correlation between the continuum discharge trajectory models assumed and DEM (Rocky) simulations at the reduced belt speed of 2.6m/s. Similarly as noted in the previous analysis the free flowing bulk solid material stream predicted by the DEM exhibits greater divergence compared to the continuum model results.

7.2.6 Summary

In addition to results presented in Figure 7-37 to Figure 7-52, results obtained from the DEM (Rocky) simulations and the continuum model for both the free flowing and cohesive materials are also shown in Appendix B. Figure B-1 to Figure B-12 show comparisons between the DEM and continuum models in both the front and top views at transition lengths of 6.35m, 3.175m and 1.5m. Furthermore, particle co-ordinates obtained from DEM (Rocky) simulations in the front and top views are also presented in Appendix B, Figure B-13 to Figure B-24.

In general, results presented in Section 7.2.2 to Section 7.2.5, and Appendix B, Figure B-1 to Figure B-24, indicate that the transition geometry influences the relative divergence or spread of the bulk solid material in the transverse direction, that is, across the face of the discharging material stream. Also, a shorter transition length will lead to greater dispersion of the material stream.

Results demonstrate the propensity of cohesive materials to shear in the transverse direction, ultimately splitting into three separate components, one present in the middle of the belt and two towards the outer edges. For the cohesive material analysed, the cross section at discharge can be approximated by a shear plane angle bound by the active stress state slip plane angle, $\theta_a$, calculated from Krause and Hettler’s Eqn. (2.1.28) presented in Section 2.1.5.2 and the internal angle of friction, $\phi_i$, of the bulk solid material.

At a fixed transition length, constant throughput and belt speed, the free flowing materials (characterized by low angles of internal friction and angles of repose) will disperse more when compared to a cohesive material (characterized by a higher angle of internal friction and angle of repose). For the belt speeds and geometries considered, results indicate that the assumption that the contact perimeter remains identical as the material stream travels from last idler roll set to discharge is shown to be valid. The slight over prediction of the burden bed depth from DEM (Rocky) simulations along the entire cross section is likely to be due to the large particle size selected, albeit due to the limitations of the software.

Quantifying the transverse divergence of free flowing bulk solid materials will ultimately be a function of the transition geometry, bulk solid material properties, air/drag and environmental effects. Results obtained from DEM analysis indicate that the free flowing bulk solid material will be inclined to rill outwards to a greater extent compared to the cohesive bulk solid material. Although site observations are in good agreement with the theoretical findings, these results highlight the need for further practical validation and research in this area.
Chapter 8 - Transfer Chute Flow Interactions

Transfer chutes are generally employed to capture, redirect and guide the flow of bulk solid materials from one conveyor belt towards one or more conveyors. Guidelines for their design have been based on maintaining a rapidly moving stream which is characterized by accelerated flow conditions. For this type of stream flow within the transfer, the continuum analysis approach previously described in Section 2.3 is normally applied.

The advantage of applying the continuum method is that it incorporates material properties which are obtained through standardised and established bulk solid material tests (described in Chapter 4) and a general idea of transfer functionality can be obtained within a matter of hours. The disadvantage is that without adequate assumptions, it cannot capture all of the characteristics exhibited by conditions other than at rapid accelerated flow. Phenomena associated with dispersed, decelerated stream (retardation), more than two co-ordinate planes of flow and material stream build-up is extremely difficult and is reliant on a vast number of assumptions.

Due to the associated supporting structure, it is often difficult or impossible to physically witness material stream flow within most transfer chute configurations. It is also dangerous to attempt to observe and record material stream flow through maintenance access hatches, and as such adequate feedback from site, other than observations prior to and immediately following the transfer is sparse. Other types of site feedback that is possible may include measurements from belt weighing systems indicating throughput and forces on idler rolls in the loading area. This type of verification of simulated material flow in transfer chutes and the impact forces on the receiving conveyor belt were measured by Katterfeld et al. [58].

Opportunely, DEM provides the advantage of allowing for such a visualization of the general mode of flow occurring through a transfer chute, and enables a qualitative analysis to be readily obtained. Accurate specification of DEM input parameters allows for the analysis to progress to a quantitative one from which more challenging systems and more difficult to handle materials may be analysed. The major disadvantage of DEM is that the computational time required is in days and weeks, rather than hours like the continuum method, and to achieve a realistic turnaround time the inclusion of fine particles typically associated with such systems is simply not possible.

Among the wide field of its application, DEM simulations of transfer chute flow belong to those that have enjoyed an increasingly large interest. Transfer chute bottlenecks have been recognized by industry as warranting considerable attention and as such an emphasis on guidelines and DEM simulations in the preliminary stages of design is gaining momentum.

The implementation of DEM to the analysis of transfer chutes is not a new subject matter. Innovative studies were conducted by Hustrulid and Mustoe [38] and Hustrulid [39] almost 20 years ago, including
the investigation of belt wear, stresses and velocities, in addition to obtaining a quantitative description of bulk solid movement through a transfer point. These studies were some of the first (see also Nordell [73]) to recognize the use of DEM as a transfer chute flow visualization tool.

Subsequent research by Dewicki and Mustoe [20] investigated calibration of the modelled bulk solid material by iterating DEM parameters such that the simulation results matched results observed on site. This form of calibration procedure would always be the preferred choice for such analysis, however, as mentioned previously, is fraught with the difficulties associated with obtaining qualitative and quantitative evaluation obscured by the transfer chute structure.

Typically, DEM calibration procedures, such as those presented in Chapter 4, are applied to establish initial bulk solid material input parameters. Bearing in mind that experimental calibration tests are typically performed on particle sizes in the 5-10mm particle diameter range, it is difficult to directly assign the same parameters to particles that have been scaled up without adequate validation. In the absence of any knowledge of the bulk solid material properties of the simulated material to be handled, a sensitivity analysis of the DEM parameters is applied. Generally, this leads to more and/or less conservative parameters being employed to gain an understanding on the overall behaviour of the transfer chute system.

Whilst particle size in reality may be in the range of sub-micron to 35mm, currently the procedure of simulating transfer chute flow generally results in modelling particles in the 25-50mm diameter particle size range. The exact particle size and loading stiffness used in the simulations is heavily constrained by the throughput, overall control volume dimensions, end goal of the simulation analysis, and ultimately project turnaround time.

Depending on the nature of the bulk solid material handled, the application to be analysed and outcomes sought, project/application specific calibration tests may also require development. Selection of DEM parameters for application to transfer chute simulation analysis with the use of project specific scale modelling type calibration tests have previously been studied by Kessler and Prenner [61].

In view of the above discussion, the approach presented in this chapter differs to past research as it implements two different existing analysis tools applied in parallel, namely the continuum analysis and DEM. An overview of the practical implementation of the continuum analysis approach to two real world case study examples is presented.

The first case study comprises a straight bifurcated (splitting stream) chute arrangement transferring coal from one delivery conveyor to two receiving conveyors. The structure of flow investigated is reflective of an accelerated type, where the bulk solid material stream velocity at loading is close to receiving conveyor belt speed. A brief sensitivity analysis of DEM (Rocky) parameters regarding modelled material flow is also presented.
The second case study consists of a stacker/reclaimer chute system transferring coal from one delivery conveyor to one receiving (boom) conveyor. The flow analysed is representative of transfer configurations governed by re-direction of a fast moving, high volume stream, where the primary mode of flow is governed by the internal (shear) properties of the bulk solid material. For this study, the material velocity at loading is considerably lower than the receiving conveyor belt speed. A study is performed by applying a modified version of the continuum analysis in parallel with DEM (Rocky) simulations and a comparison of results is presented.

For both case studies, qualitative comparison is made between observations acquired on site with the stream bed depth profiles obtained using continuum and DEM simulation analysis. Also, the cross sectional bulk solid material stream profiles at loading are assessed against those theorized by CEMA [6].

### 8.1 Analysis of Transfer Chute Flow Interactions

Material characterisation and flow parameter determination forms the fundamental basis for the implementation of the continuum chute flow analysis. This involves performing standardised, well established tests such as; compressibility, angle of repose, particle size distribution, internal friction and internal strength (flow function) characteristics as presented in Chapter 4. The properties obtained from these tests are directly applicable to the continuum analysis, are macroscopic in nature and include; bulk density, \( \rho \), wall friction angle, \( \phi_w \), and the internal angle of friction, \( \phi_i \).

Following the establishment of the bulk solid material properties to be used, belt carrying capacity may be evaluated based on CEMA [6] as shown in Section 2.1.3. Evaluation of the head height at discharge, critical velocity and discharge angle may be obtained using the approach outlined in Section 2.1.3.1 and Section 2.2 respectively, following which the trajectory may be plotted. Subsequent analysis consists of “following” a streamline from the delivery conveyor, down the geometry of the transfer chute and finally to loading onto the receiving conveyor. Typically this is performed on what is referred to as top, centre and bottom streamlines respectively. The top, centre and bottom streamlines referred to in the foregoing analysis are visually described in Figure 8-1 to Figure 8-4 for the discharge, free fall, hood and spoon impact and loading respectively.

![Figure 8-1 – Continuum Analysis: Trajectory Streamlines](image)
At the incidence of impact of any streamline with the transfer geometry, knowing the velocity immediately prior to impact, $V_i$, and the impact angle, $\theta_i$, allows for the calculation of velocity immediately following impact, $V_o$, as given by Eqn. (2.3.8). Once the initial velocity of flow commencement, $V_o$, is calculated, Eqn. (2.3.1) and Eqn. (2.3.2) may then be solved, thus obtaining the velocity profile of the hood or spoon with respect to the angular position respectively. In the case of a straight inclined chute Eqn. (2.3.6) is solved as a function of the distance down the incline. The pressure factor, $K_\text{p}$, may be assumed to be in the range of 0.4-0.8 as identified in Section 2.4.
The impact angle, $\theta$, previously described in Section 2.3, is the angle between the material streamline assessed and the chute (hood, spoon, chute or belt) wall at the point of incidence as shown in Figure 8-2, Figure 8-3 and Figure 8-4. Referring to Figure 8-3, assuming vertical free fall from the hood, the impact angle will be equal to the angular position of the spoon (if impacting with the spoon curvature) and will be equal to $90^\circ - \theta$ if impacting with a straight inclined chute, where $\theta$ is the inclination of the chute to the horizontal. During free fall, the velocity immediately prior to impact is calculated using Eqn. (2.3.7) and is based on the measured height of the falling material stream and knowing the initial velocity at commencement.

Based on a known throughput and bulk density, and assuming a width of the stream, allows for the determination of the stream bed depth at each position through the transfer. Generally, the width of the stream is governed by the contact perimeter at discharge and the width of the transfer chute at the location considered.

At loading, the acceleration may be calculated using the approach outlined in Section 2.4, however for simplicity, in the foregoing analysis rectangular cross sections have been assumed. The cross section of the bulk solid material stream within the skirts on the receiving conveyor is illustrated in Figure 8-5.

