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A continuum mechanics derivation of the empirical expression relating slug and particle velocities

O. Orozovic[a,*], A. Lavrinec[a], F. Georgiou[b], C.M. Wensrich[c]

[a] *Centre for Bulk Solids and Particulate Technologies, The University of Newcastle, NSW 2308, Australia*
[b] *School of Mathematical and Physical Sciences, The University of Newcastle, NSW 2308, Australia*
[c] *School of Engineering, The University of Newcastle, NSW 2308, Australia*

Abstract

The experimental discovery that slugs are in fact fluidised and not compact structures at the bulk density of the material was a significant result in the context of its discovery and on the physical mechanisms responsible for slug flow. However, these findings were never translated into a theoretical basis to explain why slugs are fluidised and why the long standing assumption of a compact structure was no longer valid. Recently, the authors inferred that the slope of the empirical linear relationship between the slug and particle velocity was the density ratio between the slug and the material bulk density. This paper provides a continuum mechanics derivation of this velocity relationship, proving that the slope is the density ratio. Further analysis and discussion are provided which imply that the fluidised state of a slug is a requirement for slug flow and characterises its range of operating conditions.

Keywords: pneumatic conveying, dense phase, slug flow, plug flow, slug velocity

1. Introduction

It was the experimental work of Niederreiter [1], who considered vertical conveying, and Lecreps [2], who considered horizontal conveying, that first explicitly demonstrated that a slug was not a compact structure at the material bulk density. Physically, it could be argued that the significance of these results was not profound, as their slugs of plastic pellets were found to only be slightly fluidised above the bulk state, where the slug porosity was above that of the bulk. Theoretically, however, the results were profound as they questioned the long standing assumption that the pressure drop within a pneumatic conveyor was due to the inter-particle transfer of axial stress into a radial stress and consequently, a shear stress at the wall, as firstly considered by Konrad [3]. This assumption was not supported by any experimental findings [4];

*Corresponding author. Tel: +61 2 40339026
Email address: ognjen.orozovic@newcastle.edu.au (O. Orozovic)

however, it inspired a host of similar approaches to be pursued, the aim of which was to predict a pressure drop as a function of these slug stresses [5–11].

Most theoretical models were based on the assumption of a compact slug structure and most experimental investigations were geared towards the measurement of the stress states for the pressure drop prediction. The difficulty of reliably measuring these stresses [9] resulted in questionable data being used as an input in pressure prediction or in the development of models for the stresses, which would themselves be used as inputs for pressure drop prediction. This meant that the pressure drop prediction was a function of several sub-models. Most of these sub-models had no experimental support and proved to be difficult to validate or trace errors in, as the only reliable measure was the pressure drop, which was the final output of the models. With time, the assumptions within the sub-models that formed the foundations of the pressure drop model were being questioned, such as the case of Yi [8] showing that the assumptions in the axial stress prediction of Mi and Wypych [12] were incorrect.

Even with these difficulties within the Konrad inspired approaches, the findings of Niederreiter [1] and Lecreps [2] did not significantly alter the direction of the field as much as may have been expected. Due to the various sub-model approaches and their high errors as inputs, the Konrad inspired approaches had a high variance in their output [13]. This expectedly resulted in cases where the models did provide good agreement with measurements and results such as this may have played a role in the reluctance to objectively question the Konrad inspired approaches. Furthermore, the results of Niederreiter [1] and Lecreps [2] were limited to only plastic pellets and required further investigation.

The direction change in modelling within the field that came as a consequence of improved computational power ended up providing a different modelling direction and findings such as those of Niederreiter [1] and Lecreps [2], which reflect the still poorly understood mechanisms of slug flow, were neglected theoretically. Computational studies that examined the slug porosity [14, 15] too found the same slightly fluidised structure found by Niederreiter [1] and Lecreps [2]. Further experimental studies conducted by Vasquez [16] and Nied [17, 18] either deduced or directly showed the same higher slug porosity; however, these too were conducted only on different types of plastic pellets.

