BEARING CAPACITY OF SURFACE STRIP FOOTINGS ON LAYERED SOILS

By

Seyednima Salimi Eshkevari

MSc. Geotechnical Eng.

A Thesis submitted in fulfilment of the requirements

for the degree of

Doctor of Philosophy

School of Engineering

The University of Newcastle

August 2018
I hereby certify that the work embodied in the thesis is my own work, conducted under normal supervision.

The thesis contains published scholarly work of which I am a co-author. For each such work a written statement, endorsed by the other authors, attesting to my contribution to the joint work has been included.

The thesis contains no material which has been accepted, or is being examined, for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. I give consent to the final version of my thesis being made available worldwide when deposited in the University’s Digital Repository, subject to the provisions of the Copyright Act 1968 and any approved embargo.
“I hereby certify that this thesis is submitted in the form of a series of published papers of which I am a joint author. I have included as part of the thesis a written statement from each co-author; and endorsed by the Faculty Assistant Dean (Research Training), attesting to my contribution to the joint publications.”

(Signed) ____________________
ACKNOWLEDGMENTS

The author gratefully acknowledges the financial assistance, provided through the receipt of an Australian Postgraduate Award during his candidature. Furthermore, he is very thankful for the additional financial assistance, provided by the ARC Centre of Excellence for Geotechnical Science and Engineering at the University of Newcastle.

The author is deeply indebted to his supervisors, Associate Professor Andrew Abbo, and George Kouretzis for their valuable guidance, critical suggestions, and limitless help throughout the period of his candidature. Their commitment and assistance are greatly appreciated.

The author would also like to thank Associate Professor Andrei Lyamin for his assistance, collaboration, advice and encouragement. His support was highly appreciated.

Finally, thank you to my parents, Marziyeh and Hossain, and my dear wife Negar who provided me with the support and encouragement to persevere with my studies.
# CONTENT

ACKNOWLEDGMENTS .......................................................................................... iii

CONTENT ................................................................................................................ iv

ABSTRACT .............................................................................................................. vi

NOTATION .............................................................................................................. ix

PREFACE .................................................................................................................. xi

STATEMENT OF CONTRIBUTION 1 ...................................................................... xii

STATEMENT OF CONTRIBUTION 2 ...................................................................... xiv

STATEMENT OF CONTRIBUTION 3 ...................................................................... xvi

CHAPTER 1- INTRODUCTION......................................................................................

1-1 Introduction .................................................................................................. 1

1-2 Research objectives .................................................................................... 3

1-3 Methodology ................................................................................................. 3

1-4 Thesis structure ............................................................................................. 4

CHAPTER 2- LITERATURE REVIEW ......................................................................

2-1 Introduction .................................................................................................. 6

2-2 Bearing capacity of shallow foundations ON homogeneous soils ............... 6

2-3 Bearing capacity of shallow foundations ON dense sand over soft clay ......... 11

2-3-1 General ....................................................................................................... 11

2-3-2 Projected area model ................................................................................ 11

2-3-3 Punching shear model ............................................................................... 13

2-3-4 Okamura et al. Model ............................................................................... 14

2-3-5 Numerical and analytical Solutions ............................................................ 16

2-3-6 Experimental studies ............................................................................... 17

2-4 Bearing capacity of shallow foundations ON dense sand over loose sand ... 17

2-4-1 General ....................................................................................................... 17

2-4-2 Punching shear model ............................................................................... 18

2-4-3 Experimental studies ............................................................................... 20
CONTENTS

2-5 Existing design guidelines ................................................................. 20
2-6 FINITE ELEMENT LIMIT ANALYSIS method ........................................ 21
2-6-1 general ......................................................................................... 21
2-6-2 limitations ..................................................................................... 22
2-6-3 Assumption of associated flow rule ................................................ 22
2-6-4 Soil anisotropy .............................................................................. 23
2-6-5 Non-CONSTANT friction angle ....................................................... 23
2-7 Concluding remarks .......................................................................... 24
CHAPTER 3- THESIS OUTLINE .................................................................. 23
3-1 Introduction ....................................................................................... 24
3-1-1 Chapter 4 ....................................................................................... 24
3-1-2 Chapter 5 ....................................................................................... 24
3-1-3 Chapter 6 ....................................................................................... 25
3-1-4 Chapter 7 ....................................................................................... 26
3-1-5 Chapter 8 ....................................................................................... 26
CHAPTER 4- PUNCHING SHEAR COEFFICIENTS FOR THE DESIGN OF WORKING PLATFORMS ........................................................................ 28
CHAPTER 5- GRAPHICAL INTERPRETATION OF UPPER BOUND FINITE ELEMENT LIMIT ANALYSIS RESULTS .................................................... 35
CHAPTER 6- BEARING CAPACITY OF STRIP FOOTINGS ON SAND OVER CLAY ................................................................. 44
response to reviewer 1 ........................................................................... 78
response to reviewer 2 ........................................................................... 79
CHAPTER 7- BEARING CAPACITY OF STRIP FOOTINGS ON CLAYS WITH INCREASING SHEAR STRENGTH ......................................................... 81
CHAPTER 8- BEARING CAPACITY OF STRIP FOOTINGS ON A STRONG SAND OVERLAYING WEAK SAND .......................................................... 88
CHAPTER 9 CONCLUSIONS ..................................................................... 111
9-1 general ............................................................................................ 112
9-2 MAIN findings ................................................................................ 112
9-2-1 Identified failure mechanisms ....................................................... 112
## CONTENTS

9-2-2 Bearing capacity models based on the observed failure mechanisms.............................. 113

9-2-3 Model verification........................................................................................................... 114

9-3 Further research.............................................................................................................. 114

CHAPTER 10 REFERENCES .................................................................................................. 116

APPENDIX A- UPPER AND LOWER BOUND FELA SOLUTIONS FOR STRIP FOOTINGS ON DENSE SAND OVER CLAY ........................................................................... 125

PREFACE........................................................................................................................... 126

APPENDIX B- UPPER AND LOWER BOUND FELA SOLUTIONS FOR STRIP FOOTINGS ON DENSE SAND OVER CLAY ........................................................................... 130

PREFACE........................................................................................................................... 131

APPENDIX C- DETAILED RESPONSE TO REVIEWER’S COMMENTS ................................. 138

PREFACE........................................................................................................................... 139

Effect of non-associativity on footing collapse load .............................................................. 139

Dependency of inclination of slip surfaces on dilation angle............................................... 140
ABSTRACT

Estimating the bearing capacity of shallow foundations on layered soils is frequently required both in off-shore and onshore geotechnical engineering applications. The conventional bearing capacity theory, developed for homogeneous soils is not valid for such layered systems. Granular working platforms for tracked plant is a good example of footings on layered soils with relatively thin top layer. This problem is currently treated in practice, using a simplified method, presented in the Building Research Establishment publication BRE470-2004. Available bearing capacity solutions are mainly based on empirical models, interpreted from experimental test results that cover limited range of material parameters. While it is generally accepted that such models may be applicable to soil properties and footing geometries outside the range tested experimentally, they offer limited insights on how the assumed failure mechanism affects their range of application.

This research considers the problem of estimating the ultimate bearing capacity of rigid strip footings on layered soil profiles, comprising a finite thickness of dense sand, overlaying deep layer of different weak soils i.e. very soft to stiff clay and very loose to medium dense sand. Rigid strip footings resting on the surface of the top layer were selected for analyses, as they resemble tracked plant. The ultimate bearing capacity of the footings were estimated from the average of rigorous upper and lower limit loads on the footing, calculated using the FELA method, developed in University of Newcastle. Adaptive remeshing was employed to obtain upper and lower bounds that closely bracket the true collapse loads.

The geometry of the collapse mechanisms was interpreted from upper bound FELA analyses to gain insight into contribution of layered soils parameters in the shape of the mechanism. Detailed investigations indicated that the assumed punching shear failure mechanism of Meyerhof (1974) cannot accurately describe the geometry of the shear planes for all studied material parameters. For layered soils with cohesive subgrades, it was found that the direction of failure planes beneath the footing drift from the assumed vertical in Meyerhof’s model, as strength of the bottom layer increases relative to the top layer. As such, a more detailed investigation was undertaken that resulted in a new bearing capacity model that considers variable geometry of the failure mechanism as a function of the key parameters of the problem.

The model provides results that are in close agreement with published experimental studies, and allows treating simple problems, such as the design of working platforms, without having to resort to numerical simulations. The model is extended to clays with increasing shear strength with depth, using expressions derived for bearing capacity factor as a simple function of the dimensionless rate of increase of soil strength.
The same methodology was followed to develop a new bearing capacity model for strip footings on strong sand overlying relatively weaker sand. The inspiration behind this model was the observation that the mechanism, assumed in the classical Hanna (1981) solution, only provides acceptable estimation of the collapse loads for limited range of material parameters, where the top sand is significantly stronger than the bottom layer. A new form of failure mechanism (transitional failure mechanism) was observed via FELA analyses where the relative strength of the top layer decreases, for which Hanna’s bearing capacity model (1981) overestimates the ultimate bearing capacity of the footing. A method was developed to predict the dominant mode of failure based on the key parameters of the problem, and a new bearing capacity factor was proposed for predicting the ultimate bearing capacity of the footing, where transitional failure mechanism applies. The new method was benchmarked against published experimental data and was shown to provide accurate estimates of the bearing capacity for a wide range of the problem parameters.

The use of FELA results in this research led to the refinement of the existing bearing capacity models for footings on two layered soils. The main novelty of these models is the consideration of the failure mechanisms of variable geometry, depending on the parameters of the problem, which allows capturing the failure mode for a wide range of problem parameters. The proposed models comprise simple formulas and design charts, hence can be efficiently used in practice for the quick design of working platforms for tracked plant without having to resort to numerical analyses.
NOTATION

All variables are defined as they have been introduced into the text. However, for convenience, a list of the most commonly used variables and their definitions are presented below.

\[\begin{align*}
\alpha & \quad \text{Angle of shear plane} \\
B & \quad \text{Footing width} \\
B' & \quad \text{Effective footing width} \\
c_0 & \quad \text{Undrained shear strength of clay on top of the layer} \\
c_u & \quad \text{Undrained shear strength} \\
\delta & \quad \text{Mobilised friction angle on the assumed failure planes} \\
\phi & \quad \text{Soil internal friction angle} \\
\gamma & \quad \text{Soil unit weight} \\
H & \quad \text{Thickness of the top sand layer} \\
H_{cr} & \quad \text{Critical depth} \\
K_p & \quad \text{Coefficient of passive lateral earth pressure} \\
K_s & \quad \text{Punching shear coefficient} \\
K_{sr} & \quad \text{Coefficient of shearing resistance} \\
N_c & \quad \text{Bearing capacity factor of Cohesion} \\
N_y & \quad \text{Bearing capacity factor of Soil self-weight} \\
N_{y'} & \quad \text{Bearing capacity factor of layered sand} \\
N_q & \quad \text{Bearing capacity factor of Surcharge} \\
\theta & \quad \text{Angle of failure plane of the top layer to vertical direction} \\
q_c & \quad \text{Ultimate bearing capacity of the uniform clay layer} \\
Q_L & \quad \text{Lower bound ultimate bearing capacity} \\
q_s & \quad \text{Ultimate bearing capacity of the uniform sand layer} \\
Q_U & \quad \text{Upper bound ultimate bearing capacity} \\
q_u & \quad \text{Ultimate bearing capacity of shallow foundation} \\
\rho & \quad \text{Rate of increase of undrained shear strength with depth}
\end{align*}\]
**NOTATION**

- $P_0$  
  Surcharge at the embedment depth

- $D_s$  
  Rate of shear power dissipation

- $P_p$  
  Passive thrust

- $q_t$  
  Ultimate bearing capacity of the top soil layer

- $q_b$  
  Ultimate bearing capacity of the bottom soil layer

- $q_1, q_2$  
  Bearing capacity of the upper and lower layers respectively.

- $\gamma_1, \gamma_2$  
  Unit weight of the upper and lower layers respectively.
The research work, presented in this thesis was conducted at the University of Newcastle in the School of Engineering during the period February 2012 through August 2018. This work was performed under the supervision of Associate Professor Andrew Abbo and George Kouretzis with the assistance of Jinsong Huang. This thesis is presented in the form of a thesis by publications, based on the five technical papers, listed below.


Salimi Eshkevari, S., Abbo, A.J. Kouretzis, G. Bearing capacity of strip footings on layered sand; Submitted and under review to be published by ASCE International Journal of Geomechanics.
STATEMENT OF CONTRIBUTION 1

Technical papers


Authors

Salimi Eshkevari S. (Candidate)

Performed numerical simulations analysed, interpreted and compiled the results. Primary author of the four manuscripts listed above.

I hereby certify that this statement of contribution is accurate.

Signed .................................................. Date. 6/8/18

Abbo A.J. (Co-supervisor)

Supervised the research, provided guidance and direction of the analysis and the interpretation of the results. Assisted in the preparation of the manuscripts and their editing.

I hereby certify that this statement of contribution is accurate.

Signed ............ Date. 6/8/18
Assistant Dean for Research Training

I hereby certify that this statement of contribution is accurate.

Name: "Grainger" Webster

Signed: ___________________________ Date: 4/23/2018
STATEMENT OF CONTRIBUTION 2

Technical papers


Salimi Eshkevari, S., Abbo, A.J., Kouretzis, G. Bearing capacity of strip footings on layered sand; Submitted to Computers and Geomechanics.

Authors

Salimi Eshkevari, S. (Candidate)

Performed numerical simulations analysed, interpreted and compiled the results. Assisted in the preparation of the manuscript and its editing.

I hereby certify that this statement of contribution is accurate.

Signed………………………………………………………Date………….

Abbo A.J. (Co-supervisor)

Supervised the research, provided guidance and direction of the analysis and the interpretation of the results. Assisted in the preparation of the manuscripts and their editing.

I hereby certify that this statement of contribution is accurate.

Signed……………………………………………………………………………..Date………….

Kouretzis, G. (Supervisor)

Supervised the research, provided guidance and direction of the analysis and the interpretation of the results. Assisted in the preparation of the manuscript and its editing.

I hereby certify that this statement of contribution is accurate.

Signed……………………………………………………………………………..Date………….
STATEMENTS OF CONTRIBUTION

Assistant Dean for Research Training

I hereby certify that this statement of contribution is accurate.

Name: A. Grant Weber

Signed: ...........................................................................................................

Date: 14/08/2018
STATEMENT OF CONTRIBUTION 3

Technical paper


Authors

Salimi Eshkevari, S. (Candidate)

Performed numerical simulations analysed, interpreted and compiled the results. Assisted in the preparation of the manuscript and its editing.

I hereby certify that this statement of contribution is accurate.

Signed ............................................... Date 06/08/18

Wilson, D.W.

Performed numerical simulations analysed, interpreted and compiled the results. Primary author of the manuscript listed above.

I hereby certify that this statement of contribution is accurate.

Signed ............................................... Date 06/08/18

Abbo A.J. (Supervisor)

Supervised the research, provided guidance and direction of the analysis and the interpretation of the results. Assisted in the preparation of the manuscript and its editing.

I hereby certify that this statement of contribution is accurate.

Signed ............................................... Date 06/08/18
Sloan S.W.
Provided technical expertise on the computational limit analysis. Assisted in the preparation of the manuscript and its editing.

Assistant Dean for Research Training
I hereby certify that this statement of contribution is accurate.

Signed............................................................Date 4/8/2018.
CHAPTER 1- INTRODUCTION
1-1 INTRODUCTION

Bearing capacity of shallow foundations is a classical stability problem in geotechnical engineering which is frequently encountered in practice. Terzaghi (1943) was the pioneer to extend the plasticity theory of Prandtl to foundation engineering and developed the conventional bearing capacity theory of shallow foundations for strip footings, for which the assumption of plane strain conditions is reasonable. However, Terzaghi’s bearing capacity theory assumes that soil is homogeneous, which doesn’t apply to many practical situations where footings are resting on layered subsoil profiles. As such, alternative solutions must be sought for footings on layered soils.

The term “layered soils” in general, covers a variety of subsoil conditions. In this research, “layered soil” refers to a two-layer soil system, comprising a relatively thin and strong top granular layer, overlying deep weak cohesive or loose granular soil layers. This is commonly encountered in practice, since replacing unsuitable shallow soils by materials with superior engineering properties (engineered fill) or constructing granular working platforms on top of weak subgrades is an efficient soil improvement solution. Engineered fills are often made of granular materials like sand and gravel mixtures or crushed rocks that behave as drained under any loading condition. As such, estimating bearing capacity of layered soils, comprising a relatively thin layer of dense granular material over weak cohesive or loose frictional soils is of high practical importance. An example of footings on layered soils is encountered in offshore engineering applications, where jack-up foundations are commonly installed on a sand layer overlying soft marine clay deposits. Guidelines for the Site-Specific Assessment of Mobile Jack-up Units (SNAME, 2002) include a simple method for estimating the ultimate bearing capacity of jack-up foundations on layered soils. Temporary working platforms for tracked plant is another typical example of layered soils. In 2004, the Building Research Establishment BRE published a “good practice” guide (BRE-470) which includes specifications relevant to the design, construction and maintenance of ground-supported working platforms.

Soft soil sites are often incapable of supporting heavy construction equipment. Temporary working platforms are required to provide a safe working environment. A typical working platform is shown in Figure 1-1 which consists of a layer of well compacted granular material, over clayey subgrade. Rigid rectangular tracks of piling rigs or crawler cranes transferring equipment loads to the ground can be assumed to act as a surface rigid strip footing on top of a layered soil. The ultimate limit state design for ground-supported working platforms is focused on preventing shear failure of the platform, which may have severe consequences, including overturning of the plant. Figure 1-2 indicates an overturning accident, which occurred due to
platform failure. It is therefore crucial to base the ultimate limit state design on robust methods to not only protect expensive equipment but also to prevent loss of life.
In current practice, working platforms for tracked plant are designed using a simplified method, which is included in BRE-470. Although BRE-470 has been proven to lead to reliable and safe platforms, some users have reported unnecessarily large platform thicknesses (Corke and Gannon, 2010). Based on the author’s practical experience in the design of working platforms with the BRE470 method but also with more advanced numerical tools, the BRE470 method can lead to conservative, but also unconservative results, depending on the conditions of the problem. These observations motivated this research.

1-2 RESEARCH OBJECTIVES

The main objective of this research was to improve and extend the field of application of current design methods for working platforms for tracked plant by providing simple bearing capacity formulas, derived from interpretation of advanced numerical solutions. The methodology, used for predicting bearing capacity of strip footings on dense fill overlying soft clay or loose sand was the Finite Element Limit Analysis (FELA) method, recently developed at the University of Newcastle. Although the focus was on the range of material parameters and geometries encountered in the design of working platforms for tracked plant, provision of dimensionless bearing capacity formulas allows extending the application of the developed methods to the design of any shallow strip footing on similar conditions. The calculated collapse loads from FELA simulations were validated against published experimental test results. The results of numerical simulations revealed the existence of complex failure mechanisms other than the ones, considered in the existing bearing capacity models.

1-3 METHODOLOGY

Applying the limit analysis theorem in geotechnical problems facilitated the estimation of the collapse load for various problems. However, it is often complicated to solve the equations analytically, for complex problem geometries. In the FELA method, the finite element technique is used as a tool to solve the equations of limit theorems of plasticity. This allows obtaining rigorous upper and lower limits of the collapse load for practical problems, incorporating complex loading conditions, realistic stratigraphy, and complicated geometries. Sloan (1988, 1989) was the first one who proposed the use of linear programming to solve the resulting optimisation problems. The technique was further developed by Lyamin and Sloan (2002a, b) and Krabbenhoft et al. (2005, 2007) who introduced non-linear programming in the solution. Adaptive re-meshing has been implemented more recently by Lyamin et al. (2013), facilitating more accurate solutions by increasing the number of elements at the vicinity of the failure surface. Further details on the FELA technique are discussed in Chapter 6 and Chapter 8.
Comprehensive parametric studies were undertaken with the FELA method to investigate the ultimate bearing capacity of a rough surface strip footing on layered soils. Rough, surface strip footings were considered to model the load applied from tracked plant. In addition to bracketing the collapse loads for each problem configuration, failure mechanisms were identified using the results of upper bound analyses. The latter provides very useful insight into the behaviour of layered soils for the wide range of material properties and problem geometries, considered in this research. Accounting for variable collapse mechanisms allowed developing refined analytical models, on which the resulting bearing capacity equations are based. Understanding the dominant modes of failure in various ranges of material parameters and geometries assisted in identifying the range of material parameters for which the existing bearing capacity models fail to accurately predict the collapse loads. Details of how interpreted failure mechanisms facilitated derivation of bearing capacity equations for layered soils in a wide range of material parameters are discussed in Chapter 6 and Chapter 8.

1-4 THEESIS STRUCTURE

This thesis consists of ten chapters including this introductory chapter. The following Chapter 2 presents a review of existing methods for estimating the bearing capacity of shallow foundations on homogeneous and layered soils, followed by a brief discussion on the current method of designing granular working platforms for tracked plant, and identifies gaps in estimating bearing capacity of footings on layered soils. Chapter 3 provides an overview of the published technical papers that form the main structure of the thesis. These papers are presented in Chapters 4 through to Chapter 8. Chapter 9 summarises the main contributions of this research, and the concluding remarks, together with recommendations on future works on this topic. Chapter 10 lists references and the results of FELA simulations are summarised in Appendix A and Appendix B. Finally, Appendix C presents a detailed response to issues raised by the Reviewers of the thesis.
CHAPTER 2- LITERATURE REVIEW
2-1 INTRODUCTION

A detailed review of the literature on methods for estimating the bearing capacity of shallow foundations on homogeneous and layered soils is presented in this chapter. The chapter begins with introducing the conventional bearing capacity theory of Terzaghi, and other solutions for bearing capacity factors, subsequently proposed by other researchers to estimate ultimate bearing capacity of shallow foundations on homogeneous soils. A comprehensive discussion on the bearing capacity of layered soils, comprising a granular layer of limited thickness overlying relatively weak soil is presented next. Studies on the bearing capacity of layered soils are presented separately depending on the properties of the bottom soil layer (i.e. undrained cohesive or purely frictional soil types). Finally, identified limitations of the BRE470 method for designing working platforms for tracked plant and other existing methods in the literature are discussed.

2-2 BEARING CAPACITY OF SHALLOW FOUNDATIONS ON HOMOGENEOUS SOILS

Several methods for estimating the bearing capacity of shallow footings on homogeneous soils have been developed since the middle of the previous century. Terzaghi (1948) was the first to introduce a comprehensive bearing capacity theory, the principles of which are still applied in geotechnical engineering. The basics of Terzaghi’s bearing capacity theory for strip footings over homogeneous soils and some major contributions by other researchers are briefly presented here.

Terzaghi (1943) based his method for estimating the bearing capacity on the limit equilibrium method. This required assuming the general shear failure of Figure 2-1a, for a shallow, rough, rigid, strip footing resting on a homogeneous soil layer. The failure mechanism comprises three zones; a triangular elastic block located immediately below the bottom of the foundation, the Prandtl radial shear fan and the Rankine passive zone. Equilibrium of the elastic wedge (Figure 2-1b) was used by Terzaghi to derive the bearing capacity equation. Considering equilibrium of the wedge ABJ in vertical direction for a unit length of the strip footing, we have:

\[ q_u \times 2b \times (1) = -W + 2c_u \sin\phi + 2P_p \]  
Eq. 2-1

where \( B=2b \) and \( W \) is the weight of the elastic wedge viz. \( \gamma b^2 \tan\phi \). Thus:

\[ 2bq_u = 2P_p + 2bc_u \tan\phi - \gamma b^2 \tan\phi \]  
Eq. 2-2
The bearing capacity of the footing can be calculated, if the passive forces \( P_p \) acting on the wedge sides AJ and BJ are found. The passive force \( P_p \) depends on the soil unit weight, cohesion, and the soil surcharge. Terzaghi suggested to estimate the mobilised passive forces \( P_p \) by an approximate method, considering separately the contribution of the abovementioned factors, which depends on the soil friction angle. According to Figure 2-2, the passive soil force \( P_p \) can be written as:

\[
P_p = \frac{1}{2} \gamma (b \tan \phi)^2 K_\gamma + c_u (b \tan \phi) K_c + q (b \tan \phi) K_q \tag{2-3}
\]

where \( K_\gamma \), \( K_c \) and \( K_q \) are coefficients that depend on internal friction angle of soil. Substituting Equation 2-3 in Equation 2-2 gives:

\[
2b q_u = 2 b c_u [\tan \phi (K_c + 1)] + 2 b q \left[ (\tan \phi K_q) \right] + b^2 \gamma \left[ \tan \phi (K_\gamma \tan \phi - 1) \right] \tag{2-4}
\]

which, when rearranged gives the bearing capacity as:

\[
q_u = c_u [\tan \phi (K_c + 1)] + q \left[ (\tan \phi K_q) \right] + \frac{\gamma^2}{2} \left[ \tan \phi (K_\gamma \tan \phi - 1) \right] \tag{2-5}
\]

Or, in a simpler form
Terzaghi calculated the passive force $P_p$ for the following three special cases relevant to soil strength, and provided the bearing capacity factors as follows:

when $\phi \neq 0, \gamma = 0, q \neq 0, c_u = 0$, it is $N_q = \frac{e^{(\frac{3\pi}{4})\frac{\phi}{2}} \tan \phi}{2\cos^2(\frac{45+\phi}{2})}$

when $\phi \neq 0, \gamma = 0, q = 0, c_u \neq 0$, it is $N_c = \cot \phi \left[ \frac{\frac{3\pi}{4}}{2} \tan \phi \right] - 1$ and

when $\phi \neq 0, \gamma \neq 0, q = 0, c_u = 0$, it is $N_\gamma = \left[ \frac{1}{2} K_p \tan^2 \phi \left( \tan \phi - \frac{\tan \phi}{2} \right) \right]$. 