![Figure 8-5 – Cross Section on Belt Assumed at Loading](image)

The general procedure for transfer chute flow interaction analysis is summarized in Figure 8-6.
The procedure outlined in Figure 8-6 is implemented via two case studies investigating the use of the continuum approach in parallel with DEM simulations. Both case studies investigated are from a coal handling facility and involve the transfer and loading of coal at a high throughput rate and fast belt speeds. The wall liner in each case is ceramic alumina tile.

The facility from which the two case study examples are presented handles a large number of different coal types with variable characteristics from a range of different mine sites. The flow properties of the various coal types may vary dramatically and calibration test work on a specific sample was not performed. However, the bulk solid material parameters selected in both the continuum and DEM analysis are based on flow property and calibration test results reflective of a typical coal which could be described as easy to moderate to handle.
8.2 Case Study: Bifurcated Transfer

The transfer chute arrangement analysed consists of a delivery conveyor transferring coal via a curved hood deflector followed by a flop gate and bifurcation to two receiving conveyors. The two receiving conveyors travel in-line in the same direction to the delivery conveyor but at an orientation of $5.9^\circ$ in plan view. A general arrangement of the transfer is depicted in Figure 8-7 below.

![Figure 8-7 – Bifurcated Transfer: 3D General View](image)

The extent of analysis considers overall transfer flow using both continuum and DEM with a global comparison presented. Observations taken from site by way of photographs showing flow and no flow through each of the bottom loading spoons through access hatches are used as a qualitative validation. As validation to on-site observations was only possible for the loading spoon sections, in depth analysis is only presented for these sections of the transfer.

For analysis of the loading spoons and loading onto each receiving conveyor belt, geometry and analysis boundaries are presented in Figure 8-8 and Figure 8-9.
From the DEM simulations, average magnitudes of the material stream velocity were analysed at the bottom of sections S2 to S6 for the loading spoon of receiving conveyor 1, and at the bottom sections S3 to S7 for the loading spoon of receiving conveyor 2.

For each receiving conveyor, the analysis boundaries 1-12 (at full width of the belt) were used to analyse average magnitudes of the material stream velocity and normal forces (pressures) during loading and acceleration of the material stream.

**8.2.1 Input Parameters**

The parameters which were used for the continuum analysis in the study are summarized below:

- Throughput = 8000t/hr
- Bulk Density = 765 - 825kg/m³
- Conveyor Belt Velocity = 5.2m/s (all three conveyors)
- Delivery Conveyor Belt Width = 2.2m
- Delivery Conveyor Head Pulley Diameter = 1.1m
- Delivery Conveyor Inclination at Discharge = 5.4°
- Delivery Conveyor Trough Angle = 35°
- Conveyor Surcharge Angle = 20°
- Receiving Conveyor 1 Width = 2.2m
- Delivery Conveyor Length of Horizontal Section of Belt, B = 0.8m
- Material Head Height at Discharge – 0.44 - 0.47m
- Material Stream Width at Discharge – 1.82 – 1.88m
- Receiving Conveyor 1 Inclination = 7.5°
- Receiving Conveyor 2 Width = 2.5m
- Receiving Conveyor 2 Inclination = 0.6°
- Receiving Conveyor 1 and 2 Trough Angle = 45°
- Receiving Conveyor 1 Inside Skirt Width, \( W_s = 1.44 \)m
- Receiving Conveyor 2 Inside Skirt Width, \( W_s = 1.64 \)m
- Receiving Conveyor 1 Length of Horizontal Section of Belt, \( B = 0.8 \)m
- Receiving Conveyor 1 Loading Chute Width = 1.2m
- Receiving Conveyor 2 Loading Chute Width = 1.4m
- Receiving Conveyor 2 Length of Horizontal Section of Belt, \( B = 0.9 \)m
- Orientation = 5.9° (plan view)
- Transfer Head Height \( \approx 11.5 \)m – 16.8m
- Head Height of Receiving Conveyor 1 Loading Chute = 1.4m
- Head Height of Receiving Conveyor 2 Loading Chute = 2.1m
- Wall Friction Angle (coal on ceramic tile) = 27°
- Restitution Factor = 0 (completely inelastic, no rebound)
- Vertical to Normal Pressure Factor, \( K_v = 0.4 – 0.8 \)
- Coefficient of Friction with Belt and Skirts - \( \mu_1, \mu_2 = 0.7 \)

For the DEM analysis, particle parameters investigated were:

- Particle Friction - \( 0.3 < \mu_p < 0.9 \),
- Rolling Friction - \( 0.1 < \mu_r < 0.5 \),
- Chute Wall Boundary Friction - \( 0.3 < \mu_w < 0.5 \),
- Belt Friction and Skirting Friction, \( \mu_{w1,2} = 0.7 \)
- Particle Density, \( \rho_p = 1430 \text{kg/m}^3 \)
- Particle Adhesion - \( \mu_{adh} = 0.5, s_{adh} = 3.0 \text{mm} \) (for all results presented)
- Particle Size – 30mm (Spheres)

8.2.2 Continuum and DEM Analysis

Continuum analysis was conducted by following the streamline of the centre of the stream from the delivery conveyor trajectory discharge, through the hood, flop gate, down each leg of the bifurcation, down the spoon section and onto each receiving conveyor. From a velocity profile analysis, and by assuming a width of the stream at each of the locations of the transfer, an indication of the stream bed depth is obtained. Comparison of stream flow analysis results obtained from continuum and DEM (Rocky) simulations for expected flow through each transfer configuration are presented in Figure 8-10 and Figure 8-12. An overlay of the two sets of analysis for receiving conveyor 1 and receiving conveyor 2 transfers are also shown in Figure 8-11 and Figure 8-13 respectively.
The particle velocity colour scheme shown in Figure 8-11 and Figure 8-13 is applicable to all DEM images presented in this case study.
Figure 8-12 – Receiving Conveyor 2 Transfer – Comparison of Flow

\[(\mu_p = 0.5, \mu_r = 0.2, \mu_w = 0.5, \mu_{adh} = 0.5, s_{adh} = 3.0\text{mm})\]

Figure 8-13 – Receiving Conveyor 2 Transfer – DEM and Continuum Overlay

\[(\mu_p = 0.5, \mu_r = 0.2, \mu_w = 0.5, \mu_{adh} = 0.5, s_{adh} = 3.0\text{mm})\]
Since feedback from site was only available for the area of the transfers visible from the access hatches circled in Figure 8-11 and Figure 8-13, the focus of the foregoing analysis is the loading chute area only. The section of the transfer analysed for receiving conveyor 1 and receiving conveyor 2 may be identified from the top of sections S3 and S4 presented in Figure 8-8 and Figure 8-9 respectively. The initial velocity at entry into the loading chute area (sections S3 and S4) of each transfer was calculated using the continuum analysis. For receiving conveyor 1 transfer, the initial velocity was calculated to be 7.8m/s. The initial velocity leading into the receiving conveyor 2 transfer was calculated to be 6.2m/s.

The velocity profile versus angular position for the spoon loading receiving conveyor 1 is presented in Figure 8-14. Results are shown for two bulk densities (lower and upper of the range considered) and also two pressure ratio, $K_v$, values. Only a slight change of velocity can be seen across the parameters investigated, however, a slightly lower velocity at exit was observed for the larger $K_v$ value of 0.8. Also presented in Figure 8-14 is the velocity profile obtained from DEM simulations with variation in particle parameters.

![Continuum and DEM Velocity Profiles](image)

**Figure 8-14 – Receiving Conveyor 1: Spoon Velocity Profile**

$\mu_p = 0.3, \mu_r = 0.1$

$\mu_p = 0.5, \mu_r = 0.2$

$\mu_p = 0.5, \mu_r = 0.5$

$\mu_w = 0.5, \mu_{adh} = 0.5, s_{adh} = 3.0 \text{mm}$

![DEM Spoon Stream Depth Profiles](image)

**Figure 8-15 – Receiving Conveyor 1: DEM Spoon Stream Depth Profile**

$\mu_w = 0.5, \mu_{adh} = 0.5, s_{adh} = 3.0 \text{mm}$

$\mu_p = 0.3, \mu_r = 0.1$

$\mu_p = 0.5, \mu_r = 0.2$

$\mu_p = 0.5, \mu_r = 0.5$
The corresponding side view of the modelled bulk solid material stream from the DEM simulations is shown in Figure 8-15. From results presented in Figure 8-14 and Figure 8-15, it can be observed that for a constant boundary friction, an increase in the particle and rolling friction will lead to a decrease in velocity of the material stream and subsequently an increase in the material stream bed depth.

In a similar manner, the velocity profiles obtained using the continuum analysis and the DEM simulations for through the chute loading receiving conveyor 2 are presented in Figure 8-16. For each set of parameters, the side view of the corresponding flow obtained from DEM simulations showing stream depth within the spoon is shown in Figure 8-17. Slight variation in velocity is once again exhibited for the continuum analysis. From the DEM simulations, lower particle and rolling frictions show faster velocities.

![Continuum and DEM Velocity Profiles](image)

**Figure 8-16 – Receiving Conveyor 2: Spoon Velocity Profile**

\((\mu_w = 0.5, \mu_{adh} = 0.5, s_{adh} = 3.0\text{mm})\)

![DEM Spoon Stream Depth Profiles](image)

**Figure 8-17 – Receiving Conveyor 2: DEM Spoon Stream Depth Profile**

\((\mu_w = 0.5, \mu_{adh} = 0.5, s_{adh} = 3.0\text{mm})\)

Referring to Figure 8-17, visually the influence of simulation particle parameters can be observed to be less pronounced compared to the results of the analysis for receiving conveyor 1 shown in Figure 8-15.
Together with a comparison of velocity profiles presented in Figure 8-14 and Figure 8-16 this indicates that particle parameters are of less influence on the overall flow at faster material stream velocities.

From the above simulated materials, results obtained using parameters of $\mu_p = 0.5$, $\mu_r = 0.2$ ($\mu_w = 0.5$, $\mu_{padh} = 0.5$, $s_{adh} = 3.0\text{mm}$) have been utilized for direct comparison to the continuum analysis and observations acquired from site. For flow to each receiving conveyor, a comparison of the continuum and DEM velocity profiles is shown in Figure 8-18 and an overlay of the calculated material stream depth within the spoon is shown in Figure 8-19. The continuum overlay component of analysis is shown in dark blue.

A comparison of continuum and DEM analysis results presented in Figure 8-18 and Figure 8-19 shows a noticeable discrepancy for flow to receiving conveyor 1. The continuum and DEM analysis of flow to receiving conveyor 2 shows good overall correlation. Comparison of both the continuum and DEM velocity profiles indicates that receiving conveyor 1 transfer shows greater loss in material stream...
velocity when compared to receiving conveyor 2. DEM analysis indicates that material stream interaction at loading onto receiving conveyor 1 will influence flow from and within the spoon. This is highly likely due to greater angular position at the spoon exit, in combination with lower clearance and greater inclination of the receiving belt, when compared to receiving conveyor 2. Based on this finding, a visual comparison of DEM flow simulation results to photographs obtained from site, through each spoon access hatch, is presented in Figure 8-20 to Figure 8-23 for receiving conveyor 1 and in Figure 8-24 to Figure 8-27 for receiving conveyor 2.