As it is hopefully now clear, the significance of the results of Niederreiter [1] and Lecreps [2] were based on the context of the field at the time of their discovery. The modelling change that occurred with the ever increasing computational studies meant that the significance of these findings was lost and limited follow-up experimental investigations were conducted. Furthermore, almost no theoretical work, aside from analogies to porous media [19] were considered for this observation.

This short communication provides a derivation relating the slug and particle velocities with slug to bulk density ratio using a continuum mechanics framework. A linear relationship is obtained between the slug and particle velocity, with the slope of the relationship being shown to be the slug to bulk density ratio. The result closely matches the empirical formula in the literature, for which the slope has consistently been measured to be less than one[18, 20, 21]. The direct significance of this result is that if the slug to bulk density ratio is less than one, there is a limited range of solids velocities for which slug flow can occur [20, 22], which is a well known observation within the field [23–25]. The provided derivation further cements the significance of the slug density and its potential in providing insights into the why and how slug flow occurs.

2. Slug and particle velocity relationship

2.1. Summary of related empirical and theoretical work

The assumption of a slug having a compact structure at the material bulk density resulted in a theoretical linear relationship with a 'slope' or gradient of one [18, 26, 27]. The 'intercept', labeled as c, of the linear relationship was considered by Konrad through directly applying the gas-liquid propagation velocity to slug flow [26] and was given as $c = 0.542\sqrt{gD}$. Empirically however, the relationship has always been measured to have a slope less than one [18, 20, 21]. Orozovic et al. [20, 22] considered the significance of a slope less than one and showed that this condition provides theoretical boundaries for the maximum particle velocity and the minimum stationary layer depth for slug flow to occur. Furthermore, the authors inferred that that the slope of the relationship was the density ratio of the slug to the material bulk density, resulting from the density difference between the slug and the stationary layers. This inference is of importance to confirm whether the density ratio is the parameter of interest which reflects the narrow bounds in material properties and operational conditions in which slug flow operates.

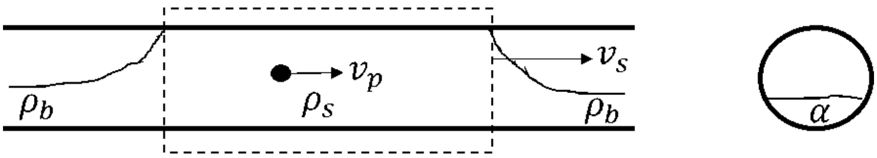

Figure 1: Densities and velocities of a slug and its stationary layers.

Figure 1 shows the densities and velocities considered by Orozovic et al. [20, 22] in the derivation of the slug length conservation of mass. As it can be seen, the slug velocity v_s corresponds to the velocity of the slug front, which is at the slug density. The slug density is below the density of the stationary layers, which are at the bulk density. The rate of change in the slug length and the propagation velocity c occur at the slug density and the material picked up, corresponding to the product of the slug velocity and layer fraction, is at the bulk density. The rate of change in slug length can therefore be shown to follow the form given by Equation 1 [28].

$$\frac{\mathrm{d}L_s}{\mathrm{d}t} = \frac{\rho_b}{\rho_s} v_s \alpha - c = \frac{v_s}{E}\alpha - c \tag{1}$$

where L_s is the slug length, ρ_b the bulk density, ρ_s the slug density, v_s the velocity of the slug front, α the layer fraction ahead of the slug (cross-sectional area of the layer divided by the cross-sectional area of the pipe), E the slug to bulk density ratio and c is the propagation velocity which will later be shown to be analogous to the speed of sound in compressible flow.

The form of the conservation of mass for the slug length given in Equation 1 shows that the density ratio E effectively increases the slug velocity through an increase in the slug length. Equation 1 and this observation will be utilised in the derivation of the velocity relationship provided in the following subsection.