Experimental tests on model footings generally confirmed Terzaghi’s assumed failure mechanism (Figure 2-1a). However, further research suggests the angle between sides of the elastic wedge is closer to $45+\phi/2$ rather than $\phi$. Meyerhof (1951) considered the failure mechanism of Figure 2-
3, in which the angle between the sides of the elastic wedge and horizontal was assumed to be 45+ϕ/2. Meyerhof’s bearing capacity theory was developed for cohesive-frictional soils and applies to both deep and shallow foundations. This theory considers a mixed shear zone, in which the shear resistance varies between radial and plane shear, depending on the depth and roughness of the foundation. Meyerhof’s failure mechanism includes an equivalent free surface (line be in Figure 2-3), over which mobilised shear stress is given by:

\[ S_0 = m(c + P_0 \tan \phi) \]  
\[ \text{Eq. 2-7} \]

in which, m is the degree of mobilisation of shear strength. Considering equilibrium of the plane bcd in Figure 2-3, Meyerhof derived the cohesion and surcharge bearing capacity factors as:

\[ N_c = \cot \phi \left[ \frac{(1+\sin \phi)e^{2\theta \tan \phi}}{1-\sin \phi \sin(2\eta+\phi)} - 1 \right] \]  
\[ \text{Eq. 2-8} \]

\[ N_q = \left[ \frac{(1+\sin \phi)e^{2\theta \tan \phi}}{1-\sin \phi \sin(2\eta+\phi)} \right] \]  
\[ \text{Eq. 2-9} \]

These bearing capacity factors are influenced by the angle of inclination of the equivalent free surface (β) and the degree of mobilisation of shear strength on this plane (m). For surface strip footings both β and m are zero. In this case, θ = π/2 and η =45- ϕ/2. Thus, \( N_q \) and \( N_c \) are given by:

\[ N_q = e^{\pi \tan \phi} \left[ \frac{(1+\sin \phi)}{1-\sin \phi} \right] \]  
\[ \text{Eq. 2-10} \]
\[ N_c = \cot \phi (N_q - 1) \quad \text{Eq. 2-11} \]

Essentially, for surface footings, Meyerhof’s solution degenerates to the solution of Reissner (1924) and Prandtl (1920).

\[ N_f \] may be found using a trial and error procedure, in which the centre of log-spiral zone is varied so that minimum passive force \((P_p)\) is obtained. The passive force is only due to weight and friction i.e. \(c\) and \(P_0\) are zero. In this case, \(N_f\) is given by:

\[
N_f = \left[ \frac{4P_p \sin \left( \frac{45 + \phi/2}{2} \right)}{y B^2} - \frac{1}{2} \tan \left( 45 + \frac{\phi}{2} \right) \right] \quad \text{Eq. 2-12}
\]

For surface strip footings, \(\beta = 0\) and \(m = 0\), thus Meyerhof suggested the following expression to approximate \(N_f\):

\[
N_f = (N_q - 1) \tan (1.4\phi) \quad \text{Eq. 2-13}
\]

Other researchers have proposed different formulas to calculate \(N_f\). Some of the most well-known solutions are presented in Table 2-1. All these expressions assume that the friction angle of soil is constant with depth. However, this is not an accurate assumption as the soil friction angle depends on level of confining stress and varies with depth. Given the sensitivity of \(N_f\) to variation of \(\phi\), these expressions should be used with caution to calculate bearing capacity of shallow foundations on granular soils.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Method of analysis</th>
<th>Proposed expression for (N_f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hansen (1970)</td>
<td>Limit Equilibrium</td>
<td>(N_f = 1.5N_c \tan^2 \phi)</td>
</tr>
<tr>
<td>Vesic (1973)</td>
<td>Limit Equilibrium</td>
<td>(N_f = 2(N_q + 1) \tan \phi)</td>
</tr>
<tr>
<td>Biarez et al. (1961)</td>
<td>Upper Bound Limit Analysis</td>
<td>(N_f = 1.8(N_q - 1) \tan \phi)</td>
</tr>
<tr>
<td>Booker (1969)</td>
<td>Slip Line Method</td>
<td>(N_f = 0.1045e^{9.6\phi})</td>
</tr>
<tr>
<td>Michalowski (1997)</td>
<td>Upper Bound Limit Analysis</td>
<td>(N_f = e^{(0.66+5.1\tan \phi)}) \tan \phi</td>
</tr>
<tr>
<td>Hjiaj et al. (2005)</td>
<td>Finite Element Limit Analysis</td>
<td>(N_f = e^{\frac{1}{5}(\pi+3\pi^2 \tan \phi)} \tan \phi^{\frac{2\pi f}{5}})</td>
</tr>
<tr>
<td>Martin (2005)</td>
<td>Method of characteristics</td>
<td>(N_f = (N_q - 1) \tan (1.32\phi))</td>
</tr>
</tbody>
</table>

* \(N_q\) and \(N_c\) are given by Eq. 2-10 and Eq.2-11, respectively.

** \(\phi\) is in radian, \(N_f = 0\) for \(\phi = 0\).*
2-3 BEARING CAPACITY OF SHALLOW FOUNDATIONS ON DENSE SAND OVER SOFT CLAY

2-3-1 GENERAL

As mentioned earlier, estimating the bearing capacity of shallow foundations on a dense granular soil overlying soft clay is a common geotechnical problem. Owing to the complexity of the problem, an exact analytical solution is not available. The ultimate bearing capacity of footings on such layered soil also depends on the properties of the bottom weak layer, unless the thickness of the top soil is large enough to accommodate the entire failure mechanism. Therefore, the conventional bearing capacity theory, discussed in previous section is not valid for this type of layered soils. Various techniques have been employed by researchers to estimate the ultimate bearing capacity of footings on this type of layered soils, which have their origins at the simplified projection method of Terzaghi (1948). However, a robust bearing capacity model that considers the behavior of layered soils for a wide range of material parameters and problem geometries encountered in practice, is yet to be developed. Some of the most influential studies on this topic are reviewed in the following section.

2-3-2 PROJECTED AREA MODEL

Terzaghi (1948) introduced the projected area model to estimate the ultimate bearing capacity of footings on sand over clay. As shown in Figure 2-4, this model assumes that the sand layer distributes uniformly the applied pressure on the actual footing to an effective footing width $B'$, resting on the upper surface of the clay layer. The ultimate bearing capacity of the footing on layered soil is then computed as that of the effective footing resting on the clay only, as:

$$ q_u = c_u N_c B'/B $$

where, $N_c$ is given from Equation 2-11 and for $\phi_u = 0$, $N_c = 5.14$ for strip footings. In doing so, this model disregards the mobilised shear resistance in sand. Referring to Figure 2-4, the effective footing width is determined by the angle ($\alpha$) at which the load is spread by the sand. Terzaghi (1948) recommended to assume that the footing pressure is spread at an inclination of one horizontal to two vertical, i.e. an angle of approximately $\alpha = 26.6^\circ$. Jacobsen (1977) conducted experimental tests on embedded circular model footings and concluded that the angle $\alpha$ is a function of relative strength of the two layers. Based on that, Jacobsen proposed Equations 2-15 and 2-16 for $\alpha$. In these equations, $q_S = 0.5\gamma B N_s S_y + \gamma D N_q S_q$ and $q_c = c_u N_c S_c$ are the bearing capacity of the same footing on the sand and the clay layer, respectively, and the $q_S/q_c$ is the relative strength of the two layers.
\[ \alpha = \tan^{-1}\left(\frac{2\beta}{\phi}\right) \quad \text{Eq. 2-15} \]

\[ \beta = 0.1125 + 0.0344\left(\frac{q_u}{q_c}\right) \quad \text{Eq. 2-16} \]

Later, Kenny and Andrawes (1997) also performed experimental tests on surface strip footings, resting on sand overlying clay. They used experimentally obtained displacement fields to determine the inclination of shear planes in the sand layer, and hence \( \alpha \). Kenny and Andrawes (1997) reported that \( \alpha \) varies as function of the ratio of sand thickness to footing width \( \frac{H}{B} \), which is not compatible with Terzaghi’s assumption. Burd & Frydman (1997) back-calculated \( \alpha \), from the results of finite element numerical analysis and showed that \( \alpha \) is a function of \( \frac{c_u}{\gamma H} \) in addition to the sand friction angle, while \( \frac{H}{B} \) has negligible effect on it. The importance of \( \frac{c_u}{\gamma H} \) has been noted earlier by Craig and Chua, (1990). Ballard et al., (2011) also used the Limitstate:GEO software which is based on Discontinuity Layout Optimization (DLO) method and back-calculated \( \alpha \) as function of \( \frac{c_u}{\gamma H} \). Spread angle \( \alpha \) values, calculated by Ballard et al. (2011) are about 5° to 10° lower than those proposed by Burd & Frydman (1997) for sand friction angle \( \phi = 40° \). Therefore, there is no consensus regarding the value of spread angle \( \alpha \) despite its significant influence on the estimated bearing capacity. Moreover, this simplified model is not representative of the actual failure mechanism as the full contribution of sand to the bearing capacity is not taken into account.
2-3-3 PUNCHING SHEAR MODEL

An alternative bearing capacity model was proposed by Meyerhof (1974) based on the results of experimental tests on layered soils. Meyerhof idealised the collapse mechanism as a rigid die punching the block of sand immediately below the footing moving downwards into the clay. This punching shear mechanism, shown in Figure 2-5, provides the bearing capacity as the sum of the shearing resistance developing along the shear failure planes in sand, and the resistance offered by clay. The planes of failure in sand are assumed to be vertical in this model, and the shear resistance mobilised on the actual failure planes in sand is accounted for by passive forces acting on the assumed vertical failure planes. The bearing capacity equation is derived from equilibrium of the sand block punching into the bottom layer. For a surface strip footing is given as:

\[ q_u = c_u N_c + \frac{\gamma H^2}{B} K_p \tan \delta \leq q_s \]  

Eq. 2-17

Where, \( N_c = 5.14 \) is the conventional bearing capacity factor, \( K_p \) is the coefficient of passive earth pressure and \( \delta \) is the mobilised friction angle on the assumed planes of failure. In addition, \( q_s \) is the ultimate bearing capacity of the same footing over a uniform layer of top sand and applies where the thickness of top sand exceeds the critical depth \( (H_{cr}) \), defined as the thickness beyond which the bottom layer has no influence on the bearing capacity of the footing.

![Figure 2-5: Punching shear model adopted for surface footings after Meyerhof (1974).](image)

The formula for the coefficient of passive earth pressure \( K_p \) proposed by Caquot (1948) was used by Meyerhof to estimate the passive forces acting upon the failure planes. This coefficient and thus the forces are highly dependent on the mobilised friction angle on the assumed failure planes \( (\delta) \), as discussed by Hanna and Meyerhof (1980). Meyerhof (1974) stated that \( \delta \) varies between \( \phi/2 \) and \( 3\phi/4 \) and suggested to use an average \( \delta = 2\phi/3 \) as a reasonable estimation of passive forces on the vertical planes. As engineers are more familiar with the internal friction angle of soils, Meyerhof (1974) introduced the punching shear coefficient \( (K_s) \) which is given by:


\[ K_s \tan \phi = K_p \tan \delta \]  

Eq. 2-18

Therefore, Equation 17 can be rewritten as:

\[ q_u = c_u N_c + \frac{\gamma H^2}{B} K_s \tan \phi \leq q_s \]  

Eq. 2-19

For the estimation of \( K_s \) in equation 2-19, Hanna and Meyerhof (1980) used a limit equilibrium technique to investigate the reduction in mobilised friction angle on the assumed failure planes due to existence of the weak lower layer. They provided charts of the ratio \( \delta/\phi \) as function of \( q_c/q_s \) and calculated \( K_s \) accordingly as function of \( \phi, \delta/\phi \) and the undrained shear strength of clay (\( c_u \)). However, Burd and Frydman (1996) suggest that \( K_s \) is a function of \( \gamma \) and \( H \) in addition to the abovementioned parameters and the design charts provided by Hanna and Meyerhof are only appropriate for the particular values of \( H \) and \( \gamma \) on which they were developed for. Moreover, the punching shear model was developed for the case of a dense sand over very weak clay and as the strength of clay layer increases relative to the sand layer, the method over-predicts the ultimate bearing capacity (Shiau et al., 2003). Both Meyerhof (1974) and Hanna (1980) extended their models to the case of circular footings by introducing shape factors and a slightly different form of the bearing capacity equation.

2-3-4 OKAMURA ET AL. MODEL

Okamura et al. (1998) developed a model based on the punching shear failure mechanism of Meyerhof (1974), but considering also the inclination of failure planes in sand. For that, Okamura et al. used results of centrifuge tests on layered soils in conjunction with the limit equilibrium method. Figure 2-6 shows the failure mechanism proposed by Okamura et al. (1998). The actual curved failure planes in sand are approximated by linear planes inclined at an angle \( \alpha_c \) relative to the vertical and the mobilised shear stress on these planes is estimated to obtain the contribution of the sand layer to the ultimate bearing capacity of the footing. Accordingly, the bearing capacity equation for strip footings on sand overlying clay results as:

\[
q_u = \left( 1 + 2 \frac{H}{B} \tan \alpha_c \right) \left( c_u N_c + P'_0 + \gamma' H \right) + \frac{K_p \sin \left( \phi' - \alpha_c \right)}{\cos \phi' \cos \alpha_c} \frac{H}{B} \left( P'_0 + \gamma' H \right) - \gamma' H \left( 1 + \frac{H}{B} \tan \alpha_c \right)
\]

Eq. 2-20

where, \( P'_0 \) is the surcharge at the footing level.
The key parameters of this model are the inclination of the failure planes in sand, defined via $\alpha_c$, and the mobilised shear stresses on these planes of failure, defined via $K_p$. To estimate the mobilised coefficients of lateral earth pressure, Okamura et al. measured experimentally the horizontal stress at locations close to the side of the sand block at the peak load. The Rankine passive coefficient $K_p$ was however, adopted as the lower bound to the measured coefficients. Equation 2-21 was proposed by Okamura et al. to calculate the angle of sand failure planes to vertical, based on the geometry of Mohr circle of stresses for the small elements A and B just above and below of the sand-clay interface, adjacent to the inclined failure planes (see Figure 2-6).

$$\alpha_c = \tan^{-1}\left(\frac{\sigma_{mc}}{\sigma_{ms}}\left(1+\sin^2\phi'\right)\right)$$

$$\frac{\sigma_{mc}}{c_u} = N_c S_c \left(1 + \gamma' H + \frac{P_0}{c_u}\right)$$

$$\frac{\sigma_{ms}}{c_u} = \frac{\sigma_{mc}}{c_u} \sqrt{\left(\frac{\sigma_{mc}}{c_u}\right)^2 \cos^2\phi' - \left(\frac{(\sigma_{mc})^2}{c_u} + 1\right)}$$

$\sigma_{mc}$ and $\sigma_{ms}$ are the mean effective vertical stress of the sand element A and mean effective stress of the clay element B respectively; $S_c$ is the shape factor that is equal to one for strip footings.

Nevertheless, the angles predicted by Equation 2-21 are inconsistent with the behaviour of the layered soils observed by other researchers such as Jacobsen and Christensen (1977), Brocklehurst (1993) and Burd and Frydman (1997) who have all indicated that the angle between the failure plane and vertical direction ($\alpha_c$ in Okamura’s et al. model) increases as the strength of

Figure 2-6: Failure mechanism of Okamura et al. (1998).
sand relative to clay increases. On the contrary, Equation 2-21 yields lower \( \alpha_c \) as \( \phi \) increases for a given \( c_u \). This could be attributed to the assumption of Okamura et al. (1998) that the direction of failure plane can be calculated at interface between sand and clay, and then extended to the whole thickness of the sand layer. In addition to the discrepancies in predicting \( \alpha_c \), using Rankine’s passive coefficient to estimate horizontal stresses on the failure plane is a simplifying assumption that underestimates the mobilised resistance of the sand layer (Okamura et al. 1998). In fact, the horizontal stress acting on the failure planes is a function of lateral movement of the sand under the footing, the magnitude of which increases as the strength of underlying layer increases. This was in fact shown by Okamura et al. (1998). However, the range of material properties, on which the model tests are performed was very limited i.e. plane strain friction angle of 47.7° for sand and undrained shear strength of (21.9 - 23.0) kPa for the bottom clay layer. A comparison between the results from this model with the punching shear model of Meyerhof (1974), indicates fair agreement for \( \phi \) close to 45° that is comparable to the range of experimental tests; however, the model overestimates the ultimate bearing capacity of the footings for lower values of \( \phi \).

2-3-5 NUMERICAL AND ANALYTICAL SOLUTIONS

The availability of computers and the emergence of the non-linear finite element method enabled bearing capacity to be more comprehensively explored for a range of material parameters, using numerical methods. Griffith (1982), and later Burd and Frydman (1996), both applied the classic displacement finite element method to estimate the ultimate bearing capacity of footings on multilayered soils. More recently, the FELA formulation mentioned earlier, provided researchers a means to accurately bracket true collapse loads and conduct numerical studies considering broad ranges of material properties. The FELA method has been used extensively by researchers to study the bearing capacity of footings (Hjiaj et al. 2004 and 2005, Lyamin et al., 2007 and 2009, Merefield et al. 1999 and 2006, Sutcliffe et al. 2004 and Yamamoto et al. 2009, 2011 and 2012). Specifically for layered soils, Shiao et al. (2003) used FELA to obtain upper and lower bound solutions to the ultimate bearing capacity of a layer of sand over clay, and provided charts demonstrating the influence of footing roughness, footing embedment depth and increase in shear strength of the clay on bearing capacity of the footing.

Michalowski and Shi (1995) obtained upper bound estimates of the true collapse load of a footing on sand over clay using kinematic approaches in limit analysis. Two collapse mechanisms extending into the bottom layer were considered and results, in the form of dimensionless design charts, were presented for a broad range of material properties. Dimensionless charts that provide the critical depth were also presented, in which the critical depth was normalised by the footing
width $H/c_u/B$ and plotted versus $c_u/B$ and $\phi$. Burd and Frydman (1996) noted that the results of Michalowski and Shi (1995) overestimate the bearing capacity for cases in which sand has a relatively high friction angle. They attribute this to the assumed failure mechanisms and the assumption of associative flow for the frictional material. Huang et al. (2007) also applied the multi-rigid-block mechanism approach of limit analysis to obtain upper bound solutions for the bearing capacity of rough strip footings on layered soils. For that, Huang et al. (2007) used a modified version of the failure mechanism by Florkiewicz, et al. (1989) which provides more refined upper bound solutions compared to those by Michalowski and Shi (1995).

2-3-6 EXPERIMENTAL STUDIES

There is a limited number of experimental studies in the literature e.g. Das and Dallo (1984), Okamura et al. (1998), Kenny and Andrawes (1997), Meyerhof (1978) reporting measurements of the bearing capacity of strip footings on sand over clay. These studies cover a rather narrow range of sand and clay properties, with sand friction angles usually around $\phi' = 45^\circ$, and clay undrained shear strength of the order of $c_u = 10$ to $20$ kPa. Okamura et al. (1998) performed centrifuge tests at 50g to measure the ultimate load on strip footings with prototype scale widths varying between 1.0 and 2.0 m. The dimensionless thickness of the sand layer ranged from $H/B = 1$ to 4, while the plane strain friction angle of the sand layer and the undrained shear strength of the clay layer were reported to be $\phi' = 47^\circ$ and $c_u = 22$ kPa, respectively. Earlier, Das and Dallo (1984) performed scaled tests to measure the bearing capacity of a strip footing with width $B=0.076$ m (3 in). The dimensionless thickness of the sand layer in their experiments varied between $H/B = 1$ and 3.01. Das and Dallo report that the friction angle of sand was $\phi' = 43^\circ$ and the undrained shear strength of clay was $c_u = 12$ kPa. The mentioned experimental data were used in this research to validate the FELA solutions.

2-4 BEARING CAPACITY OF SHALLOW FOUNDATIONS ON DENSE SAND OVER LOOSE SAND

2-4-1 GENERAL

The case of shallow footings on layered sand profiles has attracted much less attention in the literature, despite its importance for the design of working platforms for tracked piling rigs and cranes on sites where relatively loose sand deposits are encountered near the ground surface. Current practice for the design of working platforms on loose sandy deposits is based on the method described in the Building Research Establishment BRE470-2004 guidelines, which is a simplified version of the semi-empirical method proposed by Hanna (1981), which in turn is based on Meyerhof’s punching shear model. Other researchers have approached the problem
analytically, using limit equilibrium and limit analysis techniques that require assuming specific, simplified failure mechanisms e.g. Ghazavi and Eghbali (2008) derived bearing capacity factors for footings on two-layered granular soils, using an extension of the failure mechanism initially proposed by Richards et al. (1993). Analytical upper bound solutions have also been obtained for the more general case of footings on two-layered cohesive-frictional soils by e.g. Purushothamaraj et al. (1974), Florkievicz (1989) and Huang and Qin (2009). More recently, Khatri et al. (2017) used the FELA method to calculate upper and lower bounds of the collapse load, using conic optimisation (Makrodimopoulos and Martin 2006, 2007, Krabbenhøft et al. 2007, 2008). Experimental results to benchmark the abovementioned solutions are sparse, however Meyerhof (1978), Das and Munoz (1984) and Kumar et al. (2007) report findings from small-scale 1g tests on footings resting on layered sand, for different problem geometries. Extrapolation of these test results to full-scale footings must be done with caution, as scale effects may lead to overestimating the collapse load, due to the dependency of sand mechanical characteristics to the level of confining stress (Zhu et al. 2001, Cerato and Lutenegger 2007). Nevertheless, results of such model tests are invaluable, as they can provide insight on the developing failure mechanisms. The most widely used methods in practice are described in more detail in the following.

2-4-2 PUNCHING SHEAR MODEL

Hanna (1981) extended the punching shear model of Meyerhof (1974), described in Section 2-3-3 to address the problem of footings on strong sand overlying weak sand. The punching shear mechanism of Figure 2-7 was considered which consists of a general shear failure surface within the bottom sand layer, underneath an inverted frustum of dense sand punching through the upper layer. In this model the curved failure planes, formed in upper sand, are approximated again by vertical shear planes which shear resistance is calculated from the theory of retaining walls, with passive thrusts acting on them. The mobilised passive thrusts are the resultants of mobilised shearing resistance within the top layer and depend on the mobilised friction angle on the assumed shear planes ($\delta$). The passive thrust $P_p$ is given by:

$$P_p = 0.5\gamma_1H^2 \left(1 + \frac{2D}{H}\right) K_p / \cos \delta$$

Eq. 2-24

The parameters are as defined in Figure 2-7. In addition, $K_p$ is the mobilised coefficient of lateral earth pressure, which is a function of $\delta$. Considering equilibrium of the sand block, the ultimate bearing capacity of strip footing is calculated as follows.

$$q_u = q_b + \frac{2}{B} (P_p \sin \delta) - \gamma_1H \leq q_t$$

Eq. 2-25
in which $q_b$ and $q_1$ are the ultimate bearing capacity of the strip footing on a deep uniform layer with the properties of the bottom and top sand, respectively. Combining Equations 2-24 and 2-25, the ultimate bearing capacity of a surface strip footing is given by:

$$q_u = q_b + \gamma H^2 \frac{K_p \tan \delta}{B} - \gamma H \leq q_t$$  \hspace{1cm} \text{Eq. 2-26}

A key parameter in Equation 2-26 is the mobilised friction angle $\delta$, which is not constant but varies along the thickness of the top layer (Meyerhof, 1974). Hanna (1981) assumed that $\delta$ varies from the friction angle of the top sand layer $\phi_1$ immediately below the footing to $\lambda \phi_2$ at the interface of the two layers. $\lambda$ is a factor smaller than one that is a function of material properties and geometry of the problem. Hanna (1981) used experimental tests to estimate the variation of $\lambda$ with the thickness of the sand layer. Based on the results of experimental tests, Hanna concluded that a parabolic distribution from $\phi_1$ to $(q_b/q_t) \phi_2$, best describes the variation of $\delta$ on the assumed vertical failure planes. The passive force acting on the shear planes ($P_p$) could then be determined using the assumed variation of $\delta$ from the following equation:

$$P_p = \int_0^H \gamma_1 K_p \cos \delta Z (Z + D) \, dZ$$  \hspace{1cm} \text{Eq. 2-27}

in which, $K_{pz}$ is the coefficient of lateral earth pressure at a certain depth $Z$ (Caquot and Kerisel, 1948). An average mobilised friction angle ($\bar{\delta}$) can be determined by equating the passive thrust calculated using Equations 2-24 and 2-27. This gives an average mobilised friction angle on the assumed failure planes which is a function of $q_2/q_1$. Referring to the definition of $q_1$ and $q_2$ and ignoring possible difference between the unit weights of the layers, we obtain a single average mobilised $\delta$ for a given set of $\phi_1$ and $\phi_2$ in Hanna’s model. Hanna published a graph that provides $\delta/\phi_1$ as function of $\phi_1$ and $\phi_2$. This average mobilised friction angle can be used in Equation 2-26 to estimate the ultimate bearing capacity of the footing.

Figure 2-7: Assumed failure mechanism for strong sand over weak sand after Hanna (1981).
To derive an easy-to-use expression in terms of soil internal friction angle instead of average mobilised friction angle on the assumed failure planes, Hanna (1981) introduced the punching shear coefficient ($K_s$), following the concept first introduced by Meyerhof (1974):

$$K_s \tan \phi_1 = K_p \tan \delta$$  \hspace{1cm} \text{Eq. 2-28}

Hanna published a chart that provides $K_s$ as a function of $\phi_1$ and $\phi_2$. Substituting Equation 3-28 in Equation 2-26, the ultimate bearing capacity of a surface strip footing on dense sand overlying loose sand will be given by:

$$q_u = q_b + \gamma H^2 \frac{K_s \tan \phi}{B} - \gamma_1 H \leq q_t$$  \hspace{1cm} \text{Eq. 2-29}

The experimental database, on which this method was developed contained only tests on very dense top sand. This maybe the reason that Hanna’s theory (1980) overestimates the ultimate bearing capacity as the strength of the bottom sand layer increases relative to the top sand. Details on dominant modes of failure and range of material parameters, for which Hanna’s model is potentially unreliable are discussed in Chapter 8.

2-4-3 EXPERIMENTAL STUDIES

Limited experimental test results are reported for footings on layered sands. Meyerhof (1978), Das and Munoz (1984) and Kumar et al. (2007) reported findings from small-scale 1g tests on footings resting on layered sand, for different problem geometries. Meyerhof (1978) loaded up to failure a 0.05 m-wide strip footing, resting on the surface of two layers of sand, with the top and bottom layers featuring plane-strain friction angles $\phi_1 = 47.7^\circ$ and $\phi_2 = 34^\circ$, respectively. The dry density of the top and bottom layer was $\gamma_1 = 16.3$ kN/m$^3$ and $\gamma_2 = 13.8$ kN/m$^3$, respectively. The relative thickness of the top layer varied from $H/B = 0$ up to $H/B = 5$. Das and Munoz (1984) tested a 0.1016 m-wide strip footing resting on the surface of a sand layer with $\phi_1 = 43^\circ$, underlain by a looser sand layer with $\phi_2 = 36^\circ$. Extrapolation of these test results to full-scale footings must be done with caution, as scale effects may lead to overestimating the collapse load, due to the dependency of sand mechanical characteristics to the level of confining stress (Zhu et al. 2001, Cerato and Lutenegger 2007). Nevertheless, results of such model tests are invaluable, as they can provide insight on the developing failure mechanisms.

2-5 EXISTING DESIGN GUIDELINES

The design of granular working platforms in practice is commonly based on a semi-empirical method included in the BRE470-2004 guidelines, which is a simplified version of the Meyerhof’s punching shear theory described in Section 2-3-3 and 2-4-2 for clayey and sandy subgrades,
respectively. Eq. 2-17 and Eq. 2-26 are used in the BRE470 method to estimate the ultimate bearing capacity of the platform, considering the following simplifying assumptions:

- Values for “$K_p \tan \delta$” (Eq. 2–26) are presented in a single graph as a function of the internal friction angle of the platform material for both cohesive and frictional subgrades.