Flow $\approx 8100 - 8330 \text{t/h}$ Following Flow

**Figure 8-20 – Receiving Conveyor 1: On Site observation – LHS**

Flow $= 8000 \text{t/h}$ ($\mu_p = 0.5, \mu_r = 0.2, \mu_w = 0.5, \mu_{adh} = 0.5, s_{adh} = 3.0 \text{mm}$)

**Figure 8-21 – Receiving Conveyor 1: DEM Simulation – LHS**

Flow $\approx 8100 - 8330 \text{t/h}$ Following Flow

**Figure 8-22 – Receiving Conveyor 1: On Site Observation - RHS**

Flow $= 8000 \text{t/h}$ ($\mu_p = 0.5, \mu_r = 0.2, \mu_w = 0.5, \mu_{adh} = 0.5, s_{adh} = 3.0 \text{mm}$)

**Figure 8-23 – Receiving Conveyor 1: DEM Simulation - RHS**
Flow ≈ 8100 - 8330t/h

Figure 8-24 - Receiving Conveyor 2: On Site observation – LHS

Flow = 8000t/h (µp = 0.5, µr = 0.2, µw = 0.5, µadh = 0.5, sadh = 3.0mm)

Figure 8-25 – Receiving Conveyor 2: DEM Simulation - LHS

Flow ≈ 8100 - 8330t/h

Figure 8-26 - Receiving Conveyor 2: On Site observation - RHS

Flow = 8000t/h (µp = 0.5, µr = 0.2, µw = 0.5, µadh = 0.5, sadh = 3.0mm)

Figure 8-27 – Receiving Conveyor 2: DEM Simulation - RHS

The comparison of observations from site and the simulations presented in Figure 8-20 to Figure 8-27 show satisfactory overall correlation, and as such further analysis is investigated with a focus on loading of each receiving conveyor. Referring to Figure 8-5 showing the bulk solid material stream profile assumed within the acceleration zone and also the continuum analysis results presented thus far, acceleration onto the belt is calculated based on the assumption that the bulk solid material is loaded onto
each receiving conveyor close to belt speed. This results in the assumption that the bulk solid material stream will not be in contact with the skirting within the loading/skirting zone. Applying Eqn. (2.3.12), the acceleration at loading was calculated to be $6.9\text{m/s}^2$.

Following discharge from the spoon, in the continuum analysis it was assumed that the bulk solid material will discharge unimpeded until making contact with the belt surface. For loading of each receiving conveyor, the impact angle at contact of the material stream and the belt, $\theta_1$ (depicted in Figure 8-4) and the velocity at the point of contact based on the centre streamline were measured and calculated. From this analysis, knowing the inclination of each receiving conveyor, the velocity immediately following impact and parallel to the direction of belt travel, $V_p$, and the corresponding normal component of velocity, $V_n$, were then determined. For receiving conveyor 1 loading velocities were calculated to be 4.8m/s and 3.1m/s respectively. For receiving conveyor 2, the velocity immediately following impact, in the direction of belt travel, $V_p$, was calculated to be 4.3m/s and the normal component, $V_n$, 3.8m/s. For both receiving conveyor 1 and receiving conveyor 2, the resulting velocity profile of the bulk solid material stream relative to the distance from the loading point is presented in Figure 8-28 and Figure 8-29. Also presented are corresponding DEM simulation results across the range of parameters considered.

![DEM Continuum/DEM Comparison](image)

**Figure 8-28 – Receiving Conveyor 1: Loading Velocity**

$(\mu_p = 0.5, \mu_r = 0.2)$

$(\mu_w = 0.5, \mu_{adh} = 0.5, s_{adh} = 3.0\text{mm})$
Results of the DEM analysis presented in Figure 8-28 show for the assumed parameters of $\mu_p = 0.5$ and $\mu_r = 0.2$, the distance required for the material stream to attain belt speed is in the order of 3.0m. The discrepancy in spoon flow velocity calculated using continuum analysis ultimately results in a much shorter prediction of the distance for the bulk solid material to attain belt speed, which was observed to be in the order of 1.0 – 1.5m. Importantly, Figure 8-28 indicates that for similar loading speeds (DEM results for parameters of $\mu_p = 0.3$ and $\mu_r = 0.1$) the results from continuum and DEM analysis are shown to be in much better agreement. A similar trend can also be observed for receiving conveyor 2 presented in Figure 8-29. Better correlation between the DEM and continuum results are observed due to the closer agreement in the calculated loading velocities.

The trends observed above regarding the acceleration distance are also evident from pressure profiles along each receiving conveyor obtained from the DEM simulations. The pressure profiles exerted by the material stream on each of the receiving conveyors are shown in Figure 8-30. For receiving conveyor 1, it can be observed that the pressure exerted by the material stream on the belt approaches steady state at a distance of approximately 3.0m. For receiving conveyor 2, the distance is in the order of 1.0 – 1.5m.
A comparison of the results presented in Figure 8-30 to impact pressure calculated for the centre streamline using Eqn. (2.3.9) of the continuum analysis shows an impact pressure of 7.4 – 7.9kPa (765kg/m$^3$ < $\rho$ < 825kg/m$^3$) for receiving conveyor 1. Similarly for receiving conveyor 2, the impact pressure was calculated to be 11.0 – 11.9kPa (765kg/m$^3$ < $\rho$ < 825kg/m$^3$). These calculations are in good correlation with the maximum pressure on each receiving belt shown in Figure 8-30. From the same figure, the pressure on receiving conveyor 1 at steady state is slightly larger due to the belt being slightly smaller in width (2.2m compared to 2.5m).

Direct output of the abrasive wear parameter, $W_a$ (refer Section 2.4) is not possible from the DEM (Rocky) software. However, from the normal pressure and also the loading velocity profiles presented in Figure 8-28 and Figure 8-29 respectively, an indicative comparison is plotted in Figure 8-31, with the friction coefficient between the belt and bulk solid material equal to 0.7. For the continuum analysis it is assumed that the normal component of loading velocity, $V_n$, will decrease in a linear manner from the loading point, up to the distance along the conveyor belt at which the material stream attains belt speed.
Results shown in Figure 8-31 show that a much better correlation between continuum and DEM analysis is observed for receiving conveyor 2. Comparison between the abrasive wear on receiving conveyor 1 and 2 obtained from DEM analysis shows that while the loading velocity predicted is considerably lower for receiving conveyor 1, the normal pressure at loading is also lower (shown in Figure 8-30) and subsequently the abrasive wear peak shown in Figure 8-31 is of similar magnitude. However, wear will occur across a greater distance along the belt as the bulk solid material is accelerated up to belt speed. For receiving conveyor 1, the higher loading velocity calculated using the continuum analysis leads to a much lower abrasive wear profile. The general offset regarding the location on the belt between the two analyses techniques observed in Figure 8-28 to Figure 8-31 is due to the continuum analysis only focusing on the centre streamline, while the DEM analysis is for the entire stream.

8.2.3 Cross Section Profile within the Skirting Area

To investigate the material stream cross section during loading on each receiving conveyor, various cut planes at the end of each analysis boundary presented in Figure 8-8 and Figure 8-9, were analysed from DEM simulations. A select number of cross sectional images at various locations along, and perpendicular to the belt, are presented in Figure 8-32 and Figure 8-33 for each receiving conveyor respectively.
Figure 8-32 – Receiving Conveyor 1: Material Stream Cross Section during Loading

The material cross section at Section 4 shown in Figure 8-32 above refers to the location on the receiving conveyor 1 immediately following the stream discharge from the loading spoon. At earlier analysis boundary sections, the bulk solid material stream is still within the loading chute. Images of the material stream at section 5 and section 6 show that only a small fraction of the material stream is in contact with the skirting (in the order of 150mm). As such the skirting friction is not likely to influence acceleration of the material onto the belt. This was also assumed in the continuum analysis.

Section 3 shown in Figure 8-33 shows the bulk solid material cross section between discharging from the loading spoon and making fully contact with the receiving belt. This is the result of greater clearance between the bottom of the spoon and the belt of receiving conveyor 2. Results show that due to a combination of wider belt, and slightly faster loading velocity, the material stream does not make contact with the skirts.
For each receiving conveyor, images presented in Figure 8-32 and Figure 8-33 are in good general agreement to CEMA [6] with respect to the cross section of bulk solid material during loading. In typical loading circumstances, where the bulk solid material stream is loaded on the belt at a velocity close to belt speed, it would be understandable that the cross section of the bulk solid material stream is loaded at a loose poured angle, such as the angle of repose. The cross sectional material stream progression from loading to after the skirting has ceased, in Figure 8-32 and Figure 8-33 above, indicates that the bulk solid material cross section will tend to decrease in cross sectional area as the bulk solid material attains the velocity of the receiving belt.

8.2.4 Influence of DEM Parameters

Extending the DEM analysis presented in Section 8.2.2 and 8.2.3 it becomes evident that reducing the coefficient of wall friction, $\mu_w$, will lead to the possibility of selecting higher particle, $\mu_p$, and rolling friction, $\mu_r$, to obtain stream loading profiles akin to those observed on site. Figure 8-34 below shows a comparison of velocity profiles through the loading chute and onto receiving conveyor 1 for two different coefficients of wall friction. The corresponding side views of the material stream flow through the loading spoon are presented in Figure 8-35.
From the results presented, it can be observed that for simulations conducted at lower coefficients of chute wall friction, due to a faster average moving stream, the influence of particle parameters becomes of lower importance. This observation is in accordance with results shown previously in Section 8.2.2.

Selecting higher particle sliding and rolling friction, or more conservative particle parameters in combination with lower chute wall friction, allows for a multitude of combinations that would produce
realistic results. Typically, the wall friction value assigned in DEM simulations is chosen based on the wall friction angle obtained through the standardised flow property tests previously described in Chapter 4. An area of uncertainty currently exists as to the validity of this procedure particularly at fast material stream flow and ultimately faster shear rates. The standard wall friction test in the Jenike direct shear apparatus described in Section 4.1.1 is conducted at a relatively slow shear rate, in the order of 2.5mm/min. The rate of shear, or velocity of flow, of the bulk solid material stream in contact with the transfer chute wall lining for the case study presented above is in the order of 4.0 - 12.0m/s. In combination with the transfer chute wall lining material being present in the installation for a considerable amount of time and subsequent polishing of the lining surface, reduction in the static wall friction value assumed based on standard testing procedures is highly plausible. Conducting wall friction measurements on a sample of the actual wall lining material installed on site, in order to ascertain a more accurate quantification would have been the preferred option, however this was not possible.