It is worth noting that there was an error in the conservation of mass of Equation 1 presented in Orozovic et al. [20, 22], where the rate of change of slug length was considered to occur at the particle density. However, this error did not affect any of the results presented in those papers as a steady-state in slug length was always considered. Additionally, the empirical form of the slug and particle velocity relationship that was obtained by Orozovic et. al. [20, 22] is given by:

$$v_s = E v_p + c \tag{2}$$

2.2. A continuum mechanics derivation

The form of the velocity equation that we are deriving is commonly obtained from systems of partial differential equations (PDEs) modelling phenomena as diverse as roll waves [29] and traffic flow [30]. In fact, the velocity equation is a consequence of hyperbolic conservation laws under generic forcing terms [30] and will be considered as such within this paper. For the derivation, 1-D Euler like equations with additional forcing terms will be considered. Euler equations are inviscid, an assumption that has previously been successfully applied to slug flow [26] and is appropriate for the core of the slug due to all particles travelling at the same velocity [11, 16, 31–34]. However, the PDEs which will be examined here are general and do not necessarily assume inviscid flow since viscosity could be considered within the generic forcing terms, which will be further discussed later in the paper. For now, we consider the mass and momentum equations given in Equation 3 and Equation 4, respectively.

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}\left(\rho v_p\right) = 0 \tag{3}$$

where ρ is the density of the flow and v_p is the flow (particle) velocity.

$$\frac{\partial v_p}{\partial t} + v_p \frac{\partial v_p}{\partial x} + \frac{1}{\rho}\frac{\partial P}{\partial x} = f \tag{4}$$

where P is a pressure related to the granular phase and f represents the forcing term(s).

Equation 3 and Equation 4 capture the flow density and velocity at all positions and times, but the density difference between the slug and stationary layers is not captured. However, as was shown through the conservation of mass for a slug in Equation 1, the change in density is directly reflected by the elongation of the slug and a consequent increase in its velocity. This elongation is not perceived by the particles within the slug, which are reflected by the particle velocity v_p and the propagation velocity c. Therefore, based on the results of Equation 1, a characteristic variable of the following form is considered:

$$X = x - \frac{v_s}{E}t \tag{5}$$

where X is the characteristic variable, x the position of a slug, v_s the slug velocity and t the time.

The partial derivatives of Equation 5 with respect to x and t are $\frac{\partial X}{\partial x} = 1$ and $\frac{\partial X}{\partial t} = -\frac{v_s}{E}$. Using these partial derivatives (further details on the derivation are provided in Appendix A) Equation 3 can be shown to be:

$$\frac{\mathrm{d}}{\mathrm{d}X}\left(\rho\left(Ev_p - v_s\right)\right) = 0 \tag{6}$$

Substituting Equation 6 into Equation 4 and simplifying gives the following ordinary differential equation:

$$\frac{\mathrm{d}v_p}{\mathrm{d}X} = \frac{Ef\left(Ev_p - v_s\right)}{\left(Ev_p - v_s\right)^2 - E^2c^2} \tag{7}$$

where $c^2 = \frac{\mathrm{d}P}{\mathrm{d}\rho}$ and is analogous to the speed of sound in compressible flow.

The denominator in Equation 7 equalling zero corresponds to the flow velocity relative to the shock equalling the speed of propagation, also known as a 'sonic' point. However, for the derivation to be defined at this point the numerator must also be equal to zero. Thomas [35] showed that this simultaneous condition of zero in both the numerator and denominator must exist. Thus, barring the trivial solution, it is not possible to simultaneously satisfy the condition $Ef\left(Ev_p - v_s\right) = 0$ and $\left(Ev_p - v_s\right)^2 - E^2c^2 = 0$ unless the forcing term, f, equals zero. We can then solve the denominator to find $\left(Ev_p - v_s\right)^2 = E^2c^2$. Then, considering only the negative propagation velocity, due to the negative propagation given by our coordinate system (Equation 5), we find the slug velocity to be:

$$v_s = E\left(v_p + c\right) \tag{8}$$

As it can be seen, this is precisely the empirical velocity relationship, showing that the slope of the slug to particle velocity relationship is the slug to bulk density ratio. It is worth noting that the same equation can be obtained in two-phase flow considering gas and solid phases [36], as the negligible density terms of the gas phase effectively result in a single phase and Equation 8.