- The mobilised friction angle $\delta$ is taken equal to $\delta = 2\phi/3$ and accordingly the term “$K_p \tan \delta$” is calculated using Coulomb’s passive earth pressure coefficient for both cohesive and frictional subgrades.

- BRE470-2004 adopts the formulas for the bearing capacity factor $N_\gamma$ proposed by Vesic (1975), which are slightly higher than those suggested by Meyerhof (1955).

- For sandy subgrades, the embedment depth $D$ is taken as zero and the surcharge term $\gamma_1 H N q_2$ is ignored in the calculation of the contribution of the bottom sand layer to the resistance, $q_b$.

2-6 FINITE ELEMENT LIMIT ANALYSIS METHOD

2-6-1 GENERAL

This section summarises the background of the computational method used in this study, followed by a discussion on its limitation.

Limit analysis (Drucker et al. 1952) is an effective technique for estimating rigorous upper and lower bounds of the collapse load of foundations. Application of this technique was initially limited to relatively simple problems, since complicated geometries result in complex equations that have to be solved analytically. More recently, the use of the finite element method to solve the equations of limit theorems has extended the field of application of this technique to realistic geometries and heterogeneous soil stratigraphies (Sloan 2013). This computational technique called Finite Element Limit Analysis (FELA) method can provide upper and lower bound estimates of the collapse load that bracket its true value, with the difference between the upper and lower bound providing a direct measure of the error in estimating the true collapse load.

The FELA formulation based upon the lower bound theorem of plasticity seeks to maximize a set of applied stresses/tractions such that the resulting admissible stress field satisfies equilibrium, fulfils the stress boundary conditions and at no location violates the yield criteria. The maximised stresses/tractions provide a safe estimate of the loads that do not exceed the actual collapse load of the structure. Conversely, the FELA formulation of the upper bound theorem of plasticity is based upon a kinematic formulation, requiring an admissible velocity field be constructed. According to upper bound theorem, if a kinematically admissible velocity field is found, for which
the rate of work of external forces exceeds the rate of internal work, the structure will collapse. Unlike traditional finite element analysis methods that require the solutions of a large set of linear equations, the FELA formulation produce a large set of inequalities which are solved using nonlinear optimization techniques. FELA method has been employed in this research to investigate the limit loads and collapse mechanisms of surface strip footings on layered soils.

2-6-2 LIMITATIONS

The limitations of FELA method and of the Mohr-Coulomb failure criterion used in this study to describe the shear strength of geomaterials are briefly discussed below. Some of these limitations such as assuming associated flow rule for materials are inherent limitations of the FELA method, while others including ignoring soil anisotropy and the stress dependency of friction angle may be accounted in future investigations.

2-6-3 ASSUMPTION OF ASSOCIATED FLOW RULE

Employing the limit analysis theorems for the estimation of the bearing capacity of foundations requires assuming that plastic flow in soil is associated i.e. the flow potential is given by the same function as the yield surface. Considering an associated flow rule in conjunction with the Mohr-Coulomb yield criterion (used in this study) implies that the dilation angle of sand is equal to its internal friction angle at failure, which is not realistic for granular materials. While this simplification affects the estimated value of the collapse load, the question of to what extent the flow rule influences the limit loads has not been yet accurately answered. Displacement finite element analyses has been applied by researchers to investigate the effect of dilation angle on the ultimate bearing capacity of shallow footings (e.g. Loukidis and Salgado, 2009). In general, the results of published studies indicate that the effect of non-associativity on failure loads becomes more significant for soil friction angles higher than 30°. Davis (1968) correlated the error introduced in the estimation of the collapse load due to considering associative flow with the degree of kinematic constraint of the problem. Drescher and Detournay (1993), proposed to use the effective strength parameters defined in Eqs. (2-30, 2-31) below and associative flow to estimate upper and lower bounds of the bearing capacity of foundations on geomaterials that exhibit coaxial non-associative flow.

\[
tan \phi^* = \frac{(cos \psi \cos \phi)}{(1-sin \psi \sin \phi)} \times tan \phi \quad \text{Eq. 2-30}
\]

\[
c^* = \frac{(cos \psi \cos \phi)}{(1-sin \psi \sin \phi)} \times c \quad \text{Eq. 2-31}
\]
where \( \phi \) is the friction angle of soil, \( c \) is its effective cohesion and \( \psi \) is the peak dilation angle while \( \phi^* = \psi^* \) and \( c^* \) are the effective friction angle, dilation angle and cohesion, respectively. However, as discussed by Krabbenhoft et al. (2012), using the above effective strength parameters results in significant underestimation of the bearing capacity of shallow foundations. For the problem at hand, evidence of the applicability of associative flow to describe the behaviour of geomaterials below surface strip footings on layered soils is drawn from comparison with experimental results. A quantitative analysis on the effects of associativity on the calculated bearing capacity is presented in Appendix C.

**2-6-4 SOIL ANISOTROPY**

In this research, but also in the previously mentioned studies, soil is assumed to be isotropic. However, Oda and Koishikawa (1979) state that the bearing capacity factor \( N_\gamma \) can be overestimated by up to 40% to 50% if the effect of sand strength anisotropy is neglected. Yuan et al. (2018) used an anisotropic yield criterion to analyse two-dimensional strip-footing problems. They examined the degree of influence of soil anisotropy on the bearing capacity of strip footings and concluded that the ultimate bearing capacity was much lower if soil strength anisotropy was considered. As such, they propose to incorporate both strength anisotropy and non-coaxiality of principal stresses and incremental strains into numerical simulations of geotechnical problems. Kimura et al. (1985) also demonstrated that bearing capacity of shallow foundations is affected by anisotropy, although the difference becomes insignificant for very loose materials. It is possible to account for soil strength anisotropy in FELA method, by adopting a different failure criterion. However, this is not addressed in the present research.

**2-6-5 NON-CONSTANT FRICTION ANGLE**

The bearing capacity of footings on frictional soils is a function of the friction angle, which depends on a number of variables that evolve continuously during the footing loading process: the sand relative density, the level of mean effective stress, the relative magnitude of the intermediate principal effective stress, and the direction of the principal effective stresses relative to the axis of sand deposition (Loukidis and Salgado, 2011). These factors are ignored when the classic Mohr-Coulomb yield criterion is used to describe the behaviour of geomaterials, as in this study and also in the analytical methods presented earlier. However, the FELA method is capable of utilising more complex models of soil behaviour, that account for e.g. the dependency of friction angle on the confining stress levels.
2-7 CONCLUDING REMARKS

A brief review of the most common methods for estimating the bearing capacity of footings on layered soils indicates that a model which balances accuracy and practicality is still missing. Simple methods such as the projected area model disregard many key parameters of the problem including relative strength of the two layers. Predictions by such basic models should be only considered as the first estimate of the collapse loads in practice. Even the punching shear theory which is more complex than the projected area model is based on many simplifying assumptions, and is applicable to a limited range of material parameters. As will be discussed in Chapters 4, 6 and 8, the punching shear failure mechanisms of Figure 2-5 and Figure 2-7 occur when the bottom layer is significantly weak relative to the top soil layer. As shear strength of the bottom layer increases, the geometry of failure mechanism will change. As such, one single collapse mechanism is unable to provide consistent accuracy for the wide range of material parameters encountered in practice. Comparison between bearing capacity predictions of semi-empirical models (e.g. Okamura et al., 1998 and Hanna, 1980) and published experimental data indicates that the accuracy of such models drops for material parameters outside the range of those for which the models were developed.

More specifically, in the case of working platforms for tracked plant, BRE470 method may be used to obtain quick estimates of the bearing capacity of the platforms; However, the following assumptions significantly affect the accuracy of the predicted collapse loads:

- The failure mode does not depend on the relative strength of the two layers.
- The mobilised friction angle is taken always to be $\delta = 2\phi/3$, regardless the problem parameters.

Considering the significance of the mobilised friction angle, adopting $\delta = 2\phi/3$ for all design conditions at best provides a rough estimation of the platform contribution to the ultimate bearing capacity. This may result in over/under-predicting the actual resistance, depending on the relative strength of the two layers. This is extremely significant for very soft clayey subgrades. Hence, BRE-470 limits its application to clays with undrained shear strength greater than 20 kPa. On the other hand, disregarding the surcharge term for frictional subgrades is a conservative assumption that results in significant underestimation of collapse loads.

The objective of this research was to develop easy to use, but accurate bearing capacity models that consider variable collapse mechanisms in a wide range of material parameters and geometries, encountered in practice. This was successfully achieved, using FELA solutions and
interpreting the collapse mechanism for the corresponding design condition, which will be discussed in detail in the following chapters.
CHAPTER 3- THESIS OUTLINE
3-1 INTRODUCTION

This chapter provides a general overview of the thesis, which consists of five papers. Each chapter corresponds to one paper, describing the development of methods for estimating the ultimate bearing capacity of shallow strip footings on dense sand overlying soft clay or loose sand subgrade layers. Although the focus has been on providing rigorous solutions to design working platform for tracked plant as a practical application, the outcomes and conclusions are not exclusive to tracked plant and can be applied to any similar bearing capacity problem. A brief overview of these chapters is presented below. Finally, the results of all numerical analyses, performed during this research are provided in Appendix A and Appendix B.

3-1-1 CHAPTER 4

Chapter 4 “Punching shear coefficients for the design of working platforms” presents punching shear coefficients $K_s$ (see Eq. 2-18, found in page 14), back-calculated from FELA solutions, assuming the simple punching shear failure mechanism of Meyerhof (1974) for layered soils comprising cohesive subgrade. It is shown in this chapter that the punching shear coefficient is a function of $c_u/\gamma H$ and $\phi$, and is independent of the footing width. Further investigations indicated that the assumed punching shear failure mechanism cannot describe the behaviour of layered soils accurately for the entire range of material parameters encountered in the design of working platforms in practice. This was initially noticed from the unexpected change in the trend of back-calculated punching shear coefficients, as strength of clay layer increased relative to the top sand. In fact, the direction of slip surfaces immediately below the footing drift from vertical as the strength of the bottom layer increases relative to the top layer. In other words, using the punching shear mechanism of Meyerhof (1974), which considers vertical failure planes in sand, results in overestimating the contribution of the bottom clay to ultimate bearing capacity of the footing, as clay becomes stronger relative to the sand. As such, further investigation was carried out to investigate the key parameters controlling the failure mechanisms and the bearing capacity of footing on layered soils.

3-1-2 CHAPTER 5

A useful aspect of the FELA method is the capability of visualising the mechanisms and quantification of their geometry from the results of upper bound analyses. Interpretation of the collapse mechanisms was a key element of this research to better understand and describe the behaviour of footings on layered soils for different ranges of material parameters and problem geometries. However, identifying the collapse mechanisms with accuracy is not always straightforward and requires some experience and judgment. New techniques that were developed
during this research to interpret the failure mechanisms from the results of upper bound solutions are presented in Chapter 5. Velocities computed at each node of the finite element mesh may be extracted from the results of upper bound FELA, from which mechanisms are inferred. However, the interpretation of those plots is subjective and important aspects of the mechanism can be easily overlooked. Furthermore, velocity plots for very fine meshes are uninterpretable due to the density of the vectors. The problem is exacerbated when using adaptive meshing techniques. Power dissipation intensity can also be contoured and used to interpret collapse mechanisms as areas of high power dissipation intensity correspond to shear planes. For problems involving multiple materials, the magnitude of power dissipation can vary significantly between materials undergoing plastic deformation and as such contour plots will typically fail to clearly show failure planes in weaker materials. The new techniques, developed during this research include plotting of scalar values derived from the velocity field such as the components of velocity, velocity magnitude and slope which are described in detail in Chapter 5. Contouring of power dissipation intensity is also discussed and how contour plots can be refined and adapted for problems involving more than one material. These techniques include the normalization of the power dissipation and the contouring of each material using a different set of contour intervals to highlight zones of plasticity. These techniques were employed to accurately quantify the geometry of failure mechanisms corresponding to the collapse load of footing on two-layered soil.

3-1-3 CHAPTER 6

Chapter 6 presents the development of a new bearing capacity model for surface strip footings on sand overlying clay. The ultimate bearing capacity of the footing was resolved into contributions of the top sand and bottom clay, using the interpreted mechanisms from FELA analyses. This allowed an accurate estimation of the contribution of the resistance provided by each layer. A significant finding of this stage of the research was that the effective footing width (the width of a virtual footing at the interface of the two layers, over which a general shear failure occurs in bottom clay layer) varies as a function of dimensionless groups of parameters $c_u/\gamma H$ and $\phi$ and can be greater, equal or smaller than the actual footing width. The observed complex shear planes in sand were simplified by approximating the actual failure planes with lines, connecting the corners of the actual and the effective footing. An expression that provides the effective footing width was derived as function of $c_u/\gamma H$ and $\phi$. Considering variable effective footing width depending on the problem parameters allows accurate estimation of the shearing resistance provided by individual layers. Based on this method accurate punching shear coefficients $(K_{sr})$ were back-calculated from numerical results that do not exhibit the unexpected trend observed for punching shear coefficients in Chapter 4 when assuming the standard punching shear failure
mechanism of Meyerhof (1974). $K_{sr}$ was found to be a function of $c/\gamma H$ and $\phi$. The above were integrated into a new bearing capacity model which utilises the derived expressions for $K_{sr}$ and $\theta$, and is based on a new bearing capacity equation that results from equilibrium of the elastic wedge under the footing. The proposed model provides results that are in close agreement with published experimental studies, and allows treating simple problems, such as the design of working platforms, without having to resort to numerical simulations. In response to the comments of the Reviewers, Chapter 6 concludes with a discussion on the effect of assuming a punching failure mechanism with vertical planes on the estimated collapse load, and certain clarifications on the notation used in this Chapter.

3-1-4 CHAPTER 7
The undrained shear strength of normally consolidated or lightly over-consolidated clays increases linearly with depth and attains very low values near the ground surface. As such, it is useful to consider the increasing shear strength profile of such subgrades in solutions for the bearing capacity of footings on layered soils. Chapter 7 presents the extension of the model presented in Chapter 6 to clayey subgrades with increasing shear strength profiles. New expressions were provided for the bearing capacity factor $N_c$ as a simple function of the dimensionless rate of increase of soil strength. The influence of the rate of increase in shear strength on the failure mechanism of footings is also discussed in this chapter.

3-1-5 CHAPTER 8
Chapter 8 presents the extension of the bearing capacity model, presented in Chapter 6 to the common case of surface footings on dense sand overlying very loose to medium dense sand. It is shown that the bearing capacity model of Hanna (1981), which is based on a punching shear failure mechanism, only provides accurate estimates of the collapse load for a limited range of material parameters i.e. when the top sand is significantly stronger than the bottom layer. A new form of failure mechanism named transitional failure mechanism was observed as the relative strength of the top layer decreases, for which Hanna’s bearing capacity model significantly overestimates the ultimate bearing capacity. It is also shown that identical failure mechanisms will dominate the bearing capacity when $\phi_1, \phi_2, \gamma_1/\gamma_2$ and $H/B$ are constant. A new method is developed to predict the dominant mode of failure based on the key parameters of the problem, and a new bearing capacity factor is introduced that predicts the ultimate bearing capacity of the footing, in case a transitional failure mechanism will develop. Results of the new bearing capacity model are compared against experimental data and the method proposed by Hanna (1981) and embraced by BRE470. It is shown that the proposed method provides accurate results for a wide range of problem parameters. In addition, it is shown that the simplified design method of
BRE470 (2004) provides conservative results in all practical cases and may lead to unnecessary thick working platforms.

Finally, Chapter 9 summarises the main outcomes of this research, and provides some recommendations for future work on this topic. A list of references is provided in Chapter 10, and the results of numerical calculations are summarised in Appendix A and B. Finally, Appendix C includes a detailed response to issues raised by the Reviewers of the thesis.
CHAPTER 4- PUNCHING SHEAR COEFFICIENTS FOR THE DESIGN OF WORKING PLATFORMS
Punching shear coefficients for the design of working platforms

S.N. Salimi Eshkevari & A.J. Abbo
Australian Research Council Centre for Geotechnical Science and Engineering, School of Engineering, University of Newcastle, Callaghan, NSW, 2308, Australia.

ABSTRACT: Working platforms provide a safe working environment for the operation of tracked plant on sites with soft clay subgrades. The ultimate bearing capacity of such platforms is determined considering a layered soil model and assumes that collapse is governed by a punching shear mode of failure. The contribution of the platform to the bearing capacity is computed using a coefficient of punching shear which reflects the mobilised shearing resistance within the granular fill from which the platform is constructed. This paper investigates the punching shear coefficients using in computing the bearing capacity of layered soils. Finite Element Limit Analysis is used to obtain upper and lower bounds on the ultimate bearing capacity of layered soil system. Punching shear coefficients are obtained by back-calculating the ultimate bearing capacity of layered soils following the punching shear model by Meyerhof. The presented coefficients predict the actual ultimate bearing capacity of working platforms for a wide range of design conditions with 5 percent error or less.

1 INTRODUCTION
Crawler cranes and other tracked plant are more frequently being utilized in heavy engineering construction and in the development of infrastructure. Sites with poor subgrades are often incapable of supporting the loads associated with such equipment and temporary working platforms are required to provide a safe working environment. The size and capacity of construction equipment has also grown with advances in technology further emphasizing the need for reliable working platforms to not only protect expensive equipment but to mitigate risk associated often with critical construction activities.

A good practice guide to the design, construction and maintenance of ground-supported working platform BR-470 has been published by Building Research Establishment (2004) to improve safety on building sites. Although the guide has been successful in providing a reliable and safe method for platform design, some users have reported unnecessarily large platform thicknesses (Corke & Gannon 2010).

In BR-470, the ultimate capacity of the platform is based upon the punching shear mechanism proposed by Meyerhof (1974) for the bearing capacity of a sand layer overlying a weak clay subgrade. This failure mechanism assumes the footing punches vertically down through the sand to impose a bearing capacity failure within the clay for a footing of the same width. This is in contrast to the load spread model of (Terzaghi & Peck 1948) in which the sand layer spreads the load and failure in the clay is that associated with a wider footing.

The contribution of the sand layer to the bearing capacity of the footing is obtained through the resistance of sand to shearing. Assuming vertical shear planes, Meyerhof (1974) incorporated shear resistance considering passive earth pressure acting on the failure planes and the average mobilised angle of shearing resistance (δ) along the planes. The mobilised angle of shearing resistance is a function of relative strength of the two layers and the peak internal friction angle of granular materials (Hanna & Meyerhof 1980). However, this dependency has been ignored by BR-470 as it adopts a mobilised friction angle of δ=2/3q,i following Meyerhof (1974).

This paper presents the results of a parametric study investigating the punching shear coefficient. The parametric study was performed using Finite Element Limit Analysis and considered a range of parameters encountered in the design of working platforms. The presented coefficients can be used to predict the actual ultimate bearing capacity of working platforms with 5 percent error or less.

2 BACKGROUND

2.1 Punching shear failure in layered soils
Meyerhof (1974) proposed a method to estimate
the ultimate bearing capacity of footings resting on a thin layer of dense sand overlying soft or very soft clay. From the results of experimental tests on layered soils, Meyerhof observed that where the thickness of dense sand is comparable to the width of footing, sand mass with an approximately pyramidal shape is punched into the soft clay as it is shown in Figure 1. In the case of general shear failure, the friction angle of the sand \( \phi \) and the undrained cohesion of the clay \( c_u \) are mobilized. Meyerhof approximated the actual curved failure planes in the upper sand layer with a vertical plane through the footing edge. It was shown by Meyerhof that the total passive earth pressure \( P_p \) inclined at an average angle \( \delta \) acting upwards on the vertical shear planes, well approximate the forces on the actual failure surface in the sand. Under this model, bearing capacity of the clay is that of a footing of equal width and as such, the ultimate bearing capacity of a strip footing resting on the surface is given as

\[
q_u = c_u N_c + \frac{\gamma H^2}{B} K_p \tan \delta
\]  

(1)

where \( N_c \) is the bearing capacity factor equal to 5.14; \( \gamma \) the unit weight of the sand; \( H \) the thickness of the sand layer; \( B \) the footing width; \( K_p \), the coefficient of passive earth pressure and \( \delta \) is the average mobilized friction angle on the assumed vertical failure plains. As a matter of convenience, Meyerhof replaced the term \((K_p \tan \delta)\) by an equivalent term of \((K_s \tan \phi)\) in which \( K_s \) is known as the Coefficient of Punching Shear.

\[
K_p \tan \delta = K_s \tan \phi
\]  

(2)

Hence, the bearing capacity of the layered system is given by

\[
q_u = c_u N_c + \frac{\gamma H^2}{B} K_s \tan \phi
\]  

(3)

Hanna & Meyerhof (1980) calculated values of \( K_s \) using limit equilibrium technique for a range of parameters and provided design charts of \( K_s \) as functions of \( \delta/\phi \), \( c_u \) and \( \phi \). The ratio of mobilised friction angle \( (\delta/\phi) \) were in turn modelled as a function of the ratio of the bearing capacity of the sand and clay layers \((q_s/q_u)\); where \( q_u \) is the ultimate bearing capacity of the footing over uniform clay and \( q_s \) is the ultimate bearing capacity of the same footing on a deep sand layer. The bearing capacity of a strip footing on a sand layer over soft clay, calculated by Equation 1 shows fairly good agreement with the results of model tests (Meyerhof 1974). However Equation 1 tends to underestimate the bearing capacity for large friction angles.

The coefficient of punching shear defined by Meyerhof is highly dependent on the mobilised friction angle of soil \( (\delta) \) on assumed failure planes. Thus the relative strength of the two layers \((q_s/q_u)\), average confining stress in the sand layer and the peak friction angle of the top layer which influence \( \delta \) can significantly affect \( K_s \). Other parameters such as undrained shear strength of the cohesive layer \((c_u)\), the footing width \((B)\), and the unit weight of sand \((\gamma)\) which contribute to the term \((q_s/q_u)\) together with the thickness of the top layer, \((H)\) that controls the average confining stress in the sand layer, are important factors that affect \( K_s \).

1.1 BR-470 Bearing Capacity

The punching shear model of Meyerhof (1974) is used as the basis for design in BR-470 with the ultimate bearing capacity calculated according to the equation 1. Design values for the term \((K_p \tan \delta)\) are specified for given friction angles of platform material for which the average mobilised friction angle \( (\delta) \) has been assumed to equal \( 2\phi/3 \) as recommended by Meyerhof. The average value of \( \delta \) was shown to be between \( \phi/2 \) to \( 3\phi/4 \) (Meyerhof 1974). Table 1 lists design values of \( K_p \tan \delta \) specified in BR-470 (refer Table A2 and Figure A4).

While BR-470 adopted the punching shear model for bearing capacity, it did not adopt Meyerhof’s coefficient of punching shear in computing the bearing capacity equation, presumably to simplify the design calculations. Table 1 lists equivalent values of the coefficient of punching shear \((K_p)\) computed using equation (2).

Table 1. BR-470 punching shear coefficients

<table>
<thead>
<tr>
<th>( \phi )</th>
<th>( K_p \tan \delta )</th>
<th>( K_s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>3.1</td>
<td>4.4</td>
</tr>
<tr>
<td>40</td>
<td>5.5</td>
<td>6.6</td>
</tr>
<tr>
<td>42.5</td>
<td>7.4</td>
<td>8.1</td>
</tr>
<tr>
<td>45</td>
<td>10.0</td>
<td>10.0</td>
</tr>
<tr>
<td>47.5</td>
<td>14.0</td>
<td>12.8</td>
</tr>
<tr>
<td>50</td>
<td>20.0</td>
<td>16.8</td>
</tr>
</tbody>
</table>

The application of BR-470 design approach is limited to subgrades with undrained shear strength greater than 20 kPa. The guideline recommends us

Figure 1. Failure mechanism of strong sand layer underlain by soft clay after Meyerhof (1974).
using more complex methods for the design of working platforms over very soft subgrades.

1.1 Finite element limit analysis

Finite Element Limit Analysis is a finite element technique based upon the limit theorems of plasticity. Unlike traditional displacement based finite element formulations, Finite Element Limit Analysis formulations, when properly implemented, provide rigorous upper and lower bound solutions that bound the true solution. The Finite Element Limit Analysis formulations implemented in this study are based on the work initially developed by Sloan (1988), further developed by Lyamin & Sloan (2002) and later Krabbenhoft et al. (2005) and Krabbenhoff et al. (2007) are implemented using the adaptive re-meshing techniques of Sloan et al. (2013). Key features of these methods include the use of linear finite elements to model the stress or velocity fields, and collapsed solid elements at all inter-element boundaries to simulate stress or velocity discontinuities. The solutions from the lower bound formulation always yield a statically admissible stress field, while those from the upper bound formulation result in a kinematically admissible velocity field. This ensures that the solutions preserve the important bounding properties of the limit theorems. Furthermore, adaptive meshing allows improved bounds to be obtained using less computation power.

2 METHODOLOGY

The bearing capacity of a footing on a layered soil is found by application of Finite Element Limit Analysis to compute upper and lower bound estimates of the true bearing capacity. These rigorous bounds, which bracket the true bearing capacity of the footing, are computed using adaptive meshing techniques. The target solution accuracy and the refinement of the mesh have been set so that the computed bounds are within 5% of the true solution.

The soil is modeled as an associated Mohr-Coulomb material. The range of soil properties and geometries considered in the parametric study are summarised in Table 2 which reflect those considered in BR-470. The unit weights are not independent variables in the study and are correlated to the friction angle (Bolton 1986).

The coefficients of punching shear are back-calculated from the bearing capacity using Equation 3. The average of the upper and lower bounds on the bearing capacity was used in the back-calculation. Salgado et al. (2004) showed the upper and lower bounds of the ultimate bearing capacity of strip footing converge at a similar rate as the number of elements in the finite element mesh is increased. As such, the average bearing capacity may be considered as providing an accurate estimate of the true bearing capacity with an error of less than 5%.

<table>
<thead>
<tr>
<th>Property</th>
<th>Range</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Friction angle (φ)</td>
<td>45°-55°</td>
<td>45°, 47.5°, 50°, 55°</td>
</tr>
<tr>
<td>Undrained shear</td>
<td>10 - 70 (kPa)</td>
<td>10, 15, 20, 25, 30, 35, 40, 50, 60, 70</td>
</tr>
<tr>
<td>Sand thickness (H)</td>
<td>0.4 - 1.6 (m)</td>
<td>0.4, 0.8, 1.2, 1.6</td>
</tr>
<tr>
<td>Unit weight</td>
<td>17.5 - 20 (kN/m³)</td>
<td>17.5, 18.1, 18.7, 20.0</td>
</tr>
</tbody>
</table>

3 RESULTS

3.1 Ultimate bearing capacity

Finite Element Limit Analysis results for a strip footing of width 0.6 m sitting on sand layer that is 0.8 m thick are plotted in Figure 2. The upper and lower bound estimates of the ultimate bearing capacity are presented for sand layers with friction angles (φ) of 47.5° and 50°. The plots show the bearing capacity of the footing has an almost linear variation with the undrained shear strength (cu) of the clay sub-grade. The upper and lower results are in good agreement and provide tight bounds on the true collapse load. Figure 2 also compares results of Finite Element Limit Analysis to the ultimate bearing capacities calculated in accordance with BR-470. For the case in which φ= 47.5°, Figure 2a shows that BR-470 over-predicts the bearing capacity when the undrained shear strength of the clay is less than approximately 55 kPa. The over-prediction of the bearing capacity by BR-470 is more evident for platforms with higher friction angles. For a sand layer with a friction angle of 50°, Figure 2b clearly shows that bearing capacities calculated to BR-470 significantly over-estimate the bearing capacity of the layer system across the range of shear strengths relevant to the design of working platforms.