8.2.5 Summary

A case study applying the continuum and DEM analysis to a bifurcated transfer chute was conducted. Results presented show that based on the assumptions made, both analysis techniques can be applied to successfully develop an understanding of expected velocities and bulk solid material cross sectional area profiles at various locations within the transfer chute. In general, a focus on the spoon flow and loading area of the transfer shows that good overall correlation is obtained between the continuum and DEM analysis for fast flowing conditions. The bulk solid material stream observed from DEM simulations was also observed to be in good correlation to corresponding areas of the transfer obtained from photographs taken on site.

Results show that the continuum analysis approach may be applied in a successful manner and will provide an overview of conveyor transfer functionality. However, results also indicate that continuum analysis is limited to fast flow conditions and cannot produce results where transition from a fast to slow material flow regime occurs, or where interaction of more than one type of flow is exhibited. DEM analysis allows for greater insight into these specific areas of the transfer chute where the continuum analysis becomes limited in its application. Application of DEM as a valuable flow visualization tool is also demonstrated.

Material stream cross sectional profiles at loading and within the skirting area obtained from the DEM simulations show good general agreement to CEMA [6]. The type of loading exhibited would be evaluated as typical of applications where the bulk solid material stream loading velocity is close to the receiving conveyor belt speed.
8.3 Case Study: Stacker Transfer

The application and validation of continuum and DEM analysis to a stacker transfer chute was previously conducted and presented in Ilic et al [44]. An extension of this work is now presented in an effort to highlight the influence of transfer chute configuration on flow through the transfer and subsequent loading onto the receiving conveyor. A study of the relationship between the continuum analysis approach and DEM is presented, including comparison to observations obtained from site.

The stacker transfer analysed is shown in Figure 8-36 below and consists of a delivery conveyor transferring coal via a curved hood, through an axi-symmetric funnel section and spoon onto a receiving boom conveyor. During normal operation the receiving boom conveyor, spoon and skirting arrangement slew (rotates) around the centreline of the funnel and also luffs (inlines/declines). However, for the analysis presented herein, only one inclination of the boom and one slewing position is analysed. The configuration analysed in this case study is when the boom receiving conveyor is at 75° to the delivery conveyor in plan view and also is inclined at 12° to the horizontal.

The geometry and analysis boundaries used in the DEM analysis of the hood, spoon and boom conveyor are shown in Figure 8-37 to Figure 8-39. In a similar manner to the previous case study, average magnitudes of the material stream velocity were analysed at the top of sections h1 to h5 for the hood and at the top, centre and bottom of sections S1 to S4 for the spoon (increments of 10°). For the boom conveyor, analysis boundaries 1-11 (at full width of the belt) were used to analyse average magnitudes of the material stream velocity and normal forces during loading and acceleration of the material stream.
8.3.1 Input Parameters

The parameters which were used for the continuum analysis in the study are summarized below:

- Throughput = 8000t/hr
- Bulk Density = 825kg/m³
- Delivery Belt Velocity = 5.2m/s
- Receiving (Boom) Belt Velocity = 6.0m/s
- Delivery and Receiving Conveyor Belt Width = 2.0m
- Delivery and Receiving Conveyor Horizontal Section of Belt Length, \( B = 0.74 \text{m} \)
- Conveyor Surcharge Angle = 20°
- Material Head Height at Discharge – 0.45m
- Material Stream Width at Discharge – 1.8m
- Receiving Conveyor Inside Skirt Width, \( W_s = 1.326 \text{m} \)
- End Pulley Diameter = 1.3m
- Delivery Conveyor Inclination at Discharge = 16.6°
- Plan Angle = 75°
- Receiving Conveyor Inclination = 12°
- Delivery and Receiving Trough Angle = 35°
- Wall Friction Angle (Coal on Ceramic Tile) = 27°
- Restitution Factor = 0 (completely inelastic, no rebound)
- Internal Angle of Friction = 50°
- Internal Friction Coefficient = 0.77 (based on internal angle of friction)
- Vertical to Normal Pressure Factor, \( K_v = 0.4 – 0.8 \)
- Coefficient of Friction with Belt and Skirts - \( \mu_1, \mu_2 = 0.7 \)
- Horizontal Distance to Slew Centreline = 1.55m
- Transfer Head Height \( \approx 5.3 \text{m} \)
- Head Height of Spoon = 1.2m
- Hood Entry Width = 2.2m
- Hood Exit Width = 1.44m
- Spoon Entry Width = 1.54m
- Spoon Exit Width = 1.14m
- Funnel Opening = 1.4m

For the DEM component of analysis, the following particle parameters were investigated:

- Particle Friction - \( 0.3 < \mu_p < 0.9 \)
- Rolling Friction - \( 0.1 < \mu_r < 0.9 \)
- Chute Wall Boundary Friction - \( 0.3 < \mu_w < 0.5 \)
- Belt and Skirting Friction - \( \mu_{w1,2} = 0.7 \)
- Particle Density, \( \rho_p = 1430 \text{kg/m}^3 \)
- Particle Adhesion - \( \mu_{adh} = 0.5, s_{adh} = 3.0 \text{mm} \)
- Particle Size – 30mm (Spheres)

8.3.2 **Continuum and DEM Analysis**

A screenshot obtained from the DEM simulation at steady state with particle parameters identical to those for the previous case study presented in Section 8.2 is shown in Figure 8-40. The particle velocity colour
scheme shown is applicable to all subsequent DEM images presented in this section unless noted otherwise.

Continuum analysis was conducted by assessing both the top and centre streamlines of discharge from the delivery conveyor, through the curved hood and spoon and onto the receiving conveyor belt. A description of the streamlines referred to for each component of the subsequent continuum analysis was presented in Figure 8-1 to Figure 8-4 previously.

Based on calculations performed on the top and centre streamlines, the velocity profiles through the hood obtained using continuum analysis are presented in Figure 8-41. The corresponding DEM simulation velocity profiles across the range of particle parameters simulated is shown in Figure 8-42. Presented are the average velocities of the entire material stream and the 200mm centre section of the stream at each angular position analysed. Also presented in Figure 8-42 is a comparison of the continuum and DEM velocity profiles for particle parameters of $\mu_p = 0.5$, $\mu_r = 0.2$, $\mu_w = 0.5$, $\mu_{adh} = 0.5$, and $s_{adh} = 3.0\text{mm}$.
Based on the assumptions made in the continuum analysis, good overall correlation may be observed to the velocity profile from DEM. For the continuum analysis, the influence of the pressure ratio, $K_v$, was found to be negligible. Similarly, in the DEM analysis, the variation of particle parameters on the hood velocity profile shows that the variation in the average material stream exit velocity is small and within 0.5m/s.
Good overall correlation in velocity profiles presented in Figure 8-42 is further illustrated in the overlay of continuum and DEM (parameters of $\mu_p = 0.5$, $\mu_r = 0.2$, $\mu_{adh} = 0.5$ and $s_{adh} = 3.0\text{mm}$) analysis presented in Figure 8-43. Shown is the DEM flow only, overlay of continuum analysis results (based on centre streamline) and DEM for the entire stream, and also the centre section of the stream only (200mm).

Good agreement of the stream calculated using continuum analysis based on the centre streamline may be observed to the DEM results showing only the centre portion of the stream. To further demonstrate the variation in material stream velocity from the outer edges to the centre of the stream, corresponding cross sections of the material stream through the hood are also presented in Figure 8-44.
Sectional hood stream images presented in Figure 8-44 show that the bulk solid material stream will have a faster velocity in the centre of the bulk solid material stream, when compared to the outer edges of the material stream in contact with the hood wall lining. Collective results of flow through the hood presented above show the continuum analysis based on the centre streamline is in very good agreement to the flow exhibited from the DEM if only considering the centre of the material stream. This indicates that bulk solid material interaction due to the outside edges of the stream with the converging width of the hood presents other phenomena not adequately described by the continuum analysis.

In view of these findings, continuum analysis was extended to analyse subsequent flow through the spoon and onto the boom conveyor belt. Based on the centre streamline following discharge from the hood at a velocity calculated to be 4.7m/s, the bulk solid material inner, centre and outer streamlines were measured to fall vertically heights of 3.3m, 4.0m and 4.2m respectively, prior to impacting with the spoon surface. These streamlines were previously described in Figure 8-3 and the angles of impact associated were measured to be 21°, 59° and 80° respectively. Due to significantly large impact angles associated with the centre and outer streamlines, continuum analysis showing the spoon velocity profile was only conducted based on the inner streamline and is shown in Figure 8-45. Also shown is the calculated material stream bed depth profile based on a width of the material stream equal to the width of the spoon at each angular position.

From a comparison of the stream bed depth profile shown above, to the mode of flow demonstrated in DEM simulation analysis presented in Figure 8-40, a large discrepancy is observed. To visualize this further, flow through the transfer was investigated with DEM, but without the boom in place, solely focusing on the flow through the spoon. The axi-symmetric funnel section was also replaced by a modified encasement allowing the material stream to build up in an unrestricted manner. Upon further
investigation, it becomes evident that due to the space restrictions, the entire bulk solid material stream will not flow by failure along the back wall of the spoon.

The images presented in Figure 8-46 below show the centreline cross section of flow through the loading spoon without the boom belt present for three different sets of particle parameters. Also shown overlaid on top of the flow is the axi-symmetric funnel section which is present on site.

![Figure 8-46 – DEM Analysis: Influence of Modelled Internal Angle of Friction on Flow](image)

Referring to the shear flow images presented, it is evident that particle parameters influence the structure or mode of flow through the loading spoon. A steeper shear plane (or material stream direction of flow) can be observed with more conservative parameters. This is reflective of the relationship between particle sliding and rolling friction parameters investigated using the slump plane calibration test previously presented in Chapter 4. For results presented in Figure 8-46, a pronounced angle of internal shear flow can be observed, however, the angle is much less pronounced at the lowest particle and rolling friction ($\mu_p = 0.3$ and $\mu_r = 0.1$).

Another aspect of flow worthy of note which can be observed in Figure 8-46 is that the funnel section is likely to constrict the natural rill or flow of the material stream. Effectively the cross sectional area available to accommodate the flowing, or shearing bulk solid material stream, will be reduced to that shown. As such, at higher angles of internal friction, blockage may ensue as there is simply inadequate head room to accommodate the large material stream.

For analysis of the average magnitude of the material stream velocity obtained at angular positions from the top of the spoon section S1 to bottom of the spoon section S4, the velocity profile obtained from DEM is shown in Figure 8-47 for the range of particle parameters simulated. Presented is the average velocity profile of the entire stream and for the 200mm centre portion of the stream only.
Results show that, for the range of simulation parameters modelled, even if the initial velocity more than doubles in magnitude, the material stream velocity of the entire stream at the exit of the spoon remains relatively constant, at around 4m/s. Analysis of the centre of the material stream only shows slightly faster velocities, however, the same trend is observed; material stream velocities at the exit of the spoon are within 0.5m/s across the range of parameters simulated.