3. Discussion

Comparing the derived form of Equation 8 to the empirical form of Equation 2 proposed by Orozovic et. al. [20, 22], it can be seen that the proposed empirical form misrepresented the propagation velocity c by not considering the multiplication with E. Furthermore, the physical origin of the propagation velocity is provided as $c = \sqrt{\frac{\mathrm{d}P}{\mathrm{d}\rho}}$, where the bulk material pressure term requires further investigation. Dividing both sides of Equation 8 by E best demonstrates that the difference in density between the slugs and its stationary layers, which is represented by E, only elongates the slug and consequently increases its slug velocity. The particle and propagation velocity, which are features observed only where slugs are located in the domain/pipeline, are unaffected by this density difference and so are the flow equations given by Equation 3 and Equation 4.

The fact that slugs obey Equation 8 signifies that slugs occur at the critical, or 'sonic' point. Furthermore, this can only occur when the forcing term(s) f are zero – conditions that for slug flow likely correspond to a balance of the driving drag and retarding particle-wall forces. This is an insightful result itself on the conditions of slug flow and was possible to achieve even without specific expressions for the granular pressure and generic forcing terms. However, specific expressions of these terms are of interest to examine the range of validity of the derived expression as this is how particle properties, system parameters and operating conditions would be introduced into the models. More broadly, these expressions could be applied to study the conditions required for the different modes of flow and also their transitions.

The maximum theoretical particle velocity was considered by Orozovic et al. [20, 22] by evaluating

Equation 2 at $v_s = v_p$. For the derived case of Equation 8, the maximum theoretical particle velocity $v_{p_{max}}$ is given by:

$$v_{p_{max}} = \frac{Ec}{1-E} \tag{9}$$

The operating conditions of slug flow are known to be bounded under a certain particle velocity and as it can be seen from Equation 9, the maximum theoretical particle velocity is undefined for $E = 1$. The derivation of the slug to particle velocity relationship of Equation 8 suggested slugs occur at the sonic point and the maximum theoretical particle velocity given by Equation 9 implies that a slug density below that of the bulk is also a requirement for slug flow. Significantly, this work demonstrates the ability of continuum models to naturally capture nuanced features of slug flow and provides a novel avenue for further modelling.

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Appendix A Derivation details

Consider the characteristic variable $X = x - \frac{v_s}{E}t$ and the transformed x derivative ($\frac{\partial}{\partial x} = \frac{\partial}{\partial X}$) and t derivative ($\frac{\partial}{\partial t} = -\frac{v_s}{E}\frac{\partial}{\partial X}$). Then, applying to the conservation of mass of Equation 3 gives:

$$-\frac{v_s}{E}\frac{\mathrm{d}\rho}{\mathrm{d}X} + v_p\frac{\mathrm{d}\rho}{\mathrm{d}X} + \rho\frac{\mathrm{d}v_p}{\mathrm{d}X} = 0 \tag{10}$$

Collecting like terms and making the density the subject gives:

$$\rho = -\frac{(Ev_p - v_s)}{E}\frac{\mathrm{d}\rho}{\mathrm{d}v_p} \tag{11}$$

The conservation of momentum given in Equation 4, transforms to the following when applying the characteristic variable:

$$-\frac{v_s}{E}\frac{\mathrm{d}v_p}{\mathrm{d}X} + v_p\frac{\mathrm{d}v_p}{\mathrm{d}X} + \frac{1}{\rho}\frac{\mathrm{d}P}{\mathrm{d}X} = f \tag{12}$$

Multiplying through by the E in the denominator of the first term of Equation 12 and substituting Equation 11 into Equation 12 gives:

$$(Ev_p - v_s)\frac{\mathrm{d}v_p}{\mathrm{d}X} - \frac{E^2}{(Ev_p - v_s)}\frac{\mathrm{d}P}{\mathrm{d}\rho}\frac{\mathrm{d}v_p}{\mathrm{d}X} = Ef \tag{13}$$

Let:

$$c^2 = \frac{\mathrm{d}P}{\mathrm{d}\rho} \tag{14}$$

Multiplying through by $(Ev_p - v_s)$, substituting Equation 14 into Equation 13 and collecting the flow (particle velocity) terms together gives:

$$\left((Ev_p - v_s)^2 - E^2c^2\right)\frac{\mathrm{d}v_p}{\mathrm{d}X} = Ef\,(Ev_p - v_s) \tag{15}$$

Making the velocity derivative the subject of Equation 15 gives Equation 7.

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