3.2 Coefficient of Punching Shear

The coefficients of punching shear (Kp) back-calculated from the finite element bearing capacities are plotted in Figure 3 for footing with B= 0.6 m and H= 0.8 m. Consistent with previous research showing Kp varies with the ratio of bearing capacities (q/qc), the figure shows Kp, as a function of the undrained shears strength (cu) of the clay and friction angle (φ) of the sand layer. Figure 3 compares the back-calculated values of Kp to the equivalent shear coefficients (refer Table 1) based upon design to BR-470. While BR-470 is only applicable for cu greater than 20 kPa, it is clear that the simplified design methodology provides a poor estimate of the contribution of punching shear to bearing capacity.
The results of the parametric analysis show that the coefficient of punching shear varies very slightly by changing the footing width. Figure 4 plots coefficients of punching shear for cases in which $\phi = 50^\circ$. The figure clearly shows that coefficients back-calculated for footing widths of $B=0.6$ m and 1.8 m are for all practical purposes equal.

Results of the entire parametric study are shown in Figure 5, which plots the coefficients of punching shear determined from the Finite Element Limit Analysis predictions of the bearing capacity. The coefficients have been computed using the average of the upper and lower bounds predictions of bearing capacity. As these bounds typically brackets the true solution with a relative error of less than 5%, bearing capacities computed using the coefficients of punching shear from Figure 5 are accurate to within a 5%.
Figure 6. (a) Power dissipation intensity and (b) velocity plot for $\phi = 50^\circ$, $c_u/\gamma H = 2.0$ (B = 0.6 m, H = 0.8 m, $c_u$ = 30 kPa).

Figure 7. (a) Power dissipation intensity and (b) velocity plot $\phi = 45^\circ$, $c_u/\gamma H = 7.14$ (B = 0.6 m, H = 0.4 m, $c_u$ = 50 kPa).

Figure 5, shows for friction angles of $50^\circ$, $47.5^\circ$ and $55^\circ$ that the coefficients of punching shear increase with increasing $c_u/\gamma H$. In these cases, collapse of the footing resembles the punching shear mechanism of Meyerhof (1974) with the block of soil directly beneath the footing moving vertically downwards. Figure 6 shows the power dissipation intensity and the velocities for a footing with $\phi = 50^\circ$ and $c_u/\gamma H = 2.0$ which clearly shows the punching failure mechanism with shear planes extending close to vertical below the edges of the footing. For $\phi = 45^\circ$, Figure 5 shows that coefficients of punching shear do not increase monotonically, instead reaching a maximum value at $c_u/\gamma H = 5$. In this case, the footing collapse is associated with a punching shear failure mechanism that involves shear planes that angle back under the footing. This is clearly shown in Figure 7 for the footing with $\phi = 45^\circ$ and $c_u/\gamma H = 7.14$.

The results presented in Figure 5 are based on the assumption that the materials used in the model follow the associative flow rule (normality rule) that is not true for the sand in the top layer. The influence of using frictional soils in Limit Analysis on the limit loads has been discussed initially by (Davis 1968). It was concluded by Davis (1968) that such influence is not major unless the problem is strongly constrained in a kinematic sense. However, most geotechnical collapse loads are not strongly constrained as they involve freely deforming ground surface and a semi-infinite domain Sloan (2013). This condition also applies to the existing problem. Davis (1968) has concluded that in such cases the Limit bounds can be considered as an acceptable estimation of the true limit loads (Sloan 2013).

1 SUMMARY AND CONCLUSION

The ultimate bearing capacity of temporary working platforms has been investigated using Finite Element Limit Analysis. A parametric study was conducted that considered a range of soil parameters and geometry typically encountered in the design of
working platforms. Following the punching shear model proposed by Meyerhof (1974) for the bearing capacity of layered soils, the coefficient of punching shear was back-calculated from the limit analysis results.

Results show that for the range of parameters analyzed, the coefficient of punching shear is not dependent upon the footing width (B), at least for practical purposes, but is instead a function of the dimensionless parameter \((c_u/H)\). Most previous studies have not recognized this dependency. Burd & Frydman (1997) identified that the bearing capacity of a layered soil is a function of \((c_u/H)\) in considering a load spread model for bearing capacity of layered soils.

Finite Element Limit Analysis provided rigorous bounds that bracketed the true bearing capacity of layered soils to within 5%. Consequently, back-calculated values of the coefficient of punching shear can be used to accurately predict bearing capacity with less than 5% error. The results are summarised as a chart plotting the coefficient of punching shear as a function of friction angle of the sand, undrained cohesion of the clay and depth of the sand layer.

The punching shear model of Meyerhof (1974) assumes a simple mechanism in which the footing punches vertically through the sand layer. Results of Finite Element Limit Analysis showing the upper bound failure mechanisms are not presented in this paper. However, these show that the shear planes beneath the footing drift from vertical encroaching under the footing. Further research to investigate the mechanisms associated with the bearing capacity of the layered soils is being conducted.

Finally, the coefficients of punching shear adopted in the BR-470 guideline for the design of working platforms do not adequately model the variation of the coefficient. Results of Finite Element Limit Analysis showed that BR-470 over-predicts ultimate bearing capacities for sites with very soft clay subgrades. Also due to its simplistic nature, the coefficients provided in BR-470 may under-estimate the ultimate bearing capacity in other design conditions especially in lower sand friction angles and stiffer subgrades.

REFERENCES


CHAPTER 5- GRAPHICAL INTERPRETATION OF UPPER BOUND FINITE ELEMENT LIMIT ANALYSIS RESULTS
Graphical Interpretation of Upper Bound Finite Element Limit Analysis Results

Daniel W. Wilson*, Andrew J. Abbo*, S Salimi Eshkevari* and Scott W. Sloan*

*Australian Research Council Centre of Excellence for Geotechnical Science and Engineering
School of Engineering, University of Newcastle, Callaghan, NSW 2308, Australia
*Daniel.W.Wilson@newcastle.edu.au (corresponding author’s E-mail)

Abstract

Finite Element Limit Analysis (FELA) is used by geotechnical engineers to predict the collapse loads of structures. While the collapse load is often of primary interest, results of analyses should be studied to interpret the governing failure mechanisms. In upper bound FELA, velocities computed at each node of the finite elements can be plotted and mechanisms inferred from these vector fields. However, the interpretation of such plots is subjective and important aspects of the mechanism can be easily overlooked. Furthermore, velocity plots for very fine meshes are hard to interpret due to the density of the vectors, a problem exacerbated by the use of adaptive meshing techniques. Power dissipation intensity can also be contoured and used to interpret collapse mechanisms as areas of high power dissipation intensity correspond to shear planes. For problems involving multiple types of material, the magnitude of power dissipation can vary significantly between materials undergoing plastic deformation and thus contour plots will typically fail to clearly show failure planes in weaker materials.

This paper focuses on techniques for the graphical interpretation of upper bound FELA results that can be applied to enrich the interpretation of collapse mechanisms. These techniques include plotting of scalar values derived from the velocity field such as the components of velocity, velocity magnitude and slope. The contouring of power dissipation intensity is also discussed as well as how contour plots can be normalized for problems involving more than one material. Results are presented for the analysis of footings and tunnels to demonstrate these techniques.

Keywords: Finite Element, Limit Analysis

1. Introduction

The upper and lower bound theorems of plasticity are powerful tools when applied to geotechnical stability problems. Finite element analysis methods based upon the upper bound theorems are particularly useful as they provide velocities which can be visualized and interpreted to gain an understanding of the critical failure mechanism. With advances in computer speed, finite element meshes have become increasingly finer, especially when adaptive meshing techniques are employed. With fine meshes, it is necessary to adapt graphical techniques for the visualization of upper bound results so that they can be presented clearly, concisely and in a form that can be readily interpreted.

Early versions of FELA were limited to a low mesh density due to the computational power and less efficient optimization algorithms available at the time. For example, Sloan and Assadi [1] considered the collapsed of tunnels and illustrated failure mechanisms using vector plots of the velocity. For the coarse meshes used in this research, the vector plots were generally not very dense and individual velocity vectors can readily be distinguished. Later research included plots of power dissipation [2] which highlighted areas of plastic deformation due to the large size of the elements and power dissipation being constant within each element. While shear planes and mechanisms can be inferred from regions of plasticity, the localized nature of the failure planes are not captured in the analyses and the results provide only qualitative insight of collapse mechanisms.

At present, the challenges of displaying finite element results are distinctly different when high density meshes are used. These issues are exaggerated by the use of adaptive meshing that can generate tiny elements and greatly increase the mesh density in regions which are undergoing plastic deformation.
regular layout of the mesh can sometimes be misleading as the location of nodes can provide patterns and visual artifacts making it difficult for the human eye to interpret flow paths. An example of a plot showing power dissipation and velocities is shown in Fig. 1 for the analysis of a strip footing using adaptive meshing. The plot shows the power dissipation intensity (left) and the velocity field (right) with both plots clearly showing the expected Prandtl mechanism. The power dissipation plot in Fig. 1 was generated using approximately 20,000 elements, while the velocity plot was generated using approximately 1000 elements. This plot is typical of those currently presented in publications that have used FELA to study the collapse of structures. With the relatively coarse mesh used for displaying the velocities, the relatively simple collapse mechanism can be easily interpreted from the plot. Depending on the complexity of the problem, velocity plots can become very difficult to clearly visualize when the number of elements in the mesh exceeds around 5000 elements.

![Fig. 1. Power dissipation intensity (left) and velocity plot (right) for a strip footing in a clay.](image)

As noted above, the simplest method for effectively displaying velocity fields is to repeat the analysis with a much coarser finite element mesh and/or reduce the mesh refinement provided by adaptive meshing techniques. This is the current practice and has been used effectively in a number of papers, such as Wilson et al.[3] and Abbo et al. [4]. Figure 2 shows an example of this technique for a circular tunnel in a purely cohesive soil using structured meshing. The left side of the figure shows the power dissipation intensity and the right side of the figure shows the velocity field. While this technique helps to display the velocities, the vector plot is still subject to artifacts caused by the layout and different densities of the mesh. At first glance, the vector plot appears to show a velocity discontinuity over the outside edge of the tunnel. However it is the magnitude and direction of the velocities that needs to be observed rather than the layout of the vectors. The power dissipation intensity plot shows that the shear plane is actually much further way from the axis of symmetry. These visual artifacts are produced by the patterns, often associated with parametric mesh generation of domains within the problem, within the finite element mesh. These artifacts can occur regardless of whether or not an adaptive meshing technique is used.
Fig. 2. Plane strain analysis of a circular tunnel in a purely cohesive soil showing power dissipation intensity and the velocity field.

Advances in computer speeds and FELA solution algorithms have also led to the analysis of more complex problems, including investigations with complex loading scenarios and multiple materials. This creates a problem when trying to display the upper bound power dissipation if the cohesion of the soils is vastly different, such as a very strong clay over-laying a very weak clay or a sand over a clay. At collapse, the power dissipation in each layer can be an order of magnitude different, so that it becomes difficult to plot the power dissipation in such a way as to see what is happening in both layers. In the past, the authors have contoured each material independently and used these to generate a combined plot that allows the failure mechanism to be better visualized.

This paper discusses the graphic presentation of upper bound FELA results and presents techniques that are suitable for problems with fine finite element meshes and/or multiple materials. The techniques include the plotting of scalar values that can be derived from the velocity field, such as velocity magnitude and slope, and contouring of normalized power dissipation.

2. Finite Element Limit Analysis

The upper bound theorem is based on the notion that the imposed loads cannot be carried by the soil mass if, for any kinematically admissible velocity field, the rate of work done by the external forces exceeds the internal rate of dissipation [5]. A kinematically admissible velocity field is one which satisfies the plastic flow rule and the velocity boundary conditions. Similar to displacement-based finite elements, the results of the upper bound analysis are nodal velocities which represent the direction, and relative magnitude, in which each node is moving at failure.

Although there are a number of FELA implementations, this paper focuses on the methods originally developed by Sloan [6]-[7]. These techniques have evolved significantly over the past two decades and the current implementation includes the improvements by Lyamin and Sloan [8]-[9] and Krabbenhoft et al. [10]-[11] and the adaptive re-meshing techniques described by Sloan [12]. Key features of these methods include the use of linear finite elements to model the stress or velocity fields, and solid elements with zero thickness at all inter-element boundaries to simulate stress or velocity discontinuities. Unlike displacement-based formulations, these limit analysis models may legitimately have two or more nodes at the same geometric location. When visualizing the nodal velocities, this creates a plot in which there will be two different velocities at the same point.
3. Interpretation of FELA results

3.1 Velocity Plots

Vector plots of the nodal velocities provide a convenient mean of displaying the results and visualizing upper bound collapse mechanisms. As discussed above, high mesh densities and multiple nodal points at the same geometric location produce plots in which the velocity vectors are so dense that they obscure one another and make interpretation of the plot difficult. Contour plots of scalar values derived from the velocity field, such as the velocity components, velocity magnitude and slope are convenient as these types of plots are suitable for dense meshes. While each of the scalar values provides a limited representation of the failure mechanism as compared to a vector plot, each of the plots for scalar values is valuable and can provide additional insight into the nature of the failure mechanism. This is particularly true when seeking to determine the physical size and geometry of the mechanism.

Contour plots of the horizontal and vertical components of the nodal velocities are generally very easy to generate and can be quite useful in determining the geometry of the mechanism. For example, such plots can be inspected to determine blocks of soil moving either directly up or down, or to determine the center line about which a circular wedge is rotating. For example, Fig. 3 shows plots of the horizontal and vertical velocity components for a strip footing on a layered soil. The soil profile consists of a sand layer resting on a deep clay base. Fig. 3(a) displays contours of the horizontal velocities, all of which are positive due to symmetry, with red being the highest and dark blue being zero. The dark blue area under the footing represents a block of sand punching vertically downwards into the clay with a wedge of clay moving vertically downwards directly below. Fig. 3(b) plots contours of the vertical component of velocity. In this figure, dark blue represents the greatest downwards movement (negative), and red/yellow is upward movement (positive) while green areas have zero vertical velocity. This plot is useful as it confirms that the majority of the movement in the sand layer is vertical under the footing, and shows punching shear failure in a layered soil. This can be difficult to see from a vector plot (due to mesh density and multiple velocities at the same node) or a power dissipation intensity plot (due to the very large difference in the power dissipation between the two layers. It should be noted that the optimal contour levels are problem specific, and usually require a small amount of trial and error in order to obtain a clear visualization.

The magnitude of the velocities is typically conveyed by the length of the vectors as shown in Figure 1, 2 and 4. The velocities for the footing on the layered soils are plotted in Figure 4 as both a vector plot and as a contour plot of the velocity magnitude. As shown on the right-hand side of this figure, contouring the velocity magnitude displays the size and extent of the mechanism quite clearly. In this plot, the largest magnitudes occur near the edge of the footing are shown in red graduating to blue where the velocity is zero. The red velocities immediately adjacent to the footing are quite localized and can be generally disregarded due to the sharp change in velocities imposed at the edge of the footing. Of more significance are the regions shown in green which represent the larger displacements of the global failure mechanism. Again, the block moving vertically down under the footing can quite clearly be identified. In addition, the graduation from blue near the footing edge to green at the bottom of the mechanism show that the magnitude of velocities increases radially from a point located somewhere near the footing edge. This change in velocity magnitude is consistent with a rotational mechanism for which velocities would grow from zero at the center of rotation to a maximum value at the outside of a block or wedge that is undergoing rotational deformation.
Fig. 3. Horizontal (a) and vertical (b) velocity components for a strip footing resting on a sand layer overlaying clay.

Fig. 4. Vector plot (left) and contour plot of velocity magnitude (right) for a strip footing resting on a sand layer overlaying clay.
conjunction with plots of the velocity magnitude, these blocks can be used to interpret the failure mechanism. Fig. 5. shows a velocity gradient plot for a strip footing on a sand over clay layered soil. A change in direction of the velocities can be seen in Fig. 5, where the velocity gradients all “bend” at the boundary between the layers. The rigid block mechanism of Michalowski et al. [13] is overlaid on the velocity gradient plot. In this way, the velocity gradient plot can be used to create accurate rigid block mechanisms and reinforce the assumed shape and “fan” nature of the mechanism. The process of creating accurate rigid block mechanisms to confirm FELA results also contributes to a better understanding of exactly what is happening in the soil.

Fig. 5. Contour plot of velocity gradient for a strip footing resting on a sand layer overlying clay.

Fig 6. Shows the velocity vectors and velocity gradients for a single circular tunnel in a purely cohesive soil. This problem was examined extensively by Sloan et al. [14] and Wilson et al. [15] using both FELA and analytical rigid block upper bounds. The gradients show the failure mode of the tunnel, which encompasses all sides of the tunnels as a series of blocks. Each contour shows regions where the velocities are constant and provides a clear view of the size and shape of the failure. These contours show a mechanism very similar to a rigid block mechanism, and an increase in the number of contour levels will yield a more and more complex rigid block equivalent. Crumpton et al. [16] have performed a rigid block analysis with a similar mechanism. This velocity gradient plot confirms the mechanism consists of many rigid blocks as an approximation to a continuously deforming region.
The contour plot of the velocity gradient presented in Fig. 6 suggests a more accurate rigid block upper bound mechanism that has never been considered before. For many problems, contouring the velocity gradient is extremely useful for visualizing the collapse mechanism.

3.2 Power Dissipation Plots

Contour plots of power dissipation intensity, as shown in Figs. 1 and 2, provide a useful technique for interpreting the failure mechanism associated with the collapse of a structure. For problems involving two or more materials with large differences in the strength, contour plots calibrated to the entire domain often do not effectively display the mechanism clearly. As the power dissipation is proportional to the soil cohesion, the power dissipated in a weak soil at failure will generally be less than in a stronger material. In clay soils, a useful technique is to contour the normalized power dissipation intensity by dividing by the cohesion. Fig. 7 displays power dissipation intensity plots for a strip footing on a strong clay overlaying a weak clay. The left side of Fig. 7, which plots the power dissipation intensity directly, does not show the mechanism clearly. Conversely, the right-hand side of Fig. 7, which plots the normalized power dissipation intensity, displays the extent of the failure mechanism more clearly.
4. Conclusions

Techniques have been presented that improve the interpretation of collapse mechanisms from upper bound FELA results. The interpretation of these results can be subjective and important aspects of the mechanism can be easily overlooked. Furthermore, vector plots of nodal velocities for very fine meshes become hard to interpret due to the density of the vectors, a problem exacerbated by the use of adaptive meshing techniques. A number of techniques have been presented that assist in the interpretation and presentation of FELA results. The application of these techniques is demonstrated for a variety of problems involving the bearing capacity of footings and the collapse of tunnels. These methods include plotting of scalar values derived from the velocity field, such as the components of velocity, velocity magnitude and gradient. The contouring of power dissipation is also discussed in relation to problems involving more than one material, and the use of normalized power dissipation intensity is introduced for problems involving cohesive materials only.

Acknowledgements

The research reported in this paper was made possible by the Australian Laureate Fellowship grant FL0992039 awarded to Professor Scott Sloan by the Australian Research Council. The Authors are grateful for this support.

References

CHAPTER 6- BEARING CAPACITY OF STRIP FOOTINGS ON SAND OVER CLAY
Bearing capacity of strip footings on sand over clay

Seyednima Salimi Eshkevari, Andrew J. Abbo and George Kouretzis

Australian Research Council Centre for Geotechnical Science and Engineering
School of Engineering, University of Newcastle, Callaghan, NSW 2308, Australia

Corresponding Author: Seyednima Salimi Eshkevari
The University of Newcastle, Australia
School of Engineering
University Drive
Callaghan NSW 2308
Tel.: (02) 49215582
Fax: (02) 49216691
Email: c3158338@uon.edu.au
Abstract

Estimation of the bearing capacity of shallow foundations on layered soil profiles, such as a sand layer of finite thickness over clay, is mainly based on empirical models resulting from the interpretation of experimental test results. While it is generally accepted that such models may be applicable to soil properties and footing geometries outside the range tested experimentally, they offer limited insights on how the assumed failure mechanism affects their range of application. In particular, the contribution of the sand layer to the overall capacity is accounted for via simple considerations, which are valid only for a specific range of problem parameters. This paper addresses the estimation of the undrained bearing capacity of a rigid strip footing resting on the surface of a sand layer of finite thickness overlying clay, using finite element limit analysis (FELA). The rigorous upper and lower bound theorems of plasticity are employed to bracket the true bearing capacity of the footing and identify the geometry of possible failure mechanisms. Insights gained from FELA simulations are used to develop a new simple bearing capacity model, which captures the variation in shear resistance from the sand layer with the dimensionless undrained strength of the clay layer. The proposed model provides results that are in close agreement with published experimental studies, and allows treating simple problems, such as the design of working platforms, without having to resort to numerical simulations.

Keywords: Bearing capacity; Layered soils; Strip footings; Finite element limit analysis; Working platforms
Introduction

Geotechnical engineers are frequently challenged with the design of shallow foundations on layered soil profiles. Such profiles are not only formed as a result of natural processes (such as the deposition of an alluvial crust on a soft clay layer) but may also be man-made: Ground improvement layers and temporary working platforms are among the most common solutions for safely transferring loads from shallow foundations to soft clays via a competent coarse-grained layer. While simple methods for calculating settlement of layered soils are found in almost every geotechnical engineering textbook, exact plasticity solutions for estimating the bearing capacity of shallow foundations are limited to the case of homogenous and isotropic subsoil. For layered profiles, geotechnical engineers have to resort to either numerical methods, or to approximate bearing capacity models.

Early methods for estimating the bearing capacity of footings on sand over clay, pertinent to the design of working platforms or improvement layers in general, can be broadly classified as either following the projected area model of Terzaghi (1948) or the punching shear model of Meyerhof (1974). Terzaghi’s projected area model, depicted in Fig. 1, is based on the assumption that the sand layer distributes the pressure $q_u$ applied on its surface uniformly, to a hypothetical, equivalent footing with effective width $B'$ resting on the top of the clay layer. The capacity of the layered system is assumed equal to the bearing capacity of a footing with width $B'$, embedded in uniform clay. This (conservatively) ignores any shear resistance provided by the sand.

The bearing capacity is calculated according to the projected area model as:

$$q_u = N_c c_u \left( \frac{B'}{B} \right)$$

where $c_u$ is the undrained shear strength of the clay layer, assumed to be constant with depth, and $N_c$ is the bearing capacity factor $N_c = 5.14$ for strip footings. The effective footing width $B'$ is defined by the angle $\theta$ at which the load is distributed within the sand layer, as:
(2) \[ B' = B + 2H \tan \theta \]

Terzaghi (1948) recommended \( \theta \) to be taken approximately equal to 26.6\(^\circ\) i.e. the commonly considered 2:1 (vertical: horizontal) stress distribution. Later, Jacobsen (1977) conducted tests on embedded circular footings, from which he concluded that the load spread angle \( \theta \) is a function of the relative strength of the two layers. He proposed the following Eq. (3) to estimate it, as:

(3) \[ \theta = \tan^{-1} \left( \frac{2}{\beta} \right) \]

in which:

(4) \[ \beta = 0.1125 + 0.0344 \left( \frac{q_s}{q_c} \right) \]

while

(5) \[ q_s = 0.5 \gamma BN_s \gamma S_y + \gamma DN_s S_q \]

(6) \[ q_c = c_u N_c S_c \]

are the bearing capacity of an equivalent footing on the surface of the sand and of the clay layer, respectively. In the above expressions \( \gamma \) is the unit weight of sand, \( N_s, N_q \) and \( N_c \) are the bearing capacity factors, \( S_y, S_q, S_c \) are the shape factors and \( D \) is the embedment depth. Kenny and Andrawes (1997) also conducted physical model tests to assess the validity of various bearing capacity models for sand over clay profiles, including the projected area model. In order to determine the inclination of shear planes in the sand layer (equivalent to the load spread angle \( \theta \) when the shear resistance from sand is considered), Kenny and Andrawes used measurements of displacement fields obtained during their tests. Contrary to Terzaghi’s assumption that the angle at which the pressure is distributed within the sand layer is constant, Kenny and Andrawes (1997) observed that the inclination of the shear planes varied with the dimensionless sand thickness \( H/B \). The inclination ranged from \( \theta = 5.7^\circ \) for a thin layer of sand to nearly \( \theta = 20^\circ \) for a relatively deep, compared to the width of the footing, sand layer. Burd and Frydman (1997) approached the problem
of stress distribution within the sand layer numerically, and determined the inclination of the shear planes from results of numerical analyses. They concluded that $\theta$ is a function of the dimensionless shear strength of the clay layer, $c_u/yH$ and of the friction angle of the sand layer. This agrees with the findings of Craig and Chua (1990), who first reported that the geometry of the failure mechanism depends on the dimensionless shear strength $c_u/yH$. Burd and Frydman (1997) also suggest that the inclination of the shear planes is not sensitive to the dimensionless thickness $H/B$, but rather only on $c_u/yH$.

A different approach, inspired by experimental results, was proposed by Meyerhof (1974). He described the footing as a rigid die, punching the block of sand immediately below it downwards into the clay, hence this model is referred to as the punching shear model. The punching shear mechanism, depicted in Fig. 2, provides the bearing capacity as the sum of the shear resistance developing along the assumed vertical failure planes in sand, and of the bearing capacity of a footing embedded in the bottom clay layer.