The side view obtained from the DEM simulation analysis for particle parameters of \( \mu_p = 0.5, \mu_r = 0.2, \) (\( \mu_w = 0.5, \mu_{adh} = 0.5 \) and \( s_{adh} = 3.0\text{mm} \)) is presented in Figure 8-48 with the boom and funnel in place. Shown are the full stream and also the cross section view along the centreline of the receiving boom belt and spoon. A comparison of the corresponding spoon velocity profile for flow, with and without the boom belt in place, is plotted in Figure 8-49. From the profiles shown in Figure 8-49 it can be observed that the interaction between the bulk solid material stream and the boom belt actually influence the flow profile of the material stream within the spoon. A lower exit velocity can be observed for flow through the spoon with the boom belt in place.
For flow with and without the boom belt, it can be observed that the velocity of the material stream is faster in the centre section of the stream compared to the sections of the material stream in contact with

the spoon surface. Variation of the material stream cross section at the top, in the middle, and at the exit of the spoon obtained from the DEM simulation of the full stream flow with the boom belt in place is shown in Figure 8-50.

Stream cross sectional images of the flow through the spoon shown in Figure 8-50 above accentuate there is a significant proportion of slow moving, or dead material, present in direct contact with the spoon wall lining, and a faster stream flowing over the top.

Consequent to the above observations, the continuum analysis was modified to analyse the flow in a different manner, as the flow condition at the spoon surface is expected to deviate away from rapid accelerated flow. In further continuum analysis of the spoon flow, the material stream was assumed to change its mode of flow from an accelerated type to one governed by internal shearing. For this flow, it is assumed that the bulk solid material in contact with the spoon surface will tend to form a slow, or dead zone, which will build up to a constant inclination angle governed by the internal angle of friction of the material. Subsequent material falling from the hood is assumed to be re-directed down the incline and the flow is analysed as a straight inclined chute for which the velocity is calculated using Eqn. (2.3.7).

Following impact between the falling material stream and the inclined surface, the coefficient of internal friction of the shearing plane is based on gravity reclaim stockpile (rathole flow) and feeder flow principles of internal shearing described in Roberts [84]. These two types of flow are dependent on the shearing of the bulk solid material on itself with the coefficient of friction at the surface of the shear plane assumed to be:

$$\mu_{\text{shear}} = \sin(\phi_i)$$  \hspace{1cm} (8.1)

Where $\phi_i$ is the internal angle of friction

Applying Eqn. (2.3.8) from Section 2.3, and assuming that at the shear surface, $\mu = \mu_{\text{shear}}$, the velocity immediately following impact based on the inner, centre and outer streamlines were calculated to be
2.4 m/s, 2.6 m/s and 2.8 m/s respectively. These values correspond to a constant impact angle of 40° and drop heights of 2.8 m, 3.6 m and 4.2 m respectively. The spoon velocity profile calculated using the modified continuum approach, based on the inner and centre streamlines is shown in Figure 8-51. Also shown is a comparison of the continuum velocity profiles to both the spoon flow profiles with and without the boom belt in place.

Continuum Shear Flow, \( \mu = \sin \phi \)  

Continuum/DEM Comparison

Figure 8-51 – Spoon Velocity Profile Comparison: Continuum Shear and DEM

\[(\mu_p = 0.5, \mu_r = 0.2, \mu_o = 0.5, \mu_{adh} = 0.5, s_{adh} = 3.0 \text{mm})\]

Good overall correlation may be observed between the velocity profiles shown in Figure 8-51. The velocity profile approximated using the continuum analysis approach is within the range of velocities obtained from the DEM analysis. Stream velocities based on the inner and centre streamlines at exit of the spoon were calculated to be 3.8 m/s and 3.3 m/s respectively. Also, as mentioned previously, the velocity immediately following impact based on the outer streamline was calculated to be 2.8 m/s.

The above comparison between results from continuum and DEM analysis is further illustrated in Figure 8-52 and Figure 8-53, which show an overlay of the material burden depth calculated using the continuum analysis with internal shear and flow through the spoon, with and without the boom belt in place. The results show that the continuum analysis with internal shear better describes the flow through the spoon section without the boom belt in place. This further illustrates that the continuum analysis in the existing state is inadequate in capturing all of the flow dynamics associated with flows other than fast, accelerating flow conditions.
Following the material exiting the spoon, the stream was assumed to continue to flow at the internal angle of friction of 50° until impact with the belt. The corresponding boom loading velocities, in the direction of receiving belt travel were calculated to be 1.9m/s for the inner streamline, 1.7m/s for the centre streamline and 1.4m/s for the outer streamline. Further investigation using continuum analysis was conducted with loading velocities, $V_p$, of 1.4m/s and 1.9m/s, with corresponding normal components of velocity, $V_n$, calculated to be 2.6m/s and 3.6m/s respectively.

From the throughput and bulk density, the total cross sectional area may be calculated once a loading velocity is known. In view of the assumed cross section at loading, illustrated in Figure 8-5, the calculated loading parameters, including acceleration for each set of continuum results based on inner and outer streamlines are shown in Table 8-1.
Table 8-1 – Continuum Analysis: Parameters at Loading

<table>
<thead>
<tr>
<th>$V_p$ (m/s)</th>
<th>$A_{trough}$ (m²)</th>
<th>$A_{skirts}$ (m²)</th>
<th>$h_s$ (m)</th>
<th>$K_v$</th>
<th>$h/W_s$</th>
<th>$a$ (m/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.4</td>
<td>0.2</td>
<td>1.7</td>
<td>1.3</td>
<td>0.4</td>
<td>1.0</td>
<td>4.1</td>
</tr>
<tr>
<td>1.4</td>
<td>0.2</td>
<td>1.7</td>
<td>1.3</td>
<td>0.8</td>
<td>1.0</td>
<td>1.4</td>
</tr>
<tr>
<td>1.9</td>
<td>0.2</td>
<td>1.2</td>
<td>0.9</td>
<td>0.4</td>
<td>0.7</td>
<td>4.9</td>
</tr>
<tr>
<td>1.9</td>
<td>0.2</td>
<td>1.2</td>
<td>0.9</td>
<td>0.8</td>
<td>0.7</td>
<td>3.0</td>
</tr>
</tbody>
</table>

The loading velocities and accelerations shown in Table 8-1 were then used to calculate the expected velocity on the boom belt as a function of the distance from the loading point, the results of which are shown in Figure 8-54. The profiles shown are presented for both loading velocities and accelerations calculated with a varying pressure ratio, $K_v$. At the end of each analysis boundary, previously illustrated in Figure 8-39, the average magnitude of the material stream velocity was determined and is shown in Figure 8-54.

![Figure 8-54 – Material Stream Velocity on the Boom Belt](image)

($\mu_p = 0.5$, $\mu_r = 0.2$, $\mu_w = 0.5$, $\mu_{adh} = 0.5$, $s_{adh} = 3.0$mm)

The results presented in Figure 8-54 indicate that good overall correlation between continuum and DEM analysis can be obtained, however, it is dependent on the streamline followed and assumptions made. Furthermore, it can be observed that the acceleration of the bulk solid material stream is heavily influenced by the selection of the pressure factor, $K_v$.

DEM analysis indicates that the distance at which the bulk solid material stream will attain the velocity of the belt is in the order of 4.0 to 6.0m. Similarly, continuum analysis results presented indicate that the distance at which the bulk solid material stream attains belt speed is above 3.0m. Once again, the distance is dependent on the loading velocity and acceleration assumed.
Similarly to the pressure profiles obtained for the case study presented previously in Section 8.2, the pressure exerted by the bulk solid material stream on the boom belt is shown in Figure 8-55. Based on velocities calculated from the continuum analysis, using the inner and outer streamlines, corresponding impact pressures using Eqn. (2.3.9) were calculated to be 5.6kPa and 10.7kPa respectively.

It can be observed that the pressure associated with the faster loading velocity calculated from inner streamline calculations is in better agreement with the DEM analysis. From the pressure profile shown in Figure 8-55 and velocity profile presented in Figure 8-54, a comparison of the abrasive wear parameter, $W_a$, profile along the boom belt is presented in Figure 8-56. The abrasive wear parameter reported was calculated using Eqn. (2.3.10) in which the normal velocity component, $V_n$, was assumed to reduce in a linear manner from 2.6m/s and 3.6m/s at the loading point, to zero at the distance at which the material stream reaches belt speed.
The abrasive wear parameter profile comparison shown in Figure 8-56 indicates similar trends to those observed in Figure 8-54 and Figure 8-55. Namely, disregarding the analysis shown for the acceleration based on the outer streamline, good overall correlation may be observed between the continuum analysis and DEM simulation results. Results of the continuum analysis show that the distance along the boom belt at which abrasive wear is minimized is in the order of 3.0 to 6.0m. This is in good agreement with results obtained from DEM.

Unfortunately, observations from site do not show a great deal of further insight or validation regarding flow through the transfer. Photographs from site are shown in Figure 8-57 to Figure 8-59, showing areas of flow through the hood, spoon and on the boom conveyor respectively. The most useful observation regarding validation of the continuum and DEM analysis is presented in Figure 8-58 which shows coal remaining on the spoon surface following cessation of flow through the transfer. The general shape of the top surface of the remaining dead quantity of coal may be observed to be planar and inclined, from the exit of the spoon, towards the bottom of the funnel. The shape and position of the surface, and the quantity of the dead material remaining are all in good overall agreement to what was observed from the general mode of flow in the DEM simulation results. Feedback received from site, indicates that this transfer is currently delivering tonnages up to 8300t/h, dependent on the type of coal handled.

Figure 8-57 – Hood Discharge and Hood Surface Following Flow
To investigate the material stream cross sectional shape during loading and acceleration up to belt speed, sectional views perpendicular to the boom belt were examined at the end of each analysis boundary section 1-11. The corresponding sections are shown in Figure 8-60.

The images presented show for the transfer configuration analysed, the top surface of the bulk solid material will exhibit a flat profile rather than the curved type theorized by CEMA [6] and illustrated in Section 2.4. The bulk solid material cross sections shown are also different to the profiles exhibited in the case study presented in Section 8.2, with a greater height of bulk solid material in contact with the skirts during acceleration. This is more pronounced in the initial loading area, illustrated in section 1 to section 5.
8.3.4 Influence of Wall Friction

An investigation of the flow through the spoon with a reduced coefficient of friction for both the hood and the spoon indicates that for this particular transfer, the influence on the material stream velocity profiles will not be of great influence. Flow through the spoon, without the boom belt in place and at reduced wall friction, is shown in Figure 8-61, with the corresponding velocities through both the hood and spoon shown in Figure 8-62.
Figure 8-61 – Influence of Wall Friction on Spoon Flow

\[ \mu_p = 0.5, \mu_r = 0.2, \mu_w = 0.5 \quad \mu_p = 0.5, \mu_r = 0.2, \mu_w = 0.3 \]

\[ \mu_p = 0.9, \mu_r = 0.9, \mu_w = 0.5 \quad \mu_p = 0.9, \mu_r = 0.9, \mu_w = 0.3 \]

Figure 8-62 – DEM Analysis: Influence of Wall Friction

\[ (\mu_{adh} = 0.5, s_{adh} = 3.0\text{mm}) \]

Hood

Spoon

Figure 8-62 – DEM Analysis: Influence of Wall Friction

\[ (\mu_{adh} = 0.5, s_{adh} = 3.0\text{mm}) \]
Variation of wall friction presented in Figure 8-62 shows that while the reduction of wall friction results in an increase in velocity discharging from the hood, the velocity through the spoon section will not be significantly influenced. This observation is a consequence of the limited head height available to redirect the fast moving, large volume of material.