This approximate failure mechanism formed the basis of the limit equilibrium solution developed by Meyerhof to estimate the shear resistance provided by the sand layer. The distribution of stresses along the vertical slip surfaces results from considering the passive earth pressure acting on a vertical retaining wall with interface friction angle $\delta$. As such, the direction of normal stresses forms an angle $\delta$ with the vertical planes (Fig. 2), where $\delta$ is the mobilised friction angle of sand. As a result of the above, the total bearing capacity of the footing is given as:

$$q_u = c_u N_c + \gamma H^2 K_p \tan \delta \leq q_s$$

in which $K_p$ is the coefficient of mobilised earth pressure, which depends on the mobilised friction angle, $\delta$. Meyerhof (1974) mentioned that $\delta$ varies between $\phi'/2$ and $3\phi'/4$ and recommended using an average $\delta = 2\phi'/3$ in practical applications. In addition, Meyerhof (1974) applied the solution of Caquot and Kerisel (1948), to estimate the coefficient of passive earth pressure $K_p$ behind a vertical wall retaining soil with peak friction angle $\phi'$ when the soil-wall interface friction angle is equal to $\delta = 2\phi'/3$. Accordingly, he
introduced the punching shear coefficient $K_s$ to re-write the bearing capacity equation in a simpler form, where the mobilised friction angle is eliminated by setting:

$$K_s \tan \phi' = K_p \tan \delta$$  \hspace{1cm} (8)

Substitution of Eq. (8) into Eq. (7) results in following expression for the bearing capacity of a footing on sand over clay:

$$q_u = c_u N_c + \frac{\gamma H^2}{B} K_s \tan \phi' \leq q_s$$  \hspace{1cm} (9)

As discussed later by Hanna and Meyerhof (1980), the mobilised friction angle is less than the peak friction angle of sand $\phi'$ due to the influence of the weak layer on vertical displacement of the footing, and to the fact that the assumed, vertical failure planes are not the actual slip surfaces. To study the reduction in passive earth pressure due to the existence of a weak layer and accordingly estimate the mobilised friction angle, Hanna and Meyerhof hypothesised a failure mechanism and used limit equilibrium considerations to develop charts that provide the ratio $\delta/\phi'$ as function of $q_c/q_s$; where $q_c$ and $q_s$ is the bearing capacity of the same footing on a uniform layer of clay and sand, respectively. They performed parametric analyses, the results of which were compiled into charts providing $K_s$ as function of $\phi'$, $\delta/\phi'$ and of the undrained shear strength of the clay layer $c_u$. However, as identified later by Craig and Chua (1990) as well as Burd and Frydman (1997), $K_s$ is also a function of $\gamma$ and $H$, suggesting that the charts in Hanna and Meyerhof (1980) are valid only for the limited range of $H$ and $\gamma$ values considered in their parametric analyses. It should be noted here that the punching shear model was developed for the case of a dense sand over soft clay. Shiau et al. (2003) have shown that as the strength of clay layer increases relative to the sand layer, the method tends to over-predict the bearing capacity.

Okamura et al. (1998) refined Meyerhof’s punching shear model to allow considering the inclined failure planes they observed in sand during centrifuge tests (Okamura et al. 1997). The model proposed by Okamura and his co-workers is illustrated in Fig. 3 and is based on the limit equilibrium method. The inclination of the shear planes in sand is defined via the angle $\theta$ and the bearing capacity is calculated as:
The mobilised coefficient of lateral earth pressure $K_p$ in Eq. (10) resulted from measurements by Okamura et al. (1997) of the horizontal stress at locations close to the side of the sand block, at the stage where the peak load is reached. It must be noted here that these measurements do not account for the effect of the soil strength parameters on $K_p$ as the range of tested material properties was limited: The plane strain friction angle of sand used in the tests was $\phi' = 47.7^\circ$ and the undrained shear strength of clay ranged between $c_u = 21.9$ and 23.0 kPa. Subsequently, the lower bound of the measured mobilised coefficient of lateral earth pressure was adopted for use with Eq. (10). For the estimation of angle $\theta$, Okamura et al. (1998) proposed to consider soil elements A and B (Fig. 3) located above and below the sand-clay interface, at the intersection with the sand shear planes. If both soil elements have reached their failure shear stress, then the orientation of peak shear stress at A (and thus angle $\theta$) can be found from Mohr’s circle as:

$$
\theta = \tan^{-1}\left(\frac{\sigma_{mc} - \sigma_{ms}}{\cos \phi' \sin \phi' \sigma_{ms}} \frac{1 + \sin^2 \phi'}{\sigma_{ms} + 1}\right)
$$

in which $\sigma_{mc}/c_u$ and $\sigma_{ms}/c_u$ are the mean effective stress at failure developing at A and B, respectively, normalised against the undrained strength of clay. These can be calculated as:

$$
\sigma_{mc} = N_c \left(1 + \frac{\gamma H}{c_u} + \frac{\rho'_0}{c_u}\right)
$$

$$
\sigma_{ms} = \frac{\sigma_{mc}}{c_u} \sqrt{\left(\frac{\sigma_{mc}}{c_u}\right)^2 - \cos^2 \phi' \left(\frac{\sigma_{mc}}{c_u}\right)^2 + 1}
$$

where $\rho'_0$ is the surcharge at the level of the footing.

The Okamura et al. (1998) model addresses an important simplification in Meyerhof’s solution, however angle $\theta$ values predicted by Eq. (11) are not consistent with the observations reported in
a number of studies. Jacobsen and Christensen (1977), Brocklehurst (1983) and Burd and Frydman (1997) found that the angle $\theta$ in Fig. 3 increases as the relative strength of the sand increases (compared to that of the clay). In contrast, Eq. (11) results in lower $\theta$ values as $\varphi'$ increases for given $c_u$. This discrepancy stems from the fact that Okamura et al. (1998) calculate $\theta$ based only on the orientation of the failure surface at the sand-clay interface, implying that the inclination of the failure surface is constant across the whole thickness of the sand layer.

Other researchers have approached the problem at hand with analytical limit analysis methods, which provide rigorous bounds of the collapse loads. In addition, upper bound solutions provide valuable insight into failure mechanisms. Michalowski and Shi (1995) obtained upper bound estimates of the true collapse loads, considering two failure mechanisms extending into the bottom clay layer. They compiled their results in the form of dimensionless charts providing the bearing capacity and the critical depth for a broad range of material parameters. Burd and Frydman (1996) noted that the solution of Michalowski and Shi (1995) overestimates the bearing capacity in cases where the friction angle of sand is relatively high. They attribute this to the devised failure mechanisms, and the assumption of associated flow rule for sand. Huang et al. (2007) also considered a multi-rigid block mechanism to obtain upper bound capacities of rough strip footings on layered soils. In their study they used a modified version of the failure mechanism originally proposed by Florkiewicz et al. (1989) to obtain more accurate upper bounds, compared to those of Michalowski and Shi (1995).

Computational methods allow addressing some of the simplifications introduced in limit equilibrium and limit analysis methods. Griffith (1982) and later Burd and Frydman (1996) used the popular displacement finite element method to estimate the bearing capacity of footings on multi-layered soils. More recently, finite element formulations based on the bound theorems of plasticity have provided researchers with the means to accurately bracket true collapse loads (Sloan 2013). These finite element limit analysis (FELA) methods have been used extensively to study the bearing capacity of foundations (Hjiaj et al. 2004 and 2005, Lyamin et al. 2007 and 2009, Merifield et al. 1999 and 2006, Sutcliffe et al. 2004 and Yamamoto et al. 2009, 2011 and 2012). Notably, Shiau et al. (2003) used FELA to obtain upper and lower bound
solutions of the bearing capacity of a footing on sand over clay, while considering the effects of footing roughness, footing embedment depth and increase in clay shear strength with depth. More recently, Ballard et al. (2011) used the commercial software Limitstate:GEO (LSG) to study the application of combined (vertical and horizontal) loads on a shallow footing on sand over clay. LSG uses the discontinuity layout optimisation (DLO) method developed by Smith and Gilbert (2007) to determine an upper bound of the collapse load. Ballard et al. (2011) also estimated the load spread angle $\theta$ that corresponds to the upper bound limit load, and showed that it varies linearly with the logarithm of the dimensionless clay strength parameter $c_u/\gamma H$. Their results also suggest that the load spread angle decreases, and even becomes negative as $c_u/\gamma H$ increases. For sand layers with friction angle $\phi' = 40^\circ$, the estimated angles are approximately $5^\circ$ to $10^\circ$ less than those computed by Burd and Frydman (1996).

More recently, Salimi and Abbo (2015) used FELA to estimate the punching shear coefficient ($K_s$) of Meyerhof (1974). They showed that $K_s$ is independent of the footing width, and is a function of the dimensionless strength parameter $c_u/\gamma H$ and of the friction angle of sand $\phi'$. While the calculated $K_s$ values were consistent with Meyerhof’s punching shear model, Salimi and Abbo (2015) showed that the shear planes in the sand are inclined, as in Fig. 3. However, depending on the problem parameters, angle $\theta$ may attain positive but also negative values, extending outwards or inwards, respectively. This is consistent with the findings of Ballard et al. (2011), who showed that the slip surfaces may be inclined inwards when the shear strength of sand is low relatively to that of the clay layer. While values of the punching shear coefficient provided by Salimi and Abbo (2015) can be used to estimate the bearing capacity of a shallow footing on sand over clay, further work on the topic by the authors demonstrated that the contribution of each individual layer to the bearing capacity cannot be uncoupled, an assumption inherited in all bearing capacity models.

In this study, which builds upon the work of Salimi and Abbo (2015), we use FELA to investigate how the geometry of the failure mechanism changes, depending on the problem parameters, and how this affects the bearing capacity of a strip footing on sand over clay. The geometry of the corresponding mechanisms
is used to estimate the width of the equivalent footing on clay first introduced by Terzaghi, via the inclination of shear planes in sand, $\theta$. We will show that the width of this equivalent footing may be greater than, or less than that of the actual footing, depending on the relative strength of the two soil layers, and is independent of the dimensionless thickness $H/B$. Results of FELA analyses are synthesised to propose a general bearing capacity model, that unifies Terzaghi’s and Meyerhofer’s solutions, and is applicable to a wide range of material parameters and problem geometries. The proposed model is capable of accurately capturing experimental data, while remaining simple and efficient enough to be used in common practical applications, such as the design of working platforms.

**Computational analysis background**

Many classical solutions in geotechnical engineering are based on limit analysis and the bound theorems of plasticity. However, as these solutions have traditionally been obtained analytically, their application is limited to relatively simple problems. Implementation of the limit analysis method within the finite element framework (FELA), used in this study, has enabled addressing problems with complex geometries and heterogeneous soil conditions.

Available FELA methods are based on both upper and lower bound theorems. As such, they may be used to bracket limit loads within rigorous upper and lower bounds. These bounds provide a direct measure of the error in estimating the true failure load. The lower bound FELA method seeks to maximise a set of applied stresses/tractors such that the resulting admissible stress field satisfies equilibrium, fulfils the stress boundary conditions and does not violate the yield criteria at any location. The maximised stresses/tractors provide a safe estimate of the loads that can be applied without exceeding the actual collapse load of the structure. Conversely, the upper bound FELA method is based on a kinematic formulation, requiring an admissible velocity field to be constructed. According to the upper bound theorem, if a kinematically admissible velocity field is found, for which the rate of work of the external forces exceeds the rate of the internal work, the structure will collapse.
Apart from bounds of the true collapse load, upper bound FELA results can also be used to identify the geometry of the collapse mechanism, from the kinematically admissible velocity field. Furthermore, the distribution of power dissipation intensity within the soil can also be used to visualise areas in which plastic deformation is accumulating, which can then be used to infer the location and shape of the failure mechanism. Unlike traditional finite element analysis methods that require solving large sets of linear equations, the FELA formulation results in a large set of inequalities, which are solved efficiently using nonlinear optimisation techniques.

The FELA software used in this study is based upon the work of Sloan (1988), which as subsequently refined by Lyamin and Sloan (2002a,b) as well as Krabbenhoft et al. (2005, 2007). Adaptive remeshing (Lyamin et al. 2013) is employed to obtain rigorous upper and lower bounds that closely bracket the true collapse load.

**Failure mechanisms**

FELA was employed in this study parametrically to identify the failure mechanisms governing the short-term bearing capacity of a strip footing on sand over clay. Total stress analyses are performed, with sand and clay modelled as Mohr-Coulomb materials, assuming that both obey the associated flow rule, while the footing is assumed to be rigid and rough. The geometry of the failure mechanism was interpreted from power dissipation contours corresponding to the stage where the ultimate footing capacity is reached. The power dissipation $D_s$ within the soil layers is defined for an infinitesimal volume of soil $dv$ as:

$$D_s = \int_v (\sigma_s d\varepsilon_s^p) dv$$

where $\sigma_s$ and $\varepsilon_s^p$ are the deviatoric stress and plastic strain, respectively. Mesh regions with non-zero power dissipation intensities correspond to the locations of shear planes within the soil mass. These in turn define blocks with rigid body movement, from which the geometry of the failure mechanism can be inferred.

Fig. 4 presents contours of power dissipation intensity corresponding to loading of a rigid strip footing on dense sand over soft clay (Fig. 4a) and on dense sand over stiff clay (Fig. 4b) up to its ultimate capacity.
Notice that a complex mechanism is formed within the sand layer, consisting of two wedges, instead of one assumed in previous studies. The width of the equivalent footing $B'$ may be greater than the width of the actual footing $B$, as hypothesised by Terzaghi (1948) and Okamura et al. (1988), or less than the width of the actual footing, as observed by Ballard et al. (2011). The ratio $B'/B$ depends on the friction angle of the sand layer $\phi'$ and the dimensionless shear strength of clay $c_u/\gamma H$. For specific combinations of $\phi'$ and $c_u/\gamma H$, the inclination of shear planes in sand $\theta$ becomes zero, and the mechanism degenerates to the punching block mechanism proposed by Meyerhof (1974). In fact, as the shear strength of clay tends to zero the inclination of the shear planes in sand tends to 2:1, as sand will still be in the elastic range when the Prandtl failure mechanism develops in the clay layer. On the other hand, as the shear strength of clay tends to infinity, the width of the equivalent footing tends to zero, as failure will develop entirely within the sand layer. The above transition is illustrated in Fig. 5, which depicts the range of failure mechanisms observed during the parametric analyses, and the effect of the relative strength of the soil layers on the geometry of the mechanism.

Note though that all the abovementioned mechanisms are approximate, and ignore the fact that the inclination of shear planes changes within the thickness of the sand layer. Adopting a single value for angle $\theta$ based on the observed width of the equivalent footing, as shown in Fig. 4, is a convenient simplification. However, ignoring the actual geometry of the punching wedges in a limit equilibrium solution will result in estimating a lower bound of the shear resistance developing in sand. The above observations suggest that existing bearing capacity models cannot describe the entire range of possible failure mechanisms and inspired the refined bearing capacity model described in the following.

**Proposed bearing capacity model**

The identified failure mechanisms that extend into the clay layer can be generally described with the simplified mechanism shown in Fig. 4, which is similar to the mechanism proposed by Okamura et al. (1998) (Fig. 3), provided we allow for negative (or zero) values of the inclination of the shear planes, $\theta$. The bearing capacity of the footing results as the sum of: i) the undrained capacity of the equivalent footing
resting on the top of the clay layer, assuming a generalised, Prandtl-type failure mechanism, and ii) the resistance developing along the shear planes in sand. Based on the above, we may formulate a new bearing capacity model, which key parameters are the inclination of failure planes in sand $\theta$ and a new coefficient $K_{sr}$ to describe the resistance developing along these shear planes.

Angle $\theta$ can attain positive or negative values, as shown in Fig. 6. Positive $\theta$ values correspond to cases where the width of the equivalent footing is larger than the width of the actual footing (Fig. 6a), while negative $\theta$ values to cases where the width of the equivalent footing is smaller than the width of the actual footing (Fig. 6b). The model allows zero $\theta$ values, as the shear planes may be vertical under certain sand/clay relative strength combinations discussed above. If angle $\theta$ is known, then the bearing capacity of the equivalent footing on clay can be calculated as:

$$Q_b = (N_c c_u + \gamma H) \left[ B + 2H \tan(\pm \theta) \right]$$

where $N_c = 5.14$ is the undrained bearing capacity factor for a strip footing on clay and angle $\theta$ can attain positive or negative values.

The resistance developing along the shear planes in sand is a function of i) the geometry of the actual failure mechanism (Fig. 4), ii) the normal stress $\sigma_n$ acting on the shear planes (Fig. 6), and iii) the mobilised friction angle along the failure planes, $\phi_m$. It is convenient to express the contribution of the above via the coefficient $K_{sr}$, which is used to quantify the vertical component of the total force acting on each shear plane ($P_v$ in Fig. 6) as:

$$P_v = \left( \frac{\gamma H^2}{2} \right) K_{sr} \tan \phi'$$

Considering now the equilibrium of the rigid sand block under the footing (Fig. 6) provides the bearing capacity of the strip footing on sand, as:
(18) \[ q_{at} \cdot B = \gamma H^2 K_{sr} \tan \varphi' + (N_c c_u + \gamma H)[B + 2H\tan(\pm \theta)] - [B + H\tan(\pm \theta)]\gamma H \leq q_i \cdot B \]

or, in a simpler form:

(19) \[ q_{at} \cdot B = \gamma H^2 K_{sr} \tan \varphi' + (N_c c_u + \gamma H)[B + 2H\tan(\pm \theta)] + \gamma H^2 \tan(\pm \theta) \leq q_i \cdot B \]

where \( q_i \) is the bearing capacity of the strip footing on uniform sand, which can be estimated analytically according to e.g. Martin (2006). This cap on the bearing capacity corresponds to cases where failure is contained with the sand layer, as depicted in Fig. 5d.

The unknown parameters of Eq. (19) viz. \( \theta \) and \( K_{sr} \) can be estimated via FELA analyses, as described in the following.

**Estimation of bearing capacity parameters**

To investigate the mechanisms governing \( \theta \) and \( K_{sr} \), a series of parametric FELA analyses were performed, considering a wide range of material properties (friction angle \( \varphi' \) and unit weight \( \gamma \) of sand; undrained shear strength of clay \( c_u \)) and problem geometries (footing width \( B \); thickness of sand layer \( H \)). These parameters are summarised in Table 1. The range of the dimensionless clay strength \( c_u / \gamma H \) corresponding to the parameters listed in Table 1 is \( c_u / \gamma H = 0.3 \) to (approximately) 10.

The main outcome of the FELA analyses are rigorous upper \( Q_U \) and lower \( Q_L \) bounds on the bearing capacity. In the study at hand, the bearing capacity of the footing is taken as the average of the two bounds (Salgado et al. 2004) and the relative error in the solution is computed as:

(20) \[ RE(\%) = \frac{Q_U - Q_L}{Q_U + Q_L} \times 100 \]
Eq. (20) provides an objective measure of how accurately we can bracket the true bearing capacity with FELA upper and lower bound analyses. Relative errors calculated in this study were generally less than 3%, with the maximum relative error being of the order of 5%.

Accordingly, angle $\theta$ is estimated graphically from power dissipation intensity contours, as shown in Fig. 4, and its variation with the normalised clay strength $c_u/\gamma H$ is shown in Fig. 7. Notice first that angle $\theta$ is not a function of the dimensionless sand thickness $H/B$. Owing to the symmetry of the problem with respect to the axis of the footing, $\theta$ is independent of the width of the footing. Results illustrated in Fig. 7 suggest that the shape of the failure mechanism is a function of the relative strength of the two soil layers. Specifically, the higher the friction angle of sand, the wider the equivalent footing becomes (for constant $c_u/\gamma H$), tending asymptotically to the Terzaghi’s 2:1 assumption that will result in the maximum resistance offered by the clay layer if sand remained in the elastic range. However, the bulk of the results shown in Fig. 7 lies below the $\theta = 0^\circ$ line, suggesting that the equivalent footing will be wider than the actual footing only if the friction angle of sand is high and the dimensionless clay strength is low. As $c_u/\gamma H$ increases the width of the equivalent footing $B'$ decreases; for a very stiff clay relatively to the top sand layer the failure surface will develop entirely within the sand layer and $B' = 0$. Finally, notice that the inclination of the shear planes $\theta$ depends also on $H$, and if the sand layer is relatively thin then $\theta$ generally will be negative. However, as the width of the equivalent footing is equal to $B' = B + 2H \tan \theta$ this suggests that the failure mechanism will be similar to that proposed by Meyerhof.

Angle $\theta$ values depicted in Fig. 7 can be fitted with the following expression, which corresponds to the solid lines drawn in Fig. 7:

$$\theta (\text{rad}) = A \cdot \ln (c_u/\gamma H) + B$$

where

$$A = 0.039 \ln (\tan \phi') - 0.164$$
$$B = 0.597 \ln (\tan \phi') - 0.051$$

(21)
Given $\theta$ from Eq. (21), the coefficient $K_{sr}$ can be calculated from Eq. (22) using the average bearing capacity of the footing calculated from FELA analyses $(Q_U + Q_L)/2$ as:

\[
(22) \quad K_{sr} = \left( \frac{1}{\gamma H^2 \tan \phi'} \right) \left[ \left( \frac{Q_U + Q_L}{2} \right) - N_c c_u \left[ B + 2H \tan(\pm \theta) \right] - \gamma H^2 \tan(\pm \theta) \right]
\]

$K_{sr}$ values calculated from the results of FELA analyses are illustrated in Fig. 8. Notice that $K_{sr}$ depends on both $\phi'$ and $c_u/\gamma H$, but not on $H/B$. Notice also that the contribution of shear resistance from the sand layer, which is quantified via $K_{sr}$, depends on the undrained strength of the clay layer, suggesting the contribution of each individual layer to the bearing capacity cannot be uncoupled. More specifically, $K_{sr}$ decreases as the undrained shear strength of the clay layer $c_u$ decreases. This suggests that Terzaghi’s simplification, who ignored conservatively the shear resistance from sand, is rather reasonable for very soft clays. On the contrary, as $c_u$ increases the shear resistance mobilised in sand increases towards the development of a surficial mechanism in the top layer. We can use the following expression (solid lines in Fig. 8) to quantify $K_{sr}$:

\[
(23) \quad K_{sr} = \frac{C (c_u/\gamma H)}{C} + 2
\]

where

\[
C = -3.48 (\tan \phi') + 8.693
\]

Summarising, the undrained bearing capacity of a strip footing on sand over clay can be calculated with the proposed method as:

- Estimate $\theta$ from Eq. (21) and $K_{sr}$ from Eq. (23), as function of the friction angle of sand $\phi'$ and of the dimensionless shear strength of clay $c_u/\gamma H$.
- Calculate $q_{ult}$ from Eq. (19).

**Validation against published experimental data**

There is a limited number of experimental studies in the literature (e.g. Das and Dallo 1984, Okamura et al. 1998) reporting measurements of the bearing capacity of strip footings on sand over clay. These studies
cover a rather narrow range of sand and clay properties, with sand friction angles usually around $\phi' = 45^\circ$, and clay undrained shear strength of the order of $c_u = 10$ to 20 kPa. Okamura et al. (1998) performed centrifuge tests at 50g to measure the ultimate load on strip footings with prototype scale widths varying between 1.0 and 2.0 m. The dimensionless thickness of the sand layer ranged from $H/B = 1$ to 4, while the plane strain friction angle of the sand layer and the undrained shear strength of the clay layer were reported to be $\phi' = 47^\circ$ and $c_u = 22$ kPa, respectively. These values correspond to a limited range of dimensionless shear strengths between $c_u / \gamma H = 0.6$ and 1.1. Earlier, Das and Dallo (1984) performed scaled tests to measure the bearing capacity of a strip footing with width $B = 0.076$ m (3 in). The dimensionless thickness of the sand layer in their experiments varied between $H/B = 1$ and 3.01. Das and Dallo report that the friction angle of sand was $\phi' = 43^\circ$ and the undrained shear strength of clay was $c_u = 12$ kPa. These scaled experiments covered a broader range of dimensionless shear strengths from $c_u / \gamma H = 3.1$ to 9.3.

Bearing capacity values predicted with the proposed model are compared against the experimental measurements of Okamura et al. (1997) and Das and Dallo (1984) in Fig. 9 and 10, respectively. Notice that failure pressures predicted by Eq. (19) are within ±20% of the measured failure pressures for all tests. The model proposed by Okamura et al. (1998) provides similar accuracy when compared with the centrifuge tests performed by Okamura et al. (1997), which is expected as this bearing capacity model was developed on the basis of the particular tests. Although the model by Okamura et al. (1998) provides results that are within ±20% of the Das and Dallo (1984) scaled test measurements, it is clear from Fig. 10 that it consistently overestimates the bearing capacity. This can be explained if we refer back to Fig. 7: The friction angle and dimensionless strength combinations in the Okamura et al. (1987) experiments lie in the positive $\theta$ range, whereas the friction angle and dimensionless strength combinations in the Das and Dallo (1984) experiments lie in the negative $\theta$ range, and therefore the failure mechanism cannot be accurately captured with the Okamura et al. (1998) bearing capacity model.

Discussion and Conclusions
The simple bearing capacity model proposed in this study can be used for estimating the bearing capacity of strip footings on sand over clay, without having to resort to numerical analysis for simple tasks, such as the design of working platforms. Indeed, the required footing width or the properties of the improvement layer can be quickly estimated (and optimised) by applying Eqs. (19), (21) and (23) iteratively. We have shown that the model degenerates to Terzaghi’s or Meyerhof’s model, for specific relative sand/clay strength combinations, and can be used to predict the bearing capacity for a wide range of problem parameters.

The model is based on a simplified version of the failure mechanism observed in FELA simulations, however the introduction of the coefficient \( K_{sr} \) allows accounting for the effect of the complex shape of the failure planes in sand indirectly. Another main simplification embraced during this study is the adoption of associated flow rule for sand, which is an inherent assumption of limit analysis methods. Frictional materials, however, are non-associated, particularly near collapse when their behaviour is non-dilatant. Assuming associated flow will generally result in upper bound values of the bearing capacity, however the quantitative effect of material non-associativity on the limit loads calculated by FELA depends on the degree of kinematic constraint of the problem (Davis 1968). Generally, the problem of bearing capacity of footings is classified as a low kinematic constraint problem, and Davis (1968) suggests that the limit bounds calculated for non-associative materials can be considered an acceptable estimation of the true bearing capacity. In the specific problem of a strip footing on sand overlying clay, this factor becomes even less significant as the kinematic constraint of the problem decreases due to the existence of the weak bottom layer. A similar argument was adopted by Ballard et al. (2011). On top of that, the stress levels in the surficial sand layer are generally low, resulting in increased dilatancy potential of the sand layer. The above are in line with the good agreement observed against two independent sets of physical model tests, and the fact that the proposed method did not result in systematic over-prediction of test data.
References


Figure captions

Fig. 1. Projected area model (Terzaghi 1948).

Fig. 2. Punching shear model (Meyerhof 1974).

Fig. 3. Failure mechanism considered by Okamura et al. (1998).

Fig. 4. Power dissipation intensity contours in sand at failure, used to determine the shape of the actual failure mechanism and define the geometry of the simplified failure mechanism (a) dense sand ($\varphi' = 45^\circ$) over soft clay ($c_u/\gamma H = 0.7$), (b) dense sand ($\varphi' = 45^\circ$) over stiff clay ($c_u/\gamma H = 2.85$).

Fig. 5. Velocity vectors and power dissipation intensity contours for different problem parameters.

Fig. 6. Simplified failure mechanisms and equilibrium of forces (a) positive inclination of shear planes $\theta$; (b) negative inclination of shear planes $\theta$.