8.3.5 Summary

Analysis of a stacker transfer chute has been performed by way of a case study incorporating both the continuum and DEM approaches. Results presented demonstrate that from parameters obtained through standard flow property tests, the continuum analysis allows an overview of transfer functionality to be obtained. From a velocity profile, typically determined by following a streamline from the delivery conveyor discharge, through the transfer and onto the receiving conveyor, critical areas of the transfer can be identified. The velocity profile was used to calculate the corresponding cross sectional bulk solid material stream profile through the transfer and was also extended to analyse the loading characteristics on the receiving conveyor belt.

Flow through the hood section of the transfer showed good agreement between results obtained from continuum and DEM analysis. Initially, flow through the spoon section based on the traditional application of the continuum analysis showed a discrepancy to predictions of the DEM simulations. In the absence of observations of bulk solid material flow interactions on site, application of DEM allowed for flow visualization to be obtained. This visualization was implemented to develop a set of assumptions, based on which, the continuum analysis was modified. Subsequently, it was assumed that the flow through the stacker transfer will switch from accelerated to an internal shear flow. The surface along which flow occurs was analysed as a straight inclined chute, the friction of which was related to the internal angle of friction of the bulk solid material handled. Results of this analysis showed good correlation to those observed in the DEM simulations. Qualitative evidence of the predicted material flow visualized in the DEM simulations was confirmed by site feedback.

Bulk solid material interactions at the interface of the loading spoon and the receiving belt were observed to influence flow through the spoon and deceleration of the material stream was observed. Similarly, the cross sectional profile of the material stream on the receiving conveyor belt within the skirting zone was found to be influenced by interactions occurring within the transfer chute configuration and at the interface with the belt. DEM analysis showed that the top surface of the bulk solid material will exhibit a flat, horizontal profile, rather than the curved profile theorized by CEMA [6]. The loading profile exhibited is a result of the material stream loading velocity being substantially lower than the receiving conveyor belt speed.

8.4 Conclusion

Interactions occurring during the transfer and re-direction of bulk solid materials from one conveyor, to one or more conveyors, were investigated in two case study examples at a coal handling facility. The
analysis undertaken consisted of the continuum approach originally proposed by Roberts [80], conducted in parallel with DEM simulations.

Results presented for each case study show that the application of the existing theoretical approach of the continuum method allows for an evaluation of transfer chute functionality to be obtained in a short turnaround time. Typically, the continuum analysis is performed based on flow property results directly measured via standardised tests. The procedure for the practical implementation of the continuum analysis was outlined and consists of following a streamline of material discharging from the delivery conveyor, through the transfer configuration and onto the receiving conveyor belt. Equations of motion summarized in Chapter 2 are applied from which a velocity profile of the material stream through the transfer is obtained. The velocity profile of the material stream is then used to calculate the corresponding burden depth at any location within the transfer including during loading and acceleration. Wear analysis on the receiving conveyor is also calculated from the velocity profile.

In contrast, the parameters used in DEM analysis are typically selected through a calibration process, as outlined previously in Chapter 4. For applications of DEM where full calibration of parameters is not possible, as in the case studies presented, parameter selection was performed by qualitative comparison between flow simulation results and observations from site. Investigation of the sensitivity of the simulation parameters was also undertaken. Under fast, accelerated flow conditions, results using the continuum analysis approach were found to be in good agreement to DEM simulations and also to observations from site.

Analysis of the loading characteristics from the two case studies presented shows the cross sectional profile of the bulk solid material stream on the outgoing belt is governed by flow through the transfer chute and the interface of the loading chute and receiving belt. In situations where the bulk solid material stream is loaded onto the receiving conveyor belt at a velocity close to the outgoing belt speed, analysis indicates the top of the material stream cross sectional profile is likely to be curved, as suggested by CEMA [6]. In other situations where the bulk solid material stream is loaded onto the receiving belt at a velocity considerably lower than the outgoing belt, results show a much flatter profile, with a significant proportion of the bulk solid material in contact with the skirting. Further results obtained from DEM analysis also show that where the space to re-direct a high volumetric flow rate of the material stream is limited, interaction between the bulk solid material and the receiving belt will influence flow through the loading spoon. In general, a reduction in material stream velocity during flow through the loading spoon and an increase in burden depth were observed.

Implementation of DEM to bulk solid interactions during re-direction and transfer proved not only a powerful flow visualization tool, but also a thorough analysis tool. In the absence of observations available within the transfer chute, visualization of flow through the use of DEM has enabled the continuum approach to be used for analysis of flow conditions other than fast accelerated flow. In particular, DEM analysis was used to modify the implementation of the continuum approach to analyse
flow through a transfer configuration where the mode of flow was governed by internal shearing of the bulk solid material, rather than failure at the wall boundary. The same procedure adopted in the internal shearing analysis has also been applied in the continuum analysis of rock box (dead box) type flow for which a study was previously presented by Donohue et al [24].
Chapter 9 - Concluding Remarks

The aim of the research presented in this thesis has been to investigate bulk solid material behaviour and interactions during the course of transportation in belt conveying systems. A number of available theoretical approaches have been presented spanning across different areas, including; transportation, discharge, re-direction and transfer, loading and acceleration of the bulk solid material. Each of these areas consists of its own sub group of analysis techniques with the underlying common factor being the characteristic nature of the bulk solid material handled.

In parallel to the investigation using existing theoretical techniques, computer simulation implementing Discrete Element Modelling (DEM) analysis was also performed. An investigation of bulk solid material properties, both physical and virtual, influencing these interactions was undertaken and the relationship between the physical and simulated particle parameters was also investigated.

Standard experimental tests were presented from which a set of macroscopic parameters were determined and the importance of understanding the nature of the physical properties of bulk solid material was demonstrated. A suite of calibration tests was developed and implemented in order to select DEM parameters reflective of real physical behaviour of the bulk solid materials tested. An overview of the DEM contact models used in simulations was presented, including parameters that influence the behaviour of the modelled bulk solid materials. From results presented, it became evident that more than one set of parameters describing the physical characteristics of the bulk solid materials tested is possible. For the purpose of modelling in this thesis, one set of characterising DEM parameters was chosen as representative of the category of bulk solid material investigated, including; free flowing, moderate to handle and difficult to handle, cohesive materials. In view of more than one set of characterising DEM parameters, a sensitivity analysis accompanied all analysis of bulk solid material interactions investigated.

For interactions occurring between the bulk solid material and the belt during conveying, theoretical analysis of active and passive stress states according to Krause and Hettler [64] was investigated in parallel with DEM simulations. Experimental measurements were performed on a test apparatus simulating the opening and closing motion of the conveyor belt by oscillating the inclined sides of the belt. The pivot point of the sides was located at the junction of the inclined sides and the horizontal section of the belt. The transverse profile of all bulk solid materials modelled indicated good correlation to the transverse bulk solid material profile observed during experimental testing. This result indicates that the transverse bulk solid material behaviour modelled using DEM simulations as mono-sized spherical particles with parameters obtained via calibration testing presented is reflective of that observed in reality.

Cohesive shearing/separation of the material cross sections simulated with DEM were found to be in close agreement to those observed in the experimental tests. Observations made during both experimental
and simulated oscillation of the bulk solid material showed the process of shearing/tearing occurs after a short period of oscillation. Despite good qualitative agreement, expected loads on the belt did not correlate with those measured via experimental testing. The particle size modelled was believed to be too large to produce more accurate results, however, simulations with significantly smaller particles were not investigated due to exorbitant computational time. A comparison of experimental results and theoretical calculations assuming that only the weight of the wedge acts on the inclined side showed good agreement.

The influence of adhesion of the bulk solid materials to the belt has not been conducted as part of this research. Calibration tests do exist for this work (such as the inclining wall friction tester presented) and thorough research work is currently underway in this area. This is another important aspect of bulk solid and conveyor belt interactions to consider; especially as any additional force between the bulk solid material and the belt would in effect pronounce the shearing and failure cracks associated with breaking up of the material cross section.

Experimental testing and DEM simulation results indicated that material with highest bulk density exhibited the greatest normal pressure on the belt surface during the opening and closing cycles. Results obtained from both the experimental tests and DEM simulations regarding the opening cycle showed good correlation to the values calculated from the theoretical approach of Krause and Hettler. This result indicates that the active stress state is initiated with relatively small angular displacement of the belt. For the closing cycle, experimental and DEM simulation results showed lower values to those calculated using Krause and Hettler’s theory for all materials investigated, with the exception of the free flowing material. Outcomes of this study showed that the theoretical approach of Krause and Hettler is only valid for free flowing bulk solid materials, with low internal angles of friction and low friction between the material and the conveyor belt. The passive stress state was not fully developed during the closing cycle for the angular displacement investigated.

Experimental measurements conducted on a re-circulating belt conveyor test facility with offset idler rolls showed that up to five different regions with distinct transverse pressure profiles were typically observed. Pressure profiles immediately prior to, and immediately following, the idler roll set were in close agreement to the pressure profile due to gravity. As the belt approaches the horizontal roll of the idler set, an increase in the pressure on the centre section of the belt was observed to reach a value exceeding the gravitational component. Pressure present in the middle section of the belt then decreases to the gravitational component as the belt travels from the horizontal roll to the inclined wing rolls. During this process, peak pressure shifts towards the location on the belt situated at the junction of the centre and wing idler rolls.

As the belt travels away from the idler roll set, pressure within the centre section of the belt was observed to decrease to values lower than the gravitational component, while peak pressure is maintained at the junction of the centre and wing idler rolls. In travelling from one idler set to the next, experimental analysis showed that the pressure profile exerted by bulk solid materials on the belt at one half of the idler
pitch will dominate the pressure profile. Consequent measurements of the sum of normal forces acting over one idler pitch showed that the peak loading force on the belt occurs at the idler roll junction.

Experimental results showed that during transportation, the percentage of the total normal force exerted by the bulk solid material on the inclined sides of the belt is in the 35-45% range. Thus, the normal force exerted by the bulk solid on the centre section of the belt is 55-65% of the total normal force acting. These proportions were observed to occur across the entire range of internal angles of friction tested. Results of DEM simulations were observed to be within the range of values obtained from experimental tests. An almost constant relationship between the normal force exerted on the inclined side of the belt and the centre of the belt was observed to occur across the range of bulk solid materials modelled. Observations from experimental and DEM simulation results show reasonable correlation to the work of Grabner et al [29], even though that work involved measuring forces due to both the bulk solid material and the belt.