Fig. 7. Variation of angle $\theta$ with the dimensionless clay strength $c_u/\gamma H$ for different sand friction angle $\varphi'$ values. Simulations corresponding to different dimensionless sand thickness values $H/B$ are presented with different symbols.

Fig. 8: Variation of coefficient $K_{sr}$ with the dimensionless clay strength $c_u/\gamma H$ for different sand friction angle $\varphi'$ values. Simulations corresponding to different dimensionless sand thickness values $H/B$ are presented with different symbols.

Fig. 9. Comparison of the proposed bearing capacity model with the experimental results of Okamura et al. (1997) and the predictions of the bearing capacity model proposed by Okamura et al. (1988). $\gamma'$ is the effective unit weight reported by Okamura et al. (1997).

Fig. 10. Comparison of the proposed bearing capacity model with the experimental results of Das and Dallo (1984) and the predictions of the bearing capacity model proposed by Okamura et al. (1988).
Fig. 1. Projected area model (Terzaghi 1948).
Fig. 2. Punching shear model (Meyerhof 1974).
Fig. 3. Failure mechanism considered by Okamura et al. (1998).
Fig. 4. Power dissipation intensity contours in sand at failure, used to determine the shape of the actual failure mechanism and define the geometry of the simplified failure mechanism (a) dense sand ($\phi' = 45^0$) over soft clay ($c_u/\gamma H = 0.7$), (b) dense sand ($\phi' = 45^0$) over stiff clay ($c_u/\gamma H = 2.85$)
Fig. 5. Velocity vectors and power dissipation intensity contours for different problem parameters.
CHAPTER 6 BEARING CAPACITY OF STRIP FOOTINGS ON SAND OVER CLAY

Fig. 6. Simplified failure mechanisms and equilibrium of forces (a) positive inclination of shear planes $\theta$; (b) negative inclination of shear planes $\theta$.
Fig. 7. Variation of angle $\theta$ with the dimensionless clay strength $c_u/\gamma H$ for different sand friction angle $\phi'$ values. Simulations corresponding to different dimensionless sand thickness values $H/B$ are presented with different symbols.
Fig. 8: Variation of coefficient $K_{sr}$ with the dimensionless clay strength $c_u/\gamma H$ for different sand friction angle $\phi'$ values. Simulations corresponding to different dimensionless sand thickness values $H/B$ are presented with different symbols.
Fig. 9. Comparison of the proposed bearing capacity model with the experimental results of Okamura et al. (1997) and the predictions of the bearing capacity model proposed by Okamura et al. (1988). $\gamma'$ is the effective unit weight reported by Okamura et al. (1997).
Fig. 10. Comparison of the proposed bearing capacity model with the experimental results of Das and Dallo (1984) and the predictions of the bearing capacity model proposed by Okamura et al. (1988).
Table 1. Soil parameters and problem geometries considered in the parametric analysis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Friction angle of sand ($\phi'$)</td>
<td>35° - 55°</td>
</tr>
<tr>
<td>Undrained shear strength of clay ($c_u$)</td>
<td>10 - 70 kPa</td>
</tr>
<tr>
<td>Footing width ($B$)</td>
<td>0.6 - 1.8 m</td>
</tr>
<tr>
<td>Sand layer thickness ($H$)</td>
<td>0.4 - 1.6 m</td>
</tr>
<tr>
<td>*Unit weight of sand ($\gamma$)</td>
<td>15.5 - 20 kN/m³</td>
</tr>
</tbody>
</table>

*The unit weight $\gamma$ is calculated according to Bolton (1986), as function of the friction angle.
RESPONSE TO REVIEWER 1

Figure 6.1 presents a comparison between the ultimate bearing capacities estimated for a typical problem with the proposed model (here referred to as FELA) and with Hanna’s model (1980). Bearing capacities are estimated parametrically, as function of the undrained shear strength of the subgrade layer. This is used to depict the effect of assuming vertical planes of failure in sand (i.e. Meyerhof’s punching shear failure mechanism, adopted by Hanna) on the ultimate bearing capacity of strip footings on layered soils.
Notice in Fig. 6.1 that assuming vertical failure planes in sand results in under-prediction of \( q_u \) for lower values of \( c_u \) and over-prediction as \( c_u \) increases, compared to the more refined model proposed in this Chapter. The overestimation of \( q_u \) becomes more pronounced for higher values of \( c_u \). As discussed above, \( \theta \) decreases with an increase in \( c_u \) i.e. the inward inclination of the planes of failure in sand increases, resulting in smaller effective footing widths (\( B' \)). This results in lower contribution of the clay shear strength to \( q_u \), compared to the Hanna’s model, where the effective footing width does not depend on \( c_u \). The opposite is observed for very low \( c_u \) values, where the effective footing becomes wider than the actual footing, and the failure mechanism resembles the projected area model of Terzaghi. The above are consistent with the results of back-calculated punching shear coefficients, discussed in Chapter 4.

**RESPONSE TO REVIEWER 2**

Different notation is used to describe the inclination of the shear planes in sand (\( \theta \), Chapter 6) and the load spread angle \( \alpha \) (Section 2.3.2 of Chapter 2). This is intentional, and was adopted to differentiate between the inclination angle of the slip surfaces, interpreted from the observed failure mechanisms (\( \theta \)) and associated with shear failure; and the load spreading angle (\( \alpha \)) used in the projection area model and associated with elastic stress distribution.
CHAPTER 7- BEARING CAPACITY OF STRIP FOOTINGS ON CLAYS WITH INCREASING SHEAR STRENGTH
BEARING CAPACITY OF STRIP FOOTINGS ON CLAYS WITH INCREASING SHEAR STRENGTH

Seyednima Salimi Eshkevari¹, Andrew J. Abbo²
¹ ² Australian Research Council Centre for Geotechnical Science and Engineering, School of Engineering, University of Newcastle, Australia

ABSTRACT

This paper considers the bearing capacity of surface strip footings sitting on a deep layer of clay soil with shear strength profile that increases linearly with depth. While an exact solution for the bearing capacity has previously been obtained using the method of characteristics, it is not in a form that is convenient for accurate evaluation. Significant research has been done previously in this area. In those works however, the focus has been on the techniques rather than practical application and the results are either presented for limited cases or summarised in graphs that are not easy to interpret in some range of parameters. In the present study, Finite Element Limit Analysis (FELA) is used to compute rigorous upper and lower limits on the ultimate bearing capacity of strip footings resting on the non-homogeneous clay layer. The results of the numerical limit analysis, which bracket the true bearing capacity within a range of typically less than 2%, compare well with the exact solutions available in the literature. New expressions are provided for the bearing capacity in which the bearing capacity factor is expressed as a simple function of the dimensionless rate of increase of soil strength.

Keywords: Bearing capacity, Non-homogenous clay, Increasing shear strength

INTRODUCTION

A nonhomogeneous shear strength profile is a common situation encountered in geotechnical engineering when considering normally consolidated or lightly over-consolidated clays. In such soils the undrained shear strength ($c_u$) tends to increase almost linearly with depth. As shown in Fig. 1, the shear strength profile of such soils is usually described in terms of the rate of increase in shear strength with depth ($\gamma$). The rate of increase may be expressed as dimensionless quantity $\frac{pB}{co}$ where $B$ is the footing width and $co$ the undrained shear strength of clay at the foundation level [1]. The rate of increase in shear strength ($\gamma$) is known to be an inherent soil property, varying in the range of (0.6~3.0 kPa/m) which may be estimated as a product of soil effective unit weight ($\gamma'$) and $\Delta c_u / \Delta z$; that is the rate of increase in $c_u$ due to the increase in the soil effective vertical consolidation stress [2]. The soil effective unit weight and $\Delta c_u / \Delta z$, vary in the range of (4~10 kN/m$^3$) and (0.15~0.3) respectively [2]. The influence of clay non-homogeneity on the bearing capacity of footings is particularly significant in the offshore engineering, where large diameter shallow foundations are installed over normally consolidated sea bed producing very high degrees of non-homogeneity. However the benefits of considering increasing shear strength profile of soil is not limited to offshore practice.

Many researchers have previously considered the influence of soil increasing shear strength on the bearing capacity of footings. The earliest recommendations on the bearing capacity of the footings over non-homogeneous clays ($q_u$) are given by Terzaghi [3] and Skempton [4] who suggested an average bearing capacity factor to be calculated over a shallow depth underneath the footing. Limit Equilibrium was utilised by Nakase [5], using a circular slip surface. This method was later repeated by Raymond [6] that in general, overestimates the true ultimate bearing capacity of the footing. Limit analysis techniques have been a popular approach amongst the researchers with many upper bound estimates of bearing capacity [7]–[14].

One of the best approaches to calculate the bearing capacity of footings over non-homogeneous soils is provided by Davis and Booker [1] who employed the method of characteristics to obtain the bearing capacity of both smooth and rough footings resting on clay of increasing shear strength with depth. They derived an exact expression for the bearing capacity of the form

$$q_u = F[(\gamma + \pi) + \frac{pB}{4}]$$

(1)

where the factor $F$ is a function of $\frac{pB}{co}$. No analytic expression was provided for $F$ with values provided in a simple graph for $\frac{pB}{co}$ ranging from zero to 30. Values of $F$ are not easy to read accurately from the graph, particularly in the lower range of $\frac{pB}{co}$. This method was later used by Houlsby and Wroth [15] who used the same techniques to obtain solutions for circular footings. The bearing capacity software of Martin [16] which
is based on the method of characteristics can also be used to find the upper and lower bounds of $q_u$ for footings on non-homogeneous clay. Tani and Craig [2] used this method to solve the problem of skirted foundations over non-homogeneous clay for both plane strain and axisymmetric conditions. Other numerical methods such as finite elements and finite difference have also been utilized to investigate bearing capacity of these soils; for example Yun and Bransby [11] and others [17]–[21].

In comparison to the number of theoretical studies, there are only a few experimental results reported in the literature due to difficulties regarding the scale rule for physical modelling [2]. The centrifuge tests performed on surface strip footings by Nakase [22] and Kimura and Takemura [23] are examples of experimental studies undertaken on footings over non-homogeneous clay. In addition, some centrifuge tests performed on skirted foundations are reported by Tani and Craig [2], Prevost et al. [24] and more recently by Gourvenec and O’loughlin [25].

Modified bearing capacity factor defined as $(N_c = \frac{\rho B}{c_0})$ is used by most researchers to present the results of the analysis which is only a function of $\rho B/c_0$. The variation of $N_c$ is either plotted versus $\rho B/c_0$ in a graph or is given in a table for limited values of $\rho B/c_0$. It is difficult to accurately interpret $N_c$ from these graphs, especially in the lower range of $\rho B/c_0$ and the values given in the tables are for limited number of $\rho B/c_0$. Presenting expressions that can well approximate $N_c$ can be helpful for engineers.

Bransby [26] proposed the following expression for rough strip footings using the results of Davis and Booker [1], Tani and Craig [2] and Houlshby and Wroth [15].

$$N_c = (2+\pi)+1.646 \left(\frac{\rho B}{c_0}\right)$$

(2)

However this expression is not a safe estimate of bearing capacity as it over-estimates the results presented by abovementioned researchers [11].

Finite Element limit Analysis (FELA) is used in the present study to investigate the bearing capacity of a rough strip footing sitting on the surface of a clay soil with increasing undrained shear strength profile. The results of the analysis provided very tight upper and lower bounds of the exact solutions for $\rho B/c_0$ ranging from zero to 30. This is done to derive accurate expressions to predict modified bearing capacity factor for a practical range of $\rho B/c_0$.

Although the exact solution of Davis and Booker [1] is already available, it is not easy to accurately obtain equivalent modified bearing capacity factor from their presented graph of F. The average of limit loads were used to calculate $N_c$ and two expressions were found using curve fitting technique to calculate $N_c$ very accurately for rough strip footings over the whole practical range of $\rho B/c_0$.

**METHODOLOGY**

A parametric study was performed over a range of material properties and footing widths that are typically encountered in the design of strip footings. The study considers values of $\rho$ between 0.1 to 3.0 kPa/m and $B$ and $c_0$ varied between 1.0 to 150 m and 10 to 60 kPa respectively. FELA is a finite element method formulated from the limit theorems of plasticity. The FELA formulation used in this research is based on the method originally developed by Sloan [27] and [28] using linear programing methods. The formulations have been further developed later, applying non-linear programing as was described by Lyamin et al. [29] and [30]. The current formulation also benefits from adaptive re-meshing technique of Lyamin et al. [31] and [32].

Soil was modelled as an associated Mohr-Coulomb material. A gradient and a reference point in the geometry was introduced to the software in addition to the absolute value of the undrained shear strength of the clay to model the increasing shear strength of the clay with depth. Adaptive re-meshing technique allowed tighter upper and lower bounds of the true solutions. The target solution accuracy, number of adaptive re-meshing and the starting number of elements were set so to obtain bounds around 1% of the exact solution by Davis and Booker [1]. The results of maximum power dissipation and direction of velocity vectors were used to capture the failure mechanism of the problem. The average of upper and lower limit loads was used to calculate the ultimate bearing capacity of the footing following Salgado et al. [33] and the bearing capacity factor $N_c$ was calculated using the average $q_u$. Finally expressions were found for $N_c$ as a function of $\rho B/c_0$ using curve fitting, which well presents the results of the analysis for all practical values of $\rho B/c_0$.

**RESULTS AND DISCUSSIONS**

The results of this study are presented in terms of the bearing capacity factor $N_c$ that can be used...
together with conventional bearing capacity theory. $N_c$ is presented as a function of $\frac{\rho B \phi_0}{c_0}$ in the graphs of Fig. 3 and Fig. 4. The range of $\frac{\rho B \phi_0}{c_0}$ less than 2 presented in Fig. 3 is more appropriate for strip footings while the range of $\frac{\rho B \phi_0}{c_0}$ between 2 and 30 shown in Fig. 4 may be used for large rectangular footings in offshore practice. Figure 2 indicates the upper and lower bounds of $N_c$ obtained in this study together with the upper bound solutions of [7], [8], [10], [11] and [14]. The range of $\frac{\rho B \phi_0}{c_0}$ shown in Fig. 2 is based on available data in the literature.

It can be seen that the upper bound of $N_c$ is the lowest of all other available upper bound solutions in the literature and are close to those obtained from method of characteristics provided by [11], [2], [15] and [16] that are presented in Table 1. The bearing capacity factors calculated by Reddy [8] and Alshamrani [10] are over predicted significantly as $\frac{\rho B \phi_0}{c_0}$ increases. The accuracy of the upper bound solutions depends on the assumed failure mechanism which will be discussed later. The lower bound calculations provide very close results to these upper bounds (maximum 2% apart).

The average of the computed upper and lower bounds of $N_c$ are plotted in Fig. 3 and Fig. 4 together with the best fits for each case. The following expressions shown by Eq. (2) and Eq. (3) were obtained using curve fitting technique which is very accurate and simple to be used in design.

$$N_c = -4.29 \, e^{0.41 \frac{\rho B \phi_0}{c_0} + 9.46} \quad \frac{\rho B \phi_0}{c_0} < 2$$  (3)

$$N_c = 1.36(\frac{\rho B \phi_0}{c_0})^{0.73} + 5.37 \quad \frac{\rho B \phi_0}{c_0} \geq 2$$  (4)

These expressions provide the almost exact value of the modified bearing capacity factors with excellent accuracy that can conveniently be used by engineers. Using separate expressions of $N_c$ for cases with $\frac{\rho B \phi_0}{c_0}$ greater than and less than two increased the accuracy of curve fitting. More over for widths of strip footings more usually encountered in engineering practice, $\frac{\rho B \phi_0}{c_0}$ will be typically less than two.
CHAPTER 7 BEARING CAPACITY OF STRIP FOOTINGS ON CLAYS WITH INCREASING SHEAR STRENGTH

Fig. 5 Bearing Capacity Factor ($N_c$) for $\rho B/c_0 < 2$.

Fig. 4 Bearing capacity factor ($N_c$) for $\rho B/c_0 \geq 2$.

Table 2 Bearing capacity factors ($N_c$).

<table>
<thead>
<tr>
<th>Nc</th>
<th>FELA (Average)</th>
<th>FELA (Average)</th>
<th>FELA (Average)</th>
<th>FELA (Average)</th>
<th>FELA (Average)</th>
<th>FELA (Average)</th>
<th>FELA (Average)</th>
<th>FELA (Average)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5.12</td>
<td>5.16</td>
<td>5.14</td>
<td>5.14</td>
<td>5.14</td>
<td>5.14</td>
<td>5.20</td>
<td>5.16</td>
</tr>
<tr>
<td>0.5</td>
<td>5.92</td>
<td>6.02</td>
<td>5.97</td>
<td>-</td>
<td>-</td>
<td>5.97</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1.0</td>
<td>6.53</td>
<td>6.66</td>
<td>6.60</td>
<td>6.84</td>
<td>6.61</td>
<td>6.70</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2.0</td>
<td>7.55</td>
<td>7.64</td>
<td>7.60</td>
<td>7.75</td>
<td>7.60</td>
<td>7.71</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3.0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>8.41</td>
<td>8.55</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>4.0</td>
<td>9.07</td>
<td>9.18</td>
<td>9.13</td>
<td>9.08</td>
<td>9.23</td>
<td>9.13</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>5.0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>9.79</td>
<td>-</td>
<td>9.82</td>
<td>-</td>
</tr>
<tr>
<td>6.0</td>
<td>10.34</td>
<td>10.44</td>
<td>10.39</td>
<td>10.37</td>
<td>10.49</td>
<td>10.42</td>
<td>10.62</td>
<td>-</td>
</tr>
<tr>
<td>7.0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>11.01</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>8.0</td>
<td>11.51</td>
<td>11.61</td>
<td>11.56</td>
<td>11.52</td>
<td>-</td>
<td>11.58</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>10.0</td>
<td>12.59</td>
<td>12.69</td>
<td>12.64</td>
<td>12.67</td>
<td>12.73</td>
<td>12.67</td>
<td>12.95</td>
<td>-</td>
</tr>
<tr>
<td>15.0</td>
<td>15.05</td>
<td>15.21</td>
<td>15.13</td>
<td>-</td>
<td>15.14</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>20.0</td>
<td>17.31</td>
<td>17.44</td>
<td>17.38</td>
<td>-</td>
<td>17.40</td>
<td>-</td>
<td>17.46</td>
<td>-</td>
</tr>
<tr>
<td>30.0</td>
<td>21.46</td>
<td>21.63</td>
<td>21.55</td>
<td>-</td>
<td>21.69</td>
<td>21.58</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

In the graphs of Fig. 3 and Fig. 4 the bearing capacity factor calculated from Eq. (1) by Bransby [26] is also plotted for comparison. It is clear that Eq. (1) overestimates $N_c$ in the range of $\rho B/c_0$ less than 2 and underestimates $N_c$ for $\rho B/c_0$ greater than 20.

The best solutions for bearing capacity of strip footings over non-homogenous clays have been obtained using method of characteristic and recently by some finite element models. Some of the most accurate results published so far are presented in Table 1, together with lower bound, upper bound and the average values of $N_c$ calculate in this study. The average values compare very well with the exact solution provided by Davis and Booker [1] and are in excellent agreement with the results presented by Houlsby and Wroth [15] and those obtained using the bearing capacity software written by Martin [16]. The limited cases presented by Gourvenec et.al [21] are very close to the average of FELA and the results by the above mentioned researchers, while calculated $N_c$ by Tani et al. [2] and Gourvenec et.al [18] slightly over estimates the true bearing capacity factors.

3 Failure mechanism

The results of maximum power dissipation from the upper bound analysis indicate the failure mechanism of the footing, providing the lowest upper bound to the ultimate load on the footing. The influence of the degree of non-homogeneity on the mode of failure has been studied here and a comparison is made between the obtained failure mechanisms from this study and the ones proposed by Yun et.al. [11]. Figure 5a indicates the failure mechanism for $\rho B/c_0 = 2$. It can be seen that the failure mechanism is limited to much shallower depth compared to Prandtl failure mechanism [34].
The optimum failure mechanism presented by Yun et al. [11] is also plotted in this figure and these are representative of the collapse mechanisms that can be interpreted from the bands of high power dissipation intensity. Figure 5b shows observed failure mechanism for a higher degree of non-homogeneity. It can be seen from both the results of Yun et al. [11] and from the FELA analysis that as \( \rho B/c_0 \) increases the failure mechanism becomes shallower to the regions of the soil profile where the soil is weaker. However, the proposed failure mechanism by Yun et al. [11] does not quite capture the failure mechanism for the case of \( \rho B/c_0 \) equal to 5 in the area directly beneath the footing as it omits the wedge directly beneath the footing. The results of upper bound analysis indicated that the failure mechanism for very high degrees of non-homogeneity (\( \rho B/c_0 \) of 20) follows the same pattern as shown in Fig. 5b, however the triangular rigid block under the footing becomes much smaller.

Fig. 1 Power dissipation intensity from upper bound analysis showing failure mechanism of Yun et al. [11] for (a) \( \rho B/c_0=2 \) (b) \( \rho B/c_0=5 \).

CONCLUSION

Rigorous upper and lower bounds on the ultimate bearing capacity of strip footing over non-homogenous clays were calculated using FELA technique. The average of the limit loads were in very close agreement with the exact solutions of Davis and Booker [1] and were used to derive expression for the modified bearing capacity factor (\( N_c \)). These expressions can conveniently be used to estimate with high accuracy the ultimate bearing capacity of strip footings over clays with increasing shear strength in depth. A comparison between the failure mechanisms observed in this study with the assumed failure mechanisms by Yun et al. [11] indicated that better agreement exists for lower degrees of non-homogeneity (\( \rho B/c_0=2 \)) and the general shape of the failure mode remains the same as \( \rho B/c_0 \) increases but smaller rigid block is seen for higher values of \( \rho B/c_0 \).

ACKNOWLEDGMENT

The authors wish to express their gratitude to Professor Andrei Lyamin for providing and maintaining the Finite Element Limit Analysis software and his guidance and support in its use.

REFERENCES

[12] Huang, M., Qin, H. & Guo, Y. Upper bound solution for bearing capacity of
CHAPTER 7 BEARING CAPACITY OF STRIP FOOTINGS ON CLAYS WITH INCREASING SHEAR STRENGTH

GEOMATE- Osaka, Nov. 16-18, 2015


[34] Prandtl, L. Über die Harte plastischer KrBrper, Nachrichten von der KBniglichen Gesellschaft der Wissenschaften, GBttingen, 1920.
CHAPTER 8- BEARING CAPACITY OF STRIP FOOTINGS ON A STRONG SAND OVERLAYING WEAK SAND
ABSTRACT

In this paper we investigate the bearing capacity of strip footings resting on the top of a relatively thin layer of dense sand, which overlays a weaker sand layer. We use the Finite Element Limit Analysis method to calculate the collapse load and identify the geometry of the failure mechanism, as well as how this geometry changes with varying problem parameters. Results are benchmarked against published experimental data and methods used in practice for the design of working platforms for tracked plant, based on the punching shear model. The observed discrepancies are attributed to the development of a transitional mechanism when the bottom layer is rather dense, and the increase in the shear strength at the interface of the two layers is not abrupt. To cover this gap, we propose a new factor to calculate the bearing capacity in cases where a transitional mechanism will govern failure of the footing.

1. INTRODUCTION

Since Terzaghi [1] published his theory for estimating the bearing capacity of shallow foundations on homogeneous soil, a number of researchers have investigated the bearing capacity of footings on stratified natural soils, or on a layer of selected coarse-grained fill placed on top of weak subgrade to increase the capacity of the foundation. However, estimating the bearing capacity of shallow foundations on layered soils remains a challenging problem as, owing to its complexity, deriving closed-form plasticity solutions is cumbersome. Some simplifying assumptions are required to develop easy-to-use expressions for use in practice, similar to the ones originally proposed by Terzaghi for homogeneous soil. The most common scenario examined in the literature is that of footings on dense sand over soft clay, which is relevant to the design of working platforms in soft soil sites. Meyerhof [2] first used a semi-empirical approach to calculate the bearing capacity of strip and circular footings on dense sand over soft clay, while he also considered the case of loose sand over stiff clay. The pioneer punching shear model proposed
by Meyerhof was later refined in a number of experimental, numerical and analytical studies, including the works of Hanna and Meyerhof [3], Griffiths [4], Das and Dallo [5], Michalowski and Shi [6], Kenny and Andrawes [7], Burd and Frydman [8], Okamura et al. [9], Shiau et al. [10], Qin and Huang [11] and, more recently, by Salimi et al. [12].

The case of shallow footings on layered sand profiles has attracted much less attention in the literature, despite its importance for the design of working platforms for tracked piling rigs and cranes on sites where relatively loose sand deposits are encountered near the ground surface. Current practice for the design of working platforms on loose sandy deposits is based on the method described in the Building Research Establishment BRE470-2004 guidelines [13]. The BRE470 method is a simplified version of the semi-empirical method proposed by Hanna [14], which in turn is based on Meyerhof’s punching shear model. Other researchers have approached this problem analytically, using limit equilibrium and limit analysis techniques based on specific, simplified failure mechanisms e.g. Ghazavi and Eghbali [15] derived bearing capacity factors for footings on two-layered granular soils, using an extension of the failure mechanism initially proposed by Richards et al. [16]. Analytical upper bound solutions have also been obtained for the more general case of footings on two-layered cohesive-frictional soils by e.g. Purushothamaraj et al. [17], Florkievicz [18] and Huang and Qin [19]. More recently, Khatri et al. [20] used the numerical Finite Element Limit Analysis (FELA) method to calculate upper and lower bounds of the collapse load, using conic optimisation [21-24]. Experimental results to benchmark the abovementioned solutions are sparse, however Hanna [14], Das and Munoz [25] and Kumar et al. [26] report findings from small-scale 1g tests on footings resting on layered sand, for different problem geometries. Extrapolation of these test results to full-scale footings must be done with caution, as scale effects may lead to overestimating the collapse load, due to the dependency of sand mechanical characteristics to the level of confining stress [27-28]. Nevertheless, results of such model tests are invaluable, as they can provide insight on the developing failure mechanisms.

This study was motivated by the observation that neither the Hanna [14] nor the BRE470 [13] method, used widely in the design of working platforms for tracked plant, compare well with more refined numerical simulations for a certain range of material parameters, encountered in practice by the authors. To identify the reason behind the observed discrepancies we performed a series of analyses considering different problem geometries and sand parameters, using the FELA technique implemented in an in-house code developed in the University of Newcastle [29-32]. Apart from rigorous lower and upper bounds of the collapse load, we were able to gain insight on the developing failure mechanisms, and how the mechanism diverges from Meyerhof’s and Hanna’s punching shear assumption as the relative strength of the sand layers and the problem geometry changes. To support our findings, we compare FELA results against the experimental results of Meyerhof [33] and Das and Munoz [25], and predictions of the methods proposed by Hanna [14] and embraced by the BRE470. We conclude by proposing a practical method to
estimate the bearing capacity of shallow strip footings in cases where Hanna’s method yields un-conservative estimates.