Investigation of simulations with an increase in idler roll troughing angle showed a corresponding increase in the total normal force exerted by the bulk solid material on the belt. The force on the centre section of the belt was observed to increase, however, the normal force exerted on the inclined side of the belt was found to decrease.

Analysis of the sum of forces acting over one idler pitch showed an increase in the total normal force exerted on the belt with a decrease in the simulated angle of internal friction. Importantly, DEM analysis showed that the total difference between the normal forces exerted on the belt during the opening and closing cycle of the troughed shape will increase with an increase in the modelled angle of internal friction. For a constant belt deflection during the trough opening and closing cycles, this indicates that the energy loss attributed to the closing of the troughed shape increases with an increase in the internal angle of friction. This outcome of the investigation was found to be in good agreement with research by Krause and Hettler [64], Spaans [90], Mulani [69], CEMA [6] and Wheeler [106], who state that greater energy loss is attributed to bulk solid materials characterized by larger angles of internal friction. Influence of wall friction variation was found to not be of great influence on the normal force exerted on the belt within the range of values investigated.

Assessment of belt deflection geometry obtained using 3D laser scans showed that during the opening and closing of the troughed cross section, inclined sides of the belt pivot about a point located towards the outer edges. This deflection profile was observed to be contrary to the existing theoretical analysis proposed by Krause and Hettler [64] and applied by Spaans [90], which assumes that the inclined sides pivot around a point located at the idler roll junction.

Investigation showed that at greater sag, the belt cycles through greater amplitude of normal force, while the average normal force was observed to remain relatively unchanged. At identical conveying speed, the displacement and velocity of the belt deflection in the vertical direction is larger when compared to a belt
with lower sag. A greater amount of work resulting in higher energy losses of the system was therefore attributed to a belt with greater sag. This finding is in accordance with the findings of Mustoe and Bin [70], Nordell [72], Behrens [5], Spaans [90], Mulani [69] and Wheeler [106].

Experimental and DEM simulation results showed that the forces exerted on the inclined sides of the belt are in the order of 1.1 to 1.9 times greater compared to gravity acting alone. Investigation of forces located on the centre section of the belt from both experimental measurements and DEM simulations showed the normal force acting on the centre idler roll to be approximately equal to the weight force of the volume of the material located directly above it. These results were found correlate reasonably well with the results observed by Behrens [5]. However, it is important to note, Behrens measured forces due to both the bulk solid material and the belt on idler rolls, while the results herein are presented for the loading due to the bulk solid material on the belt.

Evaluation of the theoretical approach of Spaans [90] showed an over prediction of forces on the inclined sides of the belt and an under prediction on the centre section of the belt. Over prediction of the total normal force exerted by the bulk solid material on the belt was also observed. Similarly, results using the approach of Mulani [69] showed an under prediction of the normal force exerted on the centre section of the belt and the total normal force acting. Both analysis techniques showed that the constituents of the total normal force acting on the inclined side and the centre section of the belt vary significantly with the internal angle of friction. This was observed to be in disagreement with experimental and DEM simulation results. Discrepancies between theoretically calculated results and those obtained using experimental testing and DEM simulations further proves that that the passive stress state is not fully initiated during the conveying of bulk solid materials. Additional analysis based on the assumption that only a fraction of the passive stress state is initiated shows that with the application of a multiplying coefficient, the theory of Spaans and Mulani provides results in closer agreement to experimental testing and DEM simulations.

For easy to moderate to handle bulk solid materials, characterized by internal angles of friction in the range of $30^\circ < \phi_i < 45^\circ$, good correlation was observed by implementing a multiplying coefficient of 0.4. Based on the assumptions made, analysis indicates that the multiplying coefficient will decrease for bulk solid materials with $\phi_i > 45^\circ$. For bulk solid materials characterized by internal angles of friction exceeding $45^\circ$, good correlation was observed using a multiplying coefficient of 0.2. Although good overall agreement to results of experimental testing and DEM simulations was obtained using the multiplying coefficients indicated, further research across a greater number of materials exhibiting variable internal angles of friction is warranted. A review of the application of earth pressure theory to belt conveyor analysis during the closing cycle of the troughed belt shape in parallel with additional 3D laser scanning profiles of fabric and steel cord belts, belts of variable width and idler inclination should also be undertaken.
Analysis of interactions within the transition zone and at discharge showed that the transition geometry of the belt influences the transverse divergence or spread of the discharging bulk solid material stream. A shorter transition length was found to lead to a greater dispersion. Results demonstrated cohesive materials will shear in the transverse direction, ultimately splitting into three separate components, one present in the middle of the belt and two towards the outer edges. For the cohesive material investigated, approximation of the shear plane angles were found to be bound by the active stress state slip plane angle, $\theta_a$, calculated using the theory of Krause and Hettler [64] and the internal angle of friction, $\phi_i$, of the bulk solid material.

For a fixed transition distance, constant throughput and belt speed, free flowing materials (characterized by a low angle of internal friction and angle of repose) were observed to disperse to a greater extent compared to a cohesive material (characterized by a higher angle of internal friction and angle of repose). At the belt speeds and geometries considered, results showed that the contact perimeter between the bulk solid material and the belt will remain constant as the material stream travels from the last idler roll set to discharge. Although site observations were found to be in good agreement with theoretical and simulation findings, results presented highlight the need for further practical validation and research in this area. The analysis should also be extended to slow speed trajectories.

Bulk solid material interactions during transfer and re-direction were investigated in two case study examples from a coal handling facility. The investigation consisted of the continuum analysis approach originally proposed by Roberts [80] conducted in parallel with DEM simulations. A set of bulk solid material parameters was assumed based on typical results obtained from standardised flow property tests. The procedure for the practical implementation was outlined and consisted of following a streamline of material discharging from the delivery conveyor, through the transfer configuration and onto the receiving conveyor belt. By calculating the velocity profile of the material stream through the transfer, the corresponding burden depth at any location through the transfer, as well as the loading, acceleration and wear profile on the receiving conveyor was determined.

The selection of parameters for DEM simulations was performed using a qualitative comparison between flow simulation results and observations from site. As such, a concise investigation of the sensitivity of the simulation parameters was also undertaken. For fast, accelerated flow conditions, results using the continuum analysis approach were found to be in good agreement to DEM simulations and also to observations from site.

Interactions during loading showed that the cross sectional profile of the bulk solid material stream on the outgoing belt is governed by flow through the transfer chute and the interface between the loading chute and receiving belt. When the bulk solid material is loaded onto the receiving belt at a velocity close to the outgoing belt speed, analysis showed the profile of the material stream cross section will be curved as predicted by CEMA [6]. When the bulk solid material is loaded onto the receiving conveyor at a velocity considerably lower to that of the outgoing belt, results demonstrated a much flatter profile, with a
significant proportion of the bulk solid material in contact with the skirting. Further results from DEM analysis showed that when space to re-direct a high volumetric flow rate of the material stream is limited, interaction between the bulk solid material and the receiving conveyor belt will influence the flow through the loading spoon. A reduction in material stream velocity and an increase in burden depth were observed.

Modelling bulk solid material interactions through transfer chutes proved a powerful flow visualization tool and also a thorough analysis tool. Visualization of flow through the use of DEM was used to modify the application of the continuum approach to conditions other than for fast and accelerated flow. Consequently, flow through a transfer configuration where the mode of flow was governed by internal shearing of the bulk solid material stream, rather than failure at the wall boundary, was able to be analysed.

In the existing form, implementation of the continuum analysis approach to slow moving, shear flows and areas of the transfer associated with large impact, severe change in direction and acceleration is approaching its extent of applicability. For these types of flows, a modified approach based on a new set of assumptions needs to be undertaken. In the absence of any quantitative observations available from site, the use of DEM in parallel with continuum analysis has been proven to be a valuable combination.

An in depth investigation of bulk solid interactions in belt conveying systems in the form of novel experimental tests, theoretical calculations and DEM simulations has been presented. This work allows greater insight into the behaviour of bulk solid materials during the loading, acceleration, transportation, discharge, re-direction and transfer of bulk solid materials. The work presented paves the way for an improved understanding of forces, energy losses, flow characteristics, reliability and functionality of belt conveying systems.
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Appendix A – DEM Calibration

5mm - $\mu_p = 0.5$

10mm - $\mu_p = 0.3$

Figure A- 1 - PFC3D Angle of Repose Co-ordinates

($\rho_p = 2590\text{kg/m}^3$, $\mu_r = 0.1$)

5mm

10mm

Figure A- 2 - PFC3D Angle of Repose Co-ordinates

($\rho_p = 2590\text{kg/m}^3$, $\mu_p = 0.5$, $\mu_r = 0.5$)

5mm

10mm

Figure A- 3 - PFC3D Angle of Repose Co-ordinates

($\rho_p = 2590\text{kg/m}^3$, $\mu_p = 0.9$, $\mu_r = 0.9$)
Figure A-4 - PFC3D Slump Test Co-Ordinates

(10mm Spheres - \( \rho_p = 2590 \text{kg/m}^3 \), \( \mu_p = 0.3, \mu_r = 0.1 \))

Figure A-5 - PFC3D Slump Test Co-Ordinates

(10mm Spheres - \( \rho_p = 2590 \text{kg/m}^3 \), \( \mu_p = 0.5, \mu_r = 0.5 \))

Figure A-6 - PFC3D Slump Test Co-Ordinates

(10mm Spheres - \( \rho_p = 2590 \text{kg/m}^3 \), \( \mu_p = 0.9, \mu_r = 0.9 \))
No Adhesion    \( \mu_{adh} = 1, s_{adh} = 0.5\text{mm} \)

Figure A- 7 - Rocky Angle of Repose Co-Ordinates

(5mm Spheres - \( \rho_p = 2590\text{kg/m}^3, \mu_p = 0.3, \mu_r = 0.1 \))

No Adhesion    \( \mu_{adh} = 1, s_{adh} = 0.5\text{mm} \)

Figure A- 8 - Rocky Angle of Repose Co-Ordinates

(5mm Spheres - \( \rho_p = 2590\text{kg/m}^3, \mu_p = 0.5, \mu_r = 0.5 \))

No Adhesion    \( \mu_{adh} = 1, s_{adh} = 0.5\text{mm} \)

Figure A- 9 - Rocky Angle of Repose Co-Ordinates

(5mm Spheres - \( \rho_p = 2590\text{kg/m}^3, \mu_p = 0.9, \mu_r = 0.9 \))
No Adhesion \( \mu_{adh} = 0.6, s_{adh} = 1.0 \text{mm} \)

Figure A-10 - Rocky Angle of Repose Co-Ordinates

(10mm Spheres - \( \rho_p = 2590 \text{kg/m}^3, \mu_p = 0.3, \mu_r = 0.1 \))

No Adhesion \( \mu_{adh} = 0.6, s_{adh} = 1.0 \text{mm} \)

Figure A-11 - Rocky Angle of Repose Co-Ordinates

(10mm Spheres - \( \rho_p = 2590 \text{kg/m}^3, \mu_p = 0.5, \mu_r = 0.5 \))

No Adhesion \( \mu_{adh} = 0.6, s_{adh} = 1.0 \text{mm} \)

Figure A-12 - Rocky Angle of Repose Co-Ordinates

(10mm Spheres - \( \rho_p = 2590 \text{kg/m}^3, \mu_p = 0.9, \mu_r = 0.9 \))
No Adhesion \( \mu_{adh} = 1, s_{adh} = 0.5\text{mm} \)

Figure A-13 - Rocky Slump Test Co-Ordinates
(5mm Spheres - \( \rho_p = 2590\text{kg/m}^3, \mu_p = 0.3, \mu_r = 0.1 \))

No Adhesion \( \mu_{adh} = 1, s_{adh} = 0.5\text{mm} \)

Figure A-14 - Rocky Slump Test Co-Ordinates
(5mm Spheres - \( \rho_p = 2590\text{kg/m}^3, \mu_p = 0.5, \mu_r = 0.5 \))

No Adhesion \( \mu_{adh} = 1, s_{adh} = 0.5\text{mm} \)

Figure A-15 - Rocky Slump Test Co-Ordinates
(5mm Spheres - \( \rho_p = 2590\text{kg/m}^3, \mu_p = 0.9, \mu_r = 0.9 \))
No Adhesion  \( \mu_{adh} = 0.6, s_{adh} = 1.0\text{mm} \)

Figure A-16 - Rocky Slump Test Co-Ordinates

(10mm Spheres - \( \rho_p = 2590\text{kg/m}^3, \mu_p = 0.3, \mu_r = 0.1 \))

No Adhesion  \( \mu_{adh} = 0.6, s_{adh} = 1.0\text{mm} \)

Figure A-17 - Rocky Slump Test Co-Ordinates

(10mm Spheres - \( \rho_p = 2590\text{kg/m}^3, \mu_p = 0.5, \mu_r = 0.5 \))

No Adhesion  \( \mu_{adh} = 0.6, s_{adh} = 1.0\text{mm} \)

Figure A-18 - Rocky Slump Test Co-Ordinates

(10mm Spheres - \( \rho_p = 2590\text{kg/m}^3, \mu_p = 0.9, \mu_r = 0.9 \))
No Adhesion \( \mu_{adh} = 0.5, s_{adh} = 3.0 \text{mm} \)

Figure A-19 - Rocky Slump Test Co-Ordinates

(30mm Spheres - \( \rho_p = 2590 \text{kg/m}^3, \mu_p = 0.3, \mu_r = 0.1 \))

No Adhesion \( \mu_{adh} = 0.5, s_{adh} = 3.0 \text{mm} \)

Figure A-20 - Rocky Slump Test Co-Ordinates

(30mm Spheres - \( \rho_p = 2590 \text{kg/m}^3, \mu_p = 0.5, \mu_r = 0.5 \))

No Adhesion \( \mu_{adh} = 0.5, s_{adh} = 3.0 \text{mm} \)

Figure A-21 - Rocky Slump Test Co-Ordinates

(30mm Spheres - \( \rho_p = 2590 \text{kg/m}^3, \mu_p = 0.9, \mu_r = 0.9 \))
Appendix B – Trajectory Analysis

6.35m Transition

*Free Flowing Material*

![Image of trajectory analysis](image)

Figure B-1 - 6.35m Transition Discharge Trajectory (Front View) - Rocky and Continuum/CAD

(30mm Spheres - $\mu_p = 0.5$, $\mu_r = 0.2$, $\mu_w = 0.7$, $\rho_p = 1350\text{kg/m}^3$)
Figure B-2 - 6.35m Transition Discharge Trajectory (Top View) - Rocky and Continuum/CAD

(30mm Spheres - $\mu_p = 0.5$, $\mu_r = 0.2$, $\mu_w = 0.7$, $\rho_p = 1350\text{kg/m}^3$)
Cohesive Material

Figure B-3 - 6.35m Transition Discharge Trajectory (Front View) - Rocky and Continuum/CAD

(30mm Spheres - \( \mu_p = 0.9 \), \( \mu_r = 0.9 \), \( \mu_m = 0.7 \), \( \mu_{adh} = 0.5 \), \( s_{adh} = 3.0 \text{mm} \), \( \rho_p = 1350 \text{kg/m}^3 \))
Figure B-4 - 6.35m Transition Discharge Trajectory (Top View) - Rocky and Continuum/CAD

(30mm Spheres - $\mu_p = 0.9$, $\mu_r = 0.9$, $\mu_m = 0.7$, $\mu_{adh} = 0.5$, $s_{adh} = 3.0mm$, $\rho_p = 1350kg/m^3$)
3.175m Transition Length

*Free Flowing Material*

![Figure B-5: 3.175m Transition Discharge Trajectory (Front View) - Rocky and Continuum/CAD](30mm Spheres - \(\mu_p = 0.5, \mu_r = 0.2, \mu_w = 0.7, \rho_p = 1350\text{kg/m}^3\))
Figure B- 6 - 3.175m Transition Discharge Trajectory (Top View) – Rocky and Continuum/CAD

(30mm Spheres - $\mu_p = 0.5$, $\mu_r = 0.2$, $\mu_w = 0.7$, $\rho_p = 1350\text{kg/m}^3$)
Figure B- 7 - 3.175m Transition Discharge Trajectory (Front View) - Rocky and Continuum/CAD

(30mm Spheres - $\mu_p = 0.9$, $\mu_r = 0.9$, $\mu_w = 0.7$, $\mu_{adh} = 0.5$, $s_{adh} = 3.0$mm, $\rho_p = 1350$kg/m$^3$)
Figure B- 8 - 3.175m Transition Discharge Trajectory (Top View) – Rocky and Continuum/CAD

(30mm Spheres - $\mu_p = 0.9$, $\mu_r = 0.9$, $\mu_m = 0.7$, $\mu_{adh} = 0.5$, $s_{adh} = 3.0\text{mm}$, $\rho_p = 1350\text{kg/m}^3$)
1.5m Transition

*Free Flowing Material*

Figure B-9 - 1.5m Transition Discharge Trajectory (Front View) – Rocky and Continuum/CAD

(30mm Spheres - $\mu_p = 0.5$, $\mu_r = 0.2$, $\mu_w = 0.7$, $\rho_p = 1350\text{kg/m}^3$)
Figure B-10 - 1.5m Transition Discharge Trajectory (Top View) - Rocky and Continuum/CAD

(30mm Spheres - $\mu_p = 0.5$, $\mu_r = 0.2$, $\mu_w = 0.7$, $\rho_p = 1350\text{kg/m}^3$)
Cohesive Material

Figure B-11 - 1.5m Transition Discharge Trajectory (Front View) – Rocky and Continuum/CAD

(30mm Spheres - $\mu_p = 0.9$, $\mu_r = 0.9$, $\mu_w = 0.7$, $\mu_{adh} = 0.5$, $s_{adh} = 3.0\text{mm}$, $\rho_p = 1350\text{kg/m}^3$)
Figure B-12 - 1.5m Transition Discharge Trajectory (Top View) - Rocky and Continuum/CAD

(30mm Spheres - $\mu_p = 0.9$, $\mu_r = 0.9$, $\mu_w = 0.7$, $\mu_{adh} = 0.5$, $s_{adh} = 0.5$mm, $\rho_p = 1350$kg/m$^3$)
Comparison of Co-ordinates

6.35m transition

Figure B- 13 - 6.35m Transition - free flowing material (Front View)
(30mm Spheres - $\mu_p = 0.5$, $\mu_s = 0.2$, $\mu_w = 0.7$, $\rho_p = 1350 \text{kg/m}^3$)

Figure B- 14 - 6.35m Transition - cohesive material (Front View)
(30mm Spheres - $\mu_p = 0.9$, $\mu_s = 0.9$, $\mu_w = 0.7$, $\mu_{adh} = 0.5$, $s_{adh} = 3.0 \text{mm}$, $\rho_p = 1350 \text{kg/m}^3$)
Figure B-15 - 6.35m Transition - free flowing material (Top View)

(30mm Spheres - $\mu_p = 0.5$, $\mu_r = 0.2$, $\mu_w = 0.7$, $\rho_p = 1350\text{kg/m}^3$)

Figure B-16 - 6.35m Transition - cohesive flowing material (Top View)

(30mm Spheres - $\mu_p = 0.9$, $\mu_r = 0.9$, $\mu_w = 0.7$, $\mu_{adh} = 0.5$, $s_{adh} = 3.0\text{mm}$, $\rho_p = 1350\text{kg/m}^3$)
3.175m transition

Figure B-17 - 3.175m Transition - free flowing material (Front View)

(30mm Spheres - $\mu_p = 0.5$, $\mu_r = 0.2$, $\mu_w = 0.7$, $\rho_p = 1350\text{kg/m}^3$)

Figure B-18 - 3.175m Transition - cohesive material (Front View)

(30mm Spheres - $\mu_p = 0.9$, $\mu_r = 0.9$, $\mu_w = 0.7$, $\mu_{adh} = 0.5$, $s_{adh} = 3.0\text{mm}$, $\rho_p = 1350\text{kg/m}^3$)
Figure B-19 - 3.175m Transition - free flowing material (Top View)

(30mm Spheres - \( \mu_p = 0.5, \mu_r = 0.2, \mu_w = 0.7, \rho_p = 1350\text{kg/m}^3 \))

Figure B-20 - 3.175m Transition - cohesive flowing material (Top View)

(30mm Spheres - \( \mu_p = 0.9, \mu_r = 0.9, \mu_w = 0.7, \mu_{ad} = 0.5, s_{adh} = 3.0\text{mm}, \rho_p = 1350\text{kg/m}^3 \))
1.5m transition

Figure B-21 - 1.5m Transition - free flowing material (Front View)

(30mm Spheres - $\mu_p = 0.5$, $\mu_r = 0.2$, $\mu_w = 0.7$, $\rho_p = 1350\text{kg/m}^3$)

Figure B-22 - 1.5m Transition - cohesive material (Front View)

(30mm Spheres - $\mu_p = 0.9$, $\mu_r = 0.9$, $\mu_w = 0.7$, $\mu_{adh} = 0.5$, $s_{adh} = 3.0\text{mm}$, $\rho_p = 1350\text{kg/m}^3$)
Figure B-23 - 1.5m Transition - free flowing material (Top View)

(30mm Spheres - $\mu_p = 0.5$, $\mu_r = 0.2$, $\mu_a = 0.7$, $\rho_p = 1350$kg/m$^3$)

Figure B-24 - 1.5m Transition - cohesive material (Top View)

(30mm Spheres - $\mu_p = 0.9$, $\mu_r = 0.9$, $\mu_a = 0.7$, $\mu_{adh} = 0.5$, $s_{adh} = 3.0$mm, $\rho_p = 1350$kg/m$^3$)