2. EXISTING METHODS FOR ESTIMATING THE BEARING CAPACITY OF FOOTINGS ON LAYERED SANDS

We begin the presentation by providing an outline of Hanna’s [14] and of the BRE470 [13] methods for estimating the bearing capacity of footings on layered sand, in order to identify the key simplifications introduced in these methods. As mentioned earlier, Hanna [14] extended the semi-empirical punching shear model of Meyerhof [2] to the case of shallow footings on dense sand overlying loose sand. Hanna considered the failure mechanism depicted in Fig. 1, where general shear failure occurs within the bottom sand layer, as the footing punches an inverted frustum of top sand into the bottom layer. The bearing capacity of the footing results as the sum of the ultimate shear resistance mobilised in the top and bottom sand layers. More specifically, the contribution of the top layer to the resistance is associated with the passive thrust \( P_p \) acting on the (assumed for simplicity) vertical failure planes shown in Fig. 1, which depends on the mobilised friction angle on the failure planes, \( \delta \). The passive thrust is provided by the following expression:

\[
P_p = 0.5 \gamma \frac{1}{H} \left( 1 + \frac{2B}{H} \right) K_p \cos \delta
\]

in which, \( H \) and \( D \) is the thickness of the top sand layer and the embedment depth of the footing, respectively. In addition, \( K_p \) is the mobilised coefficient of lateral earth pressure and \( \gamma \) is the unit weight of the top layer. The contribution of the bottom layer is taken equal to the ultimate bearing capacity of an imaginary footing on homogeneous sand, with width equal to that of the actual footing. As a result, the bearing capacity of a strip footing on layered sand can be estimated as:

\[
q_u = q_b + \frac{2}{B} (P_p \sin \delta) - \gamma H \leq q_t
\]
in which \(q_b\) and \(q_t\) is the bearing capacity of a strip footing on a homogeneous loose and dense sand layer, embedded at depth \(H+D\) or \(D\), respectively:

\[
q_b = 0.5 \gamma_2 B N_{p2} + \gamma_1 (H + D) N_{q2} \\
q_t = 0.5 \gamma_1 B N_{p1} + \gamma_1 D N_{q1}
\]

(3)

(4)

The inequality in Eq. (2) suggests that the bearing capacity cannot exceed the bearing capacity of a footing resting on the surface of homogenous dense sand, therefore there is a critical thickness \(H\), beyond which the properties of the bottom layer do not affect the capacity of the footing, and failure develops entirely within the surficial layer. The bearing capacity factors \(N_q\) and \(N_{\gamma}\), used in Eqs. (3, 4), are calculated according to Meyerhof [34]. Combining Eqs. (1) and (2) results in the following expression:

\[
q_u = q_b + \gamma H^2 \frac{K_p \tan \delta}{b} - \gamma_1 H \leq q_t
\]

(5)

A key parameter in Eq. (5) is the mobilised friction angle \(\delta\), which is not constant but varies along the thickness of the top layer [2]. Hanna [14] used experimental results to determine a parabolic distribution for \(\delta\), which is a function of the relative shear strength of the two layers. To simplify the estimation of the bearing capacity, Hanna followed Meyerhof’s approach and used the coefficient of punching shear resistance \(K_s\):

\[
K_s \tan \phi_1 = K_p \tan \delta
\]

(6)

to eliminate \(\delta\) and \(K_p\) from Eq. (5). \(K_s\) is provided from a chart (Fig. 2) as function of the friction angle of the top \((\phi_1)\) and of the bottom \((\phi_2)\) layer, and the expression providing the bearing capacity of the strip footing is simplified as:

\[
q_u = q_b + \gamma H^2 \frac{K_s \tan \phi_1}{b} - \gamma_1 H \leq q_t
\]

(7)
The method embraced by the BRE470-2004 guidelines is based on the punching shear model, described above, however a number of conservative assumptions are introduced viz.

- The embedment depth $D$ is taken as zero and the surcharge term $\gamma_1 H N_{qs}$ is ignored in the calculation of the contribution of the bottom sand layer to the resistance, $q_b$.
- The mobilised friction angle $\delta$ is taken equal to $\delta = 2/3 \phi$ and accordingly the term $K_p \tan \delta$ is calculated using Coulomb’s passive earth pressure coefficient.
- BRE470-2004 adopts the formulas for the bearing capacity factor $N_r$ proposed by Vesic [35], which are slightly higher than those suggested by Meyerhof [34].

As a result of the above, bearing capacity values, estimated via the BRE470 method are consistently lower than the corresponding values resulting from Eq. (8), for top layer thickness less than the critical thickness.

The abovementioned methods may be used to obtain quick estimates of the bearing capacity. However, they are based on the assumption that the failure mode does not depend on the relative strength of the two layers. Intuitively one would expect that there is a transitional failure mode between the punching mechanism and general failure within the surficial layer, which will govern the response for intermediate values of the relative strength of the two layers. This is investigated in the following sections of the paper.

### 3. COMPUTATIONAL METHOD

#### 3.1 Background

Limit analysis [36] is an effective technique for estimating rigorous upper and lower bounds of the collapse load of foundations. Application of this technique was initially limited to relatively simple problems, since complicated geometries result in complex equations that have to be solved.
analytically. More recently, the use of the finite element method to solve the equations of limit theorems has extended the field of application of this technique to realistic geometries and heterogeneous soil stratigraphies [37]. This computational technique called Finite Element Limit Analysis (FELA) method can provide upper and lower bound estimates of the collapse load that bracket its true value, with the difference between the upper and lower bound providing a direct measure of the error in estimating the true collapse load. This relative error is defined here as:

\[
RE(\%) = \frac{(Q_u - Q_L)}{(Q_u + Q_L)} \times 100
\]  

where, \( Q_u \) and \( Q_L \) are the upper and lower bounds estimates of the collapse load, respectively [38]. In this study we will estimate the bearing capacity of strip footings on layered sand using the in-house FELA code, developed in the University of Newcastle on the basis of the formulation originally presented by Sloan [29], and later refined by Lyamin and Sloan [30] and Krabbenhoft et al. [23, 31]. In addition, to minimise the relative error in the estimation of the bearing capacity, we employ the adaptive re-meshing technique developed by Lyamin et al. [32]. More specifically, the number of elements and of adaptive remeshing iterations in each analysis is refined so as to achieve \( RE \) of less than 5%.

Employing the limit analysis theorems for the estimation of the bearing capacity of foundations requires assuming that plastic flow in sand is associated. Considering an associated flow rule in conjunction with the Mohr-Coulomb yield criterion (used in this study) implies that the dilation angle of sand is equal to its internal friction angle at failure, which is not realistic for granular materials. While this simplification affects the estimated value of the collapse load, the question of to what extent the flow rule influences the limit loads has not been yet accurately answered. Davis [39] correlated the error introduced in the estimation of the collapse load due to considering associated flow with the degree of kinematic constraint of the problem. Drescher and Detournay [40] proposed to use the effective strength parameters defined in Eqs. (9, 10) below and associated flow to estimate upper and lower bounds of the bearing capacity of foundations on geomaterials that exhibit coaxial non-associated flow.

\[
tan \phi^* = \frac{(cos \psi cos \phi)}{(1-sin \psi sin \phi)} \times tan \phi
\]

\[
c^* = \frac{(cos \psi cos \phi)}{(1-sin \psi sin \phi)} \times c
\]

where \( \phi \) is the friction angle of soil, \( c \) is its effective cohesion and \( \psi \) is the peak dilation angle while \( \phi^* = \psi^* \) and \( c^* \) are the effective friction angle, dilation angle and cohesion, respectively. However, as discussed by Krabbenhoft et al. [41], using the above effective strength parameters results in significant underestimation of the bearing capacity of shallow foundations. As discussed
later, FELA results obtained in the following while assuming that the dilation angle is equal to the plane strain friction angle of sand agree fairly well with published experimental results.

### 3.2 FELA model

As mentioned above the two sand layers are modelled as associated Mohr-Coulomb materials. The footing is resting on the top of the surface layer (embedment depth is equal to zero in all simulations, as it will usually be the case for working platforms) and is modelled with continuum elements. This allows simulating a rough surface strip footing, corresponding to the tracked plant, by setting the cohesion of the footing elements equal to an arbitrarily high value and their friction angle equal to the friction angle of the top sand layer. To contain the relative error $RE$ to values less than 5% we employ three (3) adaptive re-meshing iterations per analysis, with initial and target mesh sizes equal to 20,000 and 30,000 elements respectively [32]. These values, as well as the required dimension of the domain to encapsulate the failure surface, resulted from a sensitivity analysis, not presented here for brevity. Finally, the boundary conditions comprise fixed horizontal and vertical displacements at the base and fixed horizontal displacements at the lateral boundaries of the domain.

Each model was solved twice to obtain both the lower and upper limit load, from which the average bearing capacity was estimated, following Salgado et al. [42]. Apart from rigorous estimates of collapse loads, the kinematically admissible velocity field obtained from upper bound analyses provides insight on the governing failure mechanism for each analysis. The mechanism is identified from plots of nodal velocity vectors and power dissipation intensity contours. Areas of high power dissipation intensity represent planes of shear failure and plots of velocity vectors computed at each node depict the collapse mechanism [42].

Following an investigation on the error in the estimated collapse loads, the numerical technique is benchmarked against the published experimental results of Meyerhof [33] and of Das and Munoz [25] and predictions of the punching shear models of Hanna [14] and BRE470-2004 [13]. Accordingly, a range of material parameters and problem geometries were analysed parametrically, to cover cases beyond the thin very dense sand-over-very loose sand profile for which the punching shear model was proposed. The range of parameters considered in the analysis is listed in Table 1, and covers most cases encountered in practice during the design of working platforms for tracked plant. Note that the unit weight of the sand layers was kept constant during the parametric analysis; the effect of this assumption on the governing failure mechanism is discussed later. In addition, $H/B$ values outside the standard range reported in Table 1 were considered in selected cases, in order to estimate the critical thickness of the top layer i.e. the thickness where failure takes place exclusively in the top sand layer.
Table 1. Problem parameters considered in the parametric analysis.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Friction angle of top layer, $\varphi_1$</td>
<td>40° to 50°</td>
</tr>
<tr>
<td>Friction angle of bottom layer, $\varphi_2$</td>
<td>27.5° to 35°</td>
</tr>
<tr>
<td>Unit weight of top layer, $\gamma_1$</td>
<td>18 kN/m$^3$</td>
</tr>
<tr>
<td>Unit weight of bottom layer, $\gamma_2$</td>
<td>15 kN/m$^3$</td>
</tr>
<tr>
<td>Improvement thickness-over-width ratio, $H/B$</td>
<td>0.2 to 2.0</td>
</tr>
</tbody>
</table>

4. VALIDATION AND DISCUSSION

4.1 Error in the estimation of collapse loads

The use of adaptive re-meshing led to upper and lower bound collapse loads being close in all simulations. Figure depicts upper and lower bound bearing capacities calculated in a series of analyses where the thickness of the top layer $H$ and the friction angle of the bottom layer $\varphi_2$ were increased while keeping the rest of the problem parameters constant. Notice that the relative error $RE$ increases as the shear strength of the bottom sand layer increases, still it remains lower than the target value of 5%. As a result, the average bearing capacity from the upper bound and lower bound simulations is considered in the following as representative of the true collapse load.

![Figure 3: Lower and upper bounds of the bearing capacity from a selected series of FELA analyses.](image-url)
4.2 Comparison with experimental results and published solutions

The aim of the comparison that follows is two-fold: To validate the results of the computational methodology, but also to identify the conditions under which existing semi-empirical solutions diverge from numerical results and later determine the reasons behind the observed discrepancies. As mentioned earlier the scaled physical model test results reported in Meyerhof [33] and Das and Munoz [25] will be used in this section. Meyerhof [33] loaded up to failure a 0.05 m-wide strip footing, resting on the surface of two layers of sand, with the top and bottom layers featuring plane-strain friction angles $\phi_1 = 47.7^\circ$ and $\phi_2 = 34^\circ$, respectively. The dry density of the top and bottom layer was $\gamma_1 = 16.3$ kN/m$^3$ and $\gamma_2 = 13.8$ kN/m$^3$, respectively. The relative thickness of the top layer varied from $H/B = 0$ up to $H/B = 5$. The comparison of FELA results against the experimental data and the corresponding results obtained by applying Hanna’s and the BRE470 method is presented in Fig. 4. In addition, Fig. 5 presents a comparison of the same solutions against the bearing capacities measured by Das and Munoz [25], who tested a 0.1016 m-wide strip footing resting on the surface of a sand layer with $\phi_1 = 43^\circ$, underlain by a looser sand layer with $\phi_2 = 36^\circ$.

Results plotted in Fig. 4 suggest that FELA and Hanna’s method provide very similar bearing capacity values, and agree well with the experimental data at least for tests where $H/B \leq 3$. BRE470 estimations are consistently lower, and in better agreement with experimental data for this range of material parameters. For thickness of the top sand layer larger than the critical depth, where failure takes place entirely within the surficial layer, all methods underestimate the capacity. This is attributed to the small scale of experiments, and the fact that all methods consider an average, constant sand friction angle with depth; in reality the friction angle will be higher near the surface, due to the lower confining stress. Similar conclusions are drawn from Fig. 5, at least for the FELA analyses. It is worth mentioning that numerical analyses capture well the critical depth (where the bearing capacity does not further increase as $H$ increases) observed in both experimental datasets, which increases as the friction angle of the top layer increases. Notice however that Hanna’s method overestimates the bearing capacity in this case, where the shear strength of the two sand layers is comparable. On the other hand, the BRE470 method results in lower capacity values, and consistently underestimates the measured bearing capacity.
Figure 4: Comparison between FELA average bearing capacities, semi-empirical solutions [13, 14] and the experimental results of Meyerhof [33].

Figure 5: Comparison between FELA average bearing capacities, semi-empirical solutions [13, 14] and the experimental results of Das and Munoz [25].
In order to shed some light on the observed discrepancies, we plot in Fig. 6 the failure mechanisms obtained from three characteristic simulations of the abovementioned experiments, corresponding to different relative strength of the two sand layers \( q_b/q_t \) and top layer thickness \( H/B \). Notice that three different mechanisms develop:

- Punching shear mechanism (Fig. 6a), that occurs where the top layer has significantly higher shear strength compared to the bottom layer, and the thickness of the former is less than the critical depth. This mechanism is compatible with the mechanism considered by Hanna [14].

- General failure mechanism (Fig. 6c), that occurs when the thickness of the top layer is larger than the critical depth, and

- An intermediate mechanism (Fig. 6b) called transitional failure mechanism in the following, that is observed where the relative strength of the two layers \( q_b/q_t \) is higher than that required for punching to develop, while the thickness of top sand layer is not sufficient to fully accommodate the failure mechanism.

Note that the fact that the failure mechanism shown Fig. 6c appears to be non-symmetric is merely an artefact of the averaging method used to create velocity contours. The above suggest that the punching shear mechanism cannot describe all possible failure modes, and as a result Hanna’s method overpredicts the bearing capacity measured in Das and Munoz [25] experiments. A new model applicable in cases where a transitional failure mechanism will develop is described in the following section.
CHAPTER 8 BEARING CAPACITY OF STRIP FOOTINGS ON LAYERED SANDS

Figure 6: Velocity magnitudes depicting the failure mode observed in three characteristic analyses: a) Punching shear mechanism (Analysis corresponding to Meyerhof [33] experiment for $H/B=1$) b) Transitional failure mechanism (Analysis corresponding to Das and Munoz [25] experiment for $H/B=1.5$) c) General shear failure mechanism (Analysis corresponding to Das and Munoz [25] experiment for $H/B=5>H_{cr}/B$)

Figure 7: Simplified punching shear (a) and transitional failure (b) mechanisms.
5 TRANSITIONAL BEARING CAPACITY MECHANISM

5.1 Conditions for development of transitional failure mechanism

A series of 400 parametric FELA analyses were performed, considering the range of parameters listed in Table 1. The aim of these analyses was to identify under which conditions the mechanism changes from punching shear (Fig. 7a) to transitional (Fig. 7b), using power dissipation intensity and velocity contours to infer the geometry of the failure mechanism from each analysis. As depicted in Figs. 6 and 7, we can use the inclination of the shear planes in the top layer \( \theta \) to describe an idealised wedge, formed below the strip footing. Inclination \( \theta \), calculated directly from power dissipation intensity contour plots, is a function of the relative strength of the sand layers and of \( H/B \). Co-evaluation of all parametric analyses resulted in the following fitting expression for \( \theta \):

\[
\theta = a \ln(\tan(\phi_2)) + b \ln \left( \frac{H}{B} \right) + c
\]

(11)

in which, \( a \), \( b \) and \( c \) are constants depending on the friction angle of the top layer and are given in Table 2. Whether a transitional mechanism will develop or not depends on the value of the equivalent width \( B' = B - 2H\tan\theta \) (Fig. 7a), which is calculated using \( \theta \) from Eq. (11). If \( B' < 0 \) then a transitional failure mechanism will develop. Note however that while \( B' > 0 \) is a necessary condition for a punching shear failure mechanism, it is not sufficient. As discussed earlier, punching shear occurs when there is an abrupt change in shear strength of the two layers. \( B' \) may be greater than zero even for relatively high \( q_b/q_t \), if the thickness of the top layer is not sufficient to fully accommodate the elastic wedge below the footing. Such failure mechanisms, that feature a geometry similar to that of transitional mechanisms yet correspond to cases where \( B' > 0 \), are referred to as “shallow transitional” mechanisms. The problem parameters corresponding to each observed mechanism are presented in Fig. 8, which provides an overview under which conditions the shallow and the standard transitional mechanisms govern the bearing capacity of the footing and can be used to identify the dominant mode of failure.
Figure 8: Observed failure mechanisms for different $q_b/q_t$ and $H/B$ combinations.

Table 1. Constants $a$, $b$, $c$ (Eq. 12).

<table>
<thead>
<tr>
<th>$\phi_1$ (deg)</th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>17.11</td>
<td>-7.49</td>
<td>26.65</td>
</tr>
<tr>
<td>47.5</td>
<td>16.35</td>
<td>-6.98</td>
<td>28.47</td>
</tr>
<tr>
<td>45</td>
<td>16.08</td>
<td>-7.9</td>
<td>31.11</td>
</tr>
<tr>
<td>42.5</td>
<td>18.12</td>
<td>-8.61</td>
<td>34.60</td>
</tr>
<tr>
<td>40</td>
<td>18.34</td>
<td>-8.79</td>
<td>38.15</td>
</tr>
</tbody>
</table>
Note that Fig. 8 suggests that the geometry of the failure mechanism is independent of i) the width of the footing when $H/B$ is constant; ii) the actual value of the unit weight of sand layers when the ratio $\gamma_1/\gamma_2$ is constant. Additional simulations were performed to confirm that, some indicative results of which are presented in Fig. 9. Velocity vectors and power dissipation intensity contours shown in Fig. 9 confirm that angle $\theta$ is independent of the abovementioned parameters.

5.2 Bearing capacity factor for transitional failure mechanism

A new bearing capacity factor $N^*_b$ can be now defined to estimate the bearing capacity in cases where a (standard or shallow) transitional mechanism is likely to govern collapse, according to the method described above. $N^*_b$ is back-calculated, using results of the parametric FELA investigation and considering the standard bearing capacity equation for surface strip footings on sands:

![Figure 9](image-url)
The upper bound $q_t$ is the bearing capacity of a footing with the same geometry, resting on the surface of a uniform layer with the properties of the top dense sand, and applies when the thickness of top layer exceeds the critical depth. In other words, the upper bound to the bearing capacity factor $N_{\gamma}^*$ is the conventional bearing capacity factor $N_{\gamma}$. Results of the parametric study suggest that $N_{\gamma}^*$ is a function of $\phi_1$, $\phi_2$, $H/B$ (as the geometry of the failure mechanism) but also $\gamma_1/\gamma_2$. As discussed earlier, the assumption of constant $\gamma_1/\gamma_2 = 1.2$ is a reasonable approximation for practical cases. This assumption was adopted while preparing Figs. 10 through 14, which can be used to estimate $N_{\gamma}^*$ and accordingly the bearing capacity from Eq. (12) in cases where a transitional mechanism is critical, as these as identified in Fig. 8.

![Graph](image)

**Figure 10**: Bearing capacity factor $N_{\gamma}^*$ for $\phi_1 = 40^\circ$. 
Figure 11: Bearing capacity factor $N^*_\gamma$ for $\phi_1 = 42.5^\circ$.

Figure 12: Bearing capacity factor $N^*_\gamma$ for $\phi_1 = 45^\circ$. 
Figure 13: Bearing capacity factor $N^*_y$ for $\varphi_1 = 47.5^\circ$.

Figure 14: Bearing capacity factor $N^*_y$ for $\varphi_1 = 50^\circ$. 
6 CONCLUDING REMARKS
We have shown that apart from the punching shear mechanism (proposed by Hanna [14] and embraced by BRE470 [13]), a transitional failure mechanism may govern the bearing capacity of strip footings on layered sands, in cases where the thickness of the top layer is not sufficient for general shear failure to be contained within it. This transitional mechanism develops when there is not an abrupt increase in the shear strength at the interface of the two sand layers, and when the thickness of the top layer (normalised against the width of the footing) is rather small. We have also shown that using bearing capacity models based on the punching shear mechanism under these conditions leads to overestimating the collapse load of the footing. To cover this gap, we have identified under which conditions a transitional mechanism will be critical (Fig. 8), and propose a new bearing capacity formula (Eq. 12 and Figs.10-14) to calculate the collapse load in these cases. This formula applies for surface rough strip footings; hence it is appropriate for the design of working platforms for tracked plant.

Results of this numerical study also suggest that the method proposed by Hanna [14] provides accurate estimates of the bearing capacity, when a punching shear mechanism develops. On the other hand, the BRE470 method provides conservative results, which diverge from the FELA-calculated bearing capacities as the relative strength of the two layers \( q_b/q_t \) increases. In cases where a transitional mechanism is expected to govern the response, use of the proposed method will result in working platform thickness about 50% higher compared to the thickness calculated from Hanna’s method, but still about 70% lower than the thickness calculated via BRE470.

REFERENCES

Bibliography


Working platforms for tracked plant. Good practice guide to the design, installation, maintenance and repair of ground-supported working platforms. BRE470 (Building research Establishment). 2004.


CHAPTER 9 CONCLUSIONS
CHAPTER 9 CONCLUSIONS

9-1 GENERAL
This research was concerned with the bearing capacity of surface strip footing on two-layered soils, comprising dense sand overlying soft clay or loose sand. The main goals of this research were to develop new bearing capacity models that account for failure mechanisms of varying characteristics, which shape depends on the relative strength of the two layers. The proposed bearing capacity equations are easy to use and provide consistent accuracy for a wide range of material parameters and geometries, encountered in practice. The proposed models are developed using the results of two comprehensive parametric studies and are based on detailed analysis of the collapse mechanisms identified from multiple simulations. The models are verified against published experimental test results, and it is shown that they provide superior predictions compared to current state-of-practice methods. This chapter summarises the main outcomes of the research and a discussion on future research that could build upon this work.

9-2 MAIN FINDINGS

9-2-1 IDENTIFIED FAILURE MECHANISMS
The interpreted collapse mechanisms, using techniques discussed in Chapter 5, indicated that the punching shear failure mechanism of Meyerhof (1974) applies to a limited range of material properties i.e. when the top layer is significantly stronger than the bottom layer. In fact, the geometry of the collapse mechanism varies as function of relative strength of the two layers. It was found that the geometry of the collapse mechanisms is a function of $c_u/\gamma H$ and $\phi$ for footings resting on sand overlying clay. For surface footings on layered sand, $\phi_1$, $\phi_2$, $\gamma_1/\gamma_2$ and $H/B$ control the geometry of the collapse mechanism. As discussed in Chapters 4 and 6, the planes of failure in the top sand layer incline inwards as the undrained shear strength of the bottom clay layer increases, which results in variable effective footing widths. Therefore, the contribution of the bottom clay to the ultimate bearing capacity of the footing is a function of relative strength of the two layers. It was shown that assuming the punching shear mechanism of Meyerhof (1974) with vertical failure planes, results in overestimating the contribution of the bottom clay to ultimate bearing capacity of the footing, as clay becomes stronger relative to the sand.

Okamura et al. (1998) first, considered the variable contribution of the bottom clay layer to bearing capacity of the footing. However, as discussed in Section 2-3-4, their model only considers planes of failure that are extended outwards from the corners of the footing. Besides, the angles predicted by Equation 2-21 are inconsistent with the behaviour of footings on layered soils observed by other researchers such as Jacobsen and Christensen (1977), Brocklehurst (1993) and Burd and Frydman (1997). The observed collapse mechanisms in this research agree with
CHAPTER 9 CONCLUSIONS

these works and suggest that inwards inclination of the assumed linear planes of failure in sand increases as \(c_u/\gamma H\) increases for a constant \(\phi\), or as \(\phi\) decreases, when other parameters are kept constant. The inward inclination continues until the critical depth is reached for a given set of parameters, where the conventional general shear failure mechanism is formed entirely within the top sand layer.

The same observation was made for footings on layered sand. However, in this case the planes of failure incline inwards for all studied material parameters i.e. the practical range of soil friction angles. It was found that the punching shear failure mechanism applies only to a limited range of material parameters, where the top sand layer is significantly stronger than the bottom one. Transitional failure mechanism governs as \(q_s/q_t\) increases before the conventional general shear failure is fully accommodated within the top sand layer.

9-2-2 BEARING CAPACITY MODELS BASED ON THE OBSERVED FAILURE MECHANISMS

The collapse mechanisms were used as basis for the development of new bearing capacity models that account for variable modes of failure, depending on the relative strength of the two layers. The observed complex shear planes in sand were simplified by approximating the actual failure planes with lines, connecting the corners of the actual and the effective footings. An expression that provides the effective footing width was derived as function of \(c_u/\gamma H\) and \(\phi\). Based on this method punching shear coefficients (Ksr) were back-calculated from numerical results, as function of \(c_u/\gamma H\) and \(\phi\). This was integrated into a new bearing capacity model for footings on sand over clay, which is based on the derived expressions for Ksr and \(\theta\). The collapse load results from equilibrium of the elastic wedge under the footing.

For strip footings on layered sand, it was found that apart from the punching shear mechanism (proposed by Hanna 1981 and embraced by BRE470), a transitional failure mechanism may govern the bearing capacity of strip footings on layered sands, in cases where the thickness of the top layer is not sufficient for general shear failure to be contained within it. This transitional mechanism develops when there is not an abrupt increase in the shear strength at the interface of the two sand layers, and when the thickness of the top layer (normalised against the width of the footing) is rather small. It was also found that using bearing capacity models based on the punching shear mechanism under these conditions leads to overestimating the collapse load of the footing. A simple method is developed to identify the dominant mode of failure and a new bearing capacity factor is introduced for the range of material, in which the collapse mechanism deviates from the punching shear model of Meyerhof (1974).
9-2-3 MODEL VERIFICATION

The proposed bearing capacity models were validated against published experimental data and results were compared against some of the commonly used existing bearing capacity models. The experimental studies of Das and Dallo (1984) and Okamura et al. (1998) were used for the case of sand overlying clay. Very good agreement was found between the model predictions and these two sets of experimental data. The predicted bearing capacities were within 10% of the experimental tests results for all, but two cases. In no cases, the predicted values diverged more than 20% from the experimental test results.

The experimental test results by Meyerhof (1978) and Das and Munoz (1984) were used to validate the results of the bearing capacity model for footings resting on layered sand. The results of Hanna’s theory (1981) and the BRE470 method were also included for comparison. From experimental tests of Meyerhof (1978), it was found that FELA and Hanna’s method provide very similar bearing capacity values and agree well with the experimental data at least for tests where H/B ≤ 3. BRE470 predictions are consistently lower, and in better agreement with experimental data for this range of material parameters. For thickness of the top sand layer larger than the critical depth, where failure takes place entirely within the surficial layer, all methods underestimate the capacity. This is attributed to the small scale of experiments, and the fact that all these methods consider constant sand friction angle with depth. Similar conclusions are drawn from experimental data reported by Das and Munoz (1984), at least for the FELA analyses. The numerical analyses captured well the critical depth observed in both experimental datasets, which increases as the friction angle of the top layer increases. Results of this numerical study suggest that the method proposed by Hanna (1981) provides accurate estimates of the bearing capacity, when a punching shear mechanism develops. On the other hand, the BRE470 method provides conservative results, which diverge from the FELA-calculated bearing capacities as the relative strength of the two layers q_0/q_t increases. In cases where a transitional mechanism is expected to govern the response, use of the proposed method for calculating the bearing capacity of working platforms for tracked plants will result in platform thickness about 50% higher compared to the thickness calculated from Hanna’s method, but still about 70% lower than the thickness calculated via BRE470.

9-3 FURTHER RESEARCH

Assuming associated flow will generally result in upper bound values of the bearing capacity, however, the effect of material non-associativity on the limit loads is yet to be quantified in a robust way, owing to the challenges associated with considering non-associative flow such as non-
uniqueness of the limit load. An attempt to shed some light on this issue is presented in Appendix C, where additional simulations are presented in response to the Reviewers comments.

This research was concerned with the bearing capacity of surface rough strip footings on dense sand overlying loose sand and soft clay. Simulating strip, rough footings resting on the surface of the top layer is compatible with modelling of tracked plant. However, the method can be extended to embedded footings and general geometries, as a three-dimensional formulation of the FELA method has already been developed in the University of Newcastle.

As discussed in previous chapters, the bearing capacity of shallow footings depends on the strength anisotropy of the foundation soil. It is possible to account for soil anisotropy in FELA method via using an appropriate constitutive model. This capability of FELA could be exploited in the future.

Finally, review of the literature indicated that very limited experimental test results are reported for bearing capacity of footings on layered soils. Full scale footing tests or centrifuge tests can provide new insights on the problem, especially for the case of footings on layered sands for which, the collapse load is significantly influenced by stress level and scale effects.
CHAPTER 10 REFERENCES


CHAPTER 10 REFERENCES


APPENDIX A - UPPER AND LOWER BOUND FELA SOLUTIONS FOR STRIP FOOTINGS ON DENSE SAND OVER CLAY
APPENDIX A NUMERICAL RESULTS USED IN CHAPTER 6

PREFACE

A comprehensive parametric study was performed to investigate ultimate bearing capacity of surface strip footings on dense sand over relatively weak clays, considering a wide range of parameters. The upper and lower bound solutions of the collapse load, applied in developing the bearing capacity model presented in Chapter 6 are summarised in Appendix A. The ultimate bearing capacity of a footing on dense sand over clays is presented in Table A1 and Table A2 for footings with widths of 0.6m and 1.8m, respectively. Additional results are presented in Table A1, for footing widths of 1.4m and 1.6m, where critical depths were reached for footing width of 0.6m. Results are presented for sand friction angles ranging from 35˚ to 55˚, with 5˚ intervals, and sand layer thicknesses ranging from 0.4m to 1.6m with 0.4m intervals. The undrained shear strength of clay varies between 10 kPa and 70 kPa with 10 kPa intervals.

Collapse mechanisms, inferred from the results of upper bound analyses were also considered in developing the proposed bearing capacity model of Chapter 6. Table A3 of Appendix A presents the measured horizontal distance between the corners of actual footing and the location of maximum power dissipation intensity at the interface of the two soil layers. The angles of the lines, connecting corners of actual and effective footings to vertical (θ) is a key parameter of the proposed bearing capacity model. Measured values of θ in the studied range of material parameters are presented in Table A3 of Appendix A.
Table A1: Calculated upper and lower limits to the ultimate bearing capacity of strip footings on dense sand over relatively weak clay, for footing width of 0.6m.

### B= 0.6 m, $\phi=35^\circ$, $\gamma = 15.5$ kN/m$^3$

<table>
<thead>
<tr>
<th>$C_s$(kPa)</th>
<th>10</th>
<th>20</th>
<th>30, [B=1.4 m]</th>
<th>40, [B=1.4 m]</th>
<th>50, [B=1.4 m]</th>
<th>60, [B=1.4 m]</th>
<th>70, [B=1.4 m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>H (m)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td>63.6</td>
<td>59.9</td>
<td>103.4</td>
<td>95.9</td>
<td>146.6</td>
<td>139.5</td>
<td>187.9</td>
</tr>
<tr>
<td>0.8</td>
<td>105.7</td>
<td>98.7</td>
<td>153.2</td>
<td>140.4</td>
<td>166.5</td>
<td>157.6</td>
<td>204.4</td>
</tr>
<tr>
<td>1.2</td>
<td>163.2</td>
<td>152.2</td>
<td>188.2</td>
<td>150.6</td>
<td>200.8</td>
<td>189.2</td>
<td>240.6</td>
</tr>
<tr>
<td>1.6</td>
<td>187.9</td>
<td>157.7</td>
<td>185.8</td>
<td>152.5</td>
<td>246.0</td>
<td>234.2</td>
<td>290.8</td>
</tr>
</tbody>
</table>

### B= 0.6 m, $\phi=40^\circ$, $\gamma = 16.5$ kN/m$^3$

<table>
<thead>
<tr>
<th>$C_s$(kPa)</th>
<th>10</th>
<th>20</th>
<th>30, [B=1.4 m]</th>
<th>40, [B=1.4 m]</th>
<th>50, [B=1.4 m]</th>
<th>60, [B=1.4 m]</th>
<th>70, [B=1.4 m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>H (m)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td>76.6</td>
<td>72.4</td>
<td>129.2</td>
<td>120.9</td>
<td>176.1</td>
<td>163.7</td>
<td>219.1</td>
</tr>
<tr>
<td>0.8</td>
<td>134.6</td>
<td>126.1</td>
<td>204.6</td>
<td>190.0</td>
<td>263.7</td>
<td>241.9</td>
<td>314.3</td>
</tr>
<tr>
<td>1.2</td>
<td>211.0</td>
<td>198.8</td>
<td>306.8</td>
<td>285.6</td>
<td>384.3</td>
<td>354.6</td>
<td>455.4</td>
</tr>
<tr>
<td>1.6</td>
<td>301.8</td>
<td>284.0</td>
<td>429.2</td>
<td>389.8</td>
<td>435.0</td>
<td>384.5</td>
<td>520.4</td>
</tr>
</tbody>
</table>

### B= 0.6 m, $\phi=45^\circ$, $\gamma = 17.5$ kN/m$^3$

<table>
<thead>
<tr>
<th>$C_s$(kPa)</th>
<th>10</th>
<th>20</th>
<th>30, [B=1.4 m]</th>
<th>40, [B=1.4 m]</th>
<th>50, [B=1.4 m]</th>
<th>60, [B=1.4 m]</th>
<th>70, [B=1.4 m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>H (m)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td>88.0</td>
<td>84.7</td>
<td>153.5</td>
<td>143.2</td>
<td>210.6</td>
<td>196.1</td>
<td>266.0</td>
</tr>
<tr>
<td>0.8</td>
<td>164.3</td>
<td>154.8</td>
<td>256.4</td>
<td>238.3</td>
<td>335.9</td>
<td>310.1</td>
<td>408.0</td>
</tr>
<tr>
<td>1.2</td>
<td>263.1</td>
<td>246.5</td>
<td>392.7</td>
<td>366.1</td>
<td>499.7</td>
<td>462.5</td>
<td>600.6</td>
</tr>
<tr>
<td>1.6</td>
<td>378.1</td>
<td>355.9</td>
<td>555.4</td>
<td>518.7</td>
<td>696.3</td>
<td>646.3</td>
<td>829.2</td>
</tr>
</tbody>
</table>

### B= 0.6 m, $\phi=50^\circ$, $\gamma = 18.7$ kN/m$^3$

<table>
<thead>
<tr>
<th>$C_s$(kPa)</th>
<th>10</th>
<th>20</th>
<th>30, [B=1.4 m]</th>
<th>40, [B=1.4 m]</th>
<th>50, [B=1.4 m]</th>
<th>60, [B=1.4 m]</th>
<th>70, [B=1.4 m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>H (m)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td>104.6</td>
<td>97.5</td>
<td>178.0</td>
<td>166</td>
<td>247.4</td>
<td>229.4</td>
<td>313.5</td>
</tr>
<tr>
<td>0.8</td>
<td>200.9</td>
<td>187</td>
<td>314.7</td>
<td>290.9</td>
<td>416</td>
<td>381.8</td>
<td>506.3</td>
</tr>
<tr>
<td>1.2</td>
<td>325.5</td>
<td>302.8</td>
<td>487.6</td>
<td>455.2</td>
<td>627.5</td>
<td>578.6</td>
<td>753.3</td>
</tr>
<tr>
<td>1.6</td>
<td>470.7</td>
<td>438.3</td>
<td>698.5</td>
<td>647.3</td>
<td>833.7</td>
<td>818.7</td>
<td>1050</td>
</tr>
</tbody>
</table>

### B= 0.6 m, $\phi=55^\circ$, $\gamma = 20.0$ kN/m$^3$

<table>
<thead>
<tr>
<th>$C_s$(kPa)</th>
<th>10</th>
<th>20</th>
<th>30, [B=1.4 m]</th>
<th>40, [B=1.4 m]</th>
<th>50, [B=1.4 m]</th>
<th>60, [B=1.4 m]</th>
<th>70, [B=1.4 m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>H (m)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td>121.2</td>
<td>111.9</td>
<td>205.4</td>
<td>189.6</td>
<td>284.4</td>
<td>262.1</td>
<td>358.9</td>
</tr>
<tr>
<td>0.8</td>
<td>242.6</td>
<td>225.1</td>
<td>381.4</td>
<td>351</td>
<td>503.6</td>
<td>458.7</td>
<td>617.6</td>
</tr>
<tr>
<td>1.2</td>
<td>400.4</td>
<td>369</td>
<td>603.4</td>
<td>556.4</td>
<td>779.4</td>
<td>716.1</td>
<td>941.1</td>
</tr>
<tr>
<td>1.6</td>
<td>588.6</td>
<td>539.2</td>
<td>869.5</td>
<td>797.3</td>
<td>1107</td>
<td>1043</td>
<td>1334</td>
</tr>
</tbody>
</table>

Note: Footing widths of 1.4m and 1.6m were used where critical depths are reached for footing width of 0.6m.
Table A2 Calculated upper and lower limits to the ultimate bearing capacity of strip footings on dense sand over relatively weak clay, for footing width of 1.8m.

<table>
<thead>
<tr>
<th>C_d(kPa)</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
</tr>
</thead>
<tbody>
<tr>
<td>H (m)</td>
<td>Q_U(kPa)</td>
<td>Q_L(kPa)</td>
<td>Q_U(kPa)</td>
<td>Q_L(kPa)</td>
<td>Q_U(kPa)</td>
<td>Q_L(kPa)</td>
<td>Q_U(kPa)</td>
</tr>
<tr>
<td>0.4</td>
<td>55.8</td>
<td>53.9</td>
<td>103.7</td>
<td>99.9</td>
<td>149.5</td>
<td>142.3</td>
<td>192.9</td>
</tr>
<tr>
<td>0.8</td>
<td>40.6</td>
<td>66.7</td>
<td>119.1</td>
<td>113.9</td>
<td>164.5</td>
<td>156.2</td>
<td>205.8</td>
</tr>
<tr>
<td>1.2</td>
<td>88.9</td>
<td>85.4</td>
<td>143.3</td>
<td>136.8</td>
<td>191.1</td>
<td>180.6</td>
<td>232.9</td>
</tr>
<tr>
<td>1.6</td>
<td>113.0</td>
<td>108.3</td>
<td>174.5</td>
<td>165.8</td>
<td>225.2</td>
<td>213.1</td>
<td>270.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>C_d(kPa)</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
</tr>
</thead>
<tbody>
<tr>
<td>H (m)</td>
<td>Q_U(kPa)</td>
<td>Q_L(kPa)</td>
<td>Q_U(kPa)</td>
<td>Q_L(kPa)</td>
<td>Q_U(kPa)</td>
<td>Q_L(kPa)</td>
<td>Q_U(kPa)</td>
</tr>
<tr>
<td>0.4</td>
<td>60.3</td>
<td>58.0</td>
<td>112.4</td>
<td>108.0</td>
<td>162.7</td>
<td>156.3</td>
<td>211.6</td>
</tr>
<tr>
<td>0.8</td>
<td>79.0</td>
<td>75.9</td>
<td>136.5</td>
<td>130.8</td>
<td>190.8</td>
<td>181.9</td>
<td>241.1</td>
</tr>
<tr>
<td>1.2</td>
<td>105.0</td>
<td>100.3</td>
<td>170.6</td>
<td>163.1</td>
<td>229.4</td>
<td>218.5</td>
<td>284.6</td>
</tr>
<tr>
<td>1.6</td>
<td>136.5</td>
<td>129.9</td>
<td>212.6</td>
<td>202.6</td>
<td>278.2</td>
<td>264.5</td>
<td>338.9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>C_d(kPa)</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
</tr>
</thead>
<tbody>
<tr>
<td>H (m)</td>
<td>Q_U(kPa)</td>
<td>Q_L(kPa)</td>
<td>Q_U(kPa)</td>
<td>Q_L(kPa)</td>
<td>Q_U(kPa)</td>
<td>Q_L(kPa)</td>
<td>Q_U(kPa)</td>
</tr>
<tr>
<td>0.4</td>
<td>64.8</td>
<td>62.2</td>
<td>120.8</td>
<td>115.7</td>
<td>174.6</td>
<td>168.2</td>
<td>227.9</td>
</tr>
<tr>
<td>0.8</td>
<td>90.0</td>
<td>85.8</td>
<td>154.6</td>
<td>147.4</td>
<td>215.9</td>
<td>204.9</td>
<td>274.5</td>
</tr>
<tr>
<td>1.2</td>
<td>123.0</td>
<td>116.8</td>
<td>200.1</td>
<td>189.4</td>
<td>268.9</td>
<td>255.1</td>
<td>334.3</td>
</tr>
<tr>
<td>1.6</td>
<td>162.3</td>
<td>153.8</td>
<td>253.2</td>
<td>240.8</td>
<td>334.6</td>
<td>316.6</td>
<td>407.9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>C_d(kPa)</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
</tr>
</thead>
<tbody>
<tr>
<td>H (m)</td>
<td>Q_U(kPa)</td>
<td>Q_L(kPa)</td>
<td>Q_U(kPa)</td>
<td>Q_L(kPa)</td>
<td>Q_U(kPa)</td>
<td>Q_L(kPa)</td>
<td>Q_U(kPa)</td>
</tr>
<tr>
<td>0.4</td>
<td>69.8</td>
<td>66.8</td>
<td>129.0</td>
<td>123.6</td>
<td>186.6</td>
<td>178.3</td>
<td>243.7</td>
</tr>
<tr>
<td>0.8</td>
<td>102.5</td>
<td>96.4</td>
<td>174.1</td>
<td>164.9</td>
<td>240.9</td>
<td>228.9</td>
<td>306.5</td>
</tr>
<tr>
<td>1.2</td>
<td>144.0</td>
<td>135.3</td>
<td>232.4</td>
<td>217.9</td>
<td>312.9</td>
<td>295.1</td>
<td>387.2</td>
</tr>
<tr>
<td>1.6</td>
<td>194.1</td>
<td>182.1</td>
<td>301.6</td>
<td>283.9</td>
<td>397.6</td>
<td>373.8</td>
<td>485.6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>C_d(kPa)</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
</tr>
</thead>
<tbody>
<tr>
<td>H (m)</td>
<td>Q_U(kPa)</td>
<td>Q_L(kPa)</td>
<td>Q_U(kPa)</td>
<td>Q_L(kPa)</td>
<td>Q_U(kPa)</td>
<td>Q_L(kPa)</td>
<td>Q_U(kPa)</td>
</tr>
<tr>
<td>0.4</td>
<td>75.7</td>
<td>72.1</td>
<td>138.0</td>
<td>131.7</td>
<td>199.3</td>
<td>189.1</td>
<td>259.2</td>
</tr>
<tr>
<td>0.8</td>
<td>117.1</td>
<td>109.8</td>
<td>196.5</td>
<td>184.1</td>
<td>270.6</td>
<td>254.6</td>
<td>342.4</td>
</tr>
<tr>
<td>1.2</td>
<td>170.6</td>
<td>159.0</td>
<td>272.1</td>
<td>253.8</td>
<td>361.6</td>
<td>339.5</td>
<td>447.6</td>
</tr>
<tr>
<td>1.6</td>
<td>234.0</td>
<td>216.1</td>
<td>358.9</td>
<td>335.5</td>
<td>474.0</td>
<td>438.1</td>
<td>574.6</td>
</tr>
</tbody>
</table>
APPENDIX A NUMERICAL RESULTS USED IN CHAPTER 6

Table A3: Measured horizontal distance between corners of actual and effective footings and corresponding calculated $\theta$.

<table>
<thead>
<tr>
<th>$C_d$(kPa)</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi=35^\circ$, $\gamma = 15.5$ KN/m$^3$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H (m)</td>
<td>$\Delta X$ (m)</td>
<td>$\theta$ (\degree)</td>
<td>$\Delta X$ (m)</td>
<td>$\theta$ (\degree)</td>
<td>$\Delta X$ (m)</td>
<td>$\theta$ (\degree)</td>
<td>$\Delta X$ (m)</td>
</tr>
<tr>
<td>0.4</td>
<td>-0.16</td>
<td>21.8</td>
<td>-0.22</td>
<td>28.8</td>
<td>-0.25</td>
<td>32.0</td>
<td>-0.28</td>
</tr>
<tr>
<td>0.8</td>
<td>-0.19</td>
<td>-13.4</td>
<td>-0.32</td>
<td>-21.8</td>
<td>-0.40</td>
<td>-26.6</td>
<td>-0.44</td>
</tr>
<tr>
<td>1.2</td>
<td>-0.17</td>
<td>-8.1</td>
<td>-0.38</td>
<td>-17.6</td>
<td>-0.48</td>
<td>-21.8</td>
<td>-0.57</td>
</tr>
<tr>
<td>1.6</td>
<td>-0.10</td>
<td>-3.6</td>
<td>-0.38</td>
<td>-13.4</td>
<td>-0.54</td>
<td>-18.7</td>
<td>-0.64</td>
</tr>
</tbody>
</table>

| $\phi=40^\circ$, $\gamma = 16.5$ KN/m$^3$|
| H (m)  | $\Delta X$ (m) | $\theta$ (\degree) | $\Delta X$ (m) | $\theta$ (\degree) | $\Delta X$ (m) | $\theta$ (\degree) | $\Delta X$ (m) | $\theta$ (\degree) | $\Delta X$ (m) | $\theta$ (\degree) |
| 0.4    | -0.09 | -12.7 | -0.14 | -19.3 | -0.17 | -23.0 | -0.19 | -25.4 | -0.20 | -26.6 | -0.21 | -27.7 | -0.22 | -28.8 |
| 0.8    | -0.07 | -5.0 | -0.18 | -12.7 | -0.25 | -17.4 | -0.28 | -19.3 | -0.31 | -21.2 | -0.34 | -23.0 | -0.36 | -24.2 |
| 1.2    | 0.00 | 0.0 | -0.18 | -8.5 | -0.27 | -12.7 | -0.35 | -16.3 | -0.39 | -18.0 | -0.42 | -19.3 | -0.46 | -21.0 |
| 1.6    | 0.10 | 3.6 | -0.14 | -5.0 | -0.28 | -9.9 | -0.36 | -12.7 | -0.44 | -15.4 | -0.50 | -17.4 | -0.52 | -18.0 |

| $\phi=45^\circ$, $\gamma = 17.5$ KN/m$^3$|
| H (m)  | $\Delta X$ (m) | $\theta$ (\degree) | $\Delta X$ (m) | $\theta$ (\degree) | $\Delta X$ (m) | $\theta$ (\degree) | $\Delta X$ (m) | $\theta$ (\degree) | $\Delta X$ (m) | $\theta$ (\degree) |
| 0.4    | -0.04 | -5.7 | -0.08 | -11.3 | -0.10 | -14.0 | -0.12 | -16.7 | -0.13 | -18.0 | -0.14 | -19.3 | -0.15 | -20.6 |
| 0.8    | 0.03 | 2.2 | -0.08 | -5.7 | -0.14 | -9.9 | -0.16 | -11.3 | -0.19 | -13.4 | -0.20 | -14.0 | -0.23 | -16.0 |
| 1.2    | 0.13 | 6.2 | -0.03 | -1.4 | -0.12 | -5.7 | -0.19 | -9.0 | -0.23 | -10.9 | -0.24 | -11.3 | -0.28 | -13.1 |
| 1.6    | 0.28 | 9.9 | -0.06 | -2.2 | -0.07 | -2.5 | -0.16 | -5.7 | -0.23 | -8.2 | -0.28 | -9.9 | -0.31 | -11.6 |

| $\phi=50^\circ$, $\gamma = 18.7$ KN/m$^3$|
| H (m)  | $\Delta X$ (m) | $\theta$ (\degree) | $\Delta X$ (m) | $\theta$ (\degree) | $\Delta X$ (m) | $\theta$ (\degree) | $\Delta X$ (m) | $\theta$ (\degree) | $\Delta X$ (m) | $\theta$ (\degree) |
| 0.4    | 0.01 | 1.4 | -0.04 | -5.7 | -0.06 | -8.5 | -0.07 | -9.9 | -0.08 | -11.3 | -0.09 | -12.7 | -0.10 | -14.0 |
| 0.8    | 0.11 | 7.8 | -0.02 | 1.4 | -0.05 | -3.6 | -0.08 | -5.7 | -0.10 | -7.1 | -0.12 | -8.5 | -0.13 | -9.2 |
| 1.2    | 0.24 | 11.3 | 0.09 | 4.3 | 0.03 | 1.4 | -0.05 | 2.4 | -0.09 | 4.3 | -0.12 | 5.7 | -0.14 | 6.7 |
| 1.6    | 0.42 | 14.7 | 0.22 | 7.8 | 0.10 | 3.6 | 0.04 | 1.4 | -0.04 | 1.4 | -0.10 | 3.6 | -0.13 | 4.7 |

| $\phi=55^\circ$, $\gamma = 20.0$ KN/m$^3$|
| H (m)  | $\Delta X$ (m) | $\theta$ (\degree) | $\Delta X$ (m) | $\theta$ (\degree) | $\Delta X$ (m) | $\theta$ (\degree) | $\Delta X$ (m) | $\theta$ (\degree) | $\Delta X$ (m) | $\theta$ (\degree) |
| 0.4    | 0.05 | 7.1 | 0.00 | 0.0 | -0.02 | -2.9 | -0.03 | -4.3 | -0.04 | -5.7 | -0.05 | -6.4 | -0.05 | -7.1 |
| 0.8    | 0.17 | 12.0 | 0.10 | 7.1 | 0.03 | 2.2 | 0.00 | 0.0 | -0.03 | 2.2 | -0.04 | 2.9 | -0.06 | 4.3 |
| 1.2    | 0.34 | 15.8 | 0.19 | 9.0 | 0.15 | 7.1 | 0.06 | 2.9 | 0.03 | 1.4 | 0.06 | 0.0 | -0.03 | 1.4 |
| 1.6    | 0.58 | 19.9 | 0.34 | 12.0 | 0.23 | 8.2 | 0.20 | 7.1 | 0.13 | 4.7 | 0.06 | 2.2 | 0.03 | 1.1 |
APPENDIX B- UPPER AND LOWER BOUND FELA SOLUTIONS FOR STRIP FOOTINGS ON DENSE SAND OVER CLAY
PREFACE

A comprehensive parametric study was performed to investigate the behaviour of surface strip footings on dense sand over relatively loose to medium dense sands. The upper and lower bound solutions of the collapse load for the studied range of material parameters are presented in Table B1 to Table B5 of Appendix B for sand unit weight ratios of 1.2. The presented values correspond to top sand friction angles ranging from 40° to 50° and bottom sand friction angles ranging from 27.5° to 35°, with 2.5° intervals.

Collapse mechanisms were inferred from the results of upper bound analyses, from which the inclination angle of approximate linear planes of failure in top sand θ were measured. Table B6 to B10 of Appendix B present the measured horizontal distance between the corners of actual footing and the locations of maximum power dissipation intensity at the interface of the two sand layers ΔX, and the corresponding θ for all examined cases.
Table B1: Calculated upper and lower limits to the ultimate bearing capacity of strip footings on dense sand over relatively loose sand for top sand friction angle of 40°.

<table>
<thead>
<tr>
<th>φ₂</th>
<th>27.5</th>
<th>30.0</th>
<th>32.5</th>
<th>35.0</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>H</td>
<td>Qᵥ (kPa)</td>
<td>Qᵤ (kPa)</td>
<td>H</td>
</tr>
<tr>
<td>H/B</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>0.6</td>
<td>384</td>
<td>400</td>
<td>0.6</td>
</tr>
<tr>
<td>0.3</td>
<td>0.9</td>
<td>460</td>
<td>479</td>
<td>0.9</td>
</tr>
<tr>
<td>0.4</td>
<td>1.2</td>
<td>545</td>
<td>567</td>
<td>1.2</td>
</tr>
<tr>
<td>0.5</td>
<td>1.5</td>
<td>621</td>
<td>646</td>
<td>1.5</td>
</tr>
<tr>
<td>0.6</td>
<td>1.6</td>
<td>472</td>
<td>492</td>
<td>1.2</td>
</tr>
<tr>
<td>0.8</td>
<td>1.6</td>
<td>597</td>
<td>622</td>
<td>1.6</td>
</tr>
<tr>
<td>1.0</td>
<td>2.0</td>
<td>733</td>
<td>765</td>
<td>2.0</td>
</tr>
<tr>
<td>1.2</td>
<td>1.2</td>
<td>444</td>
<td>472</td>
<td>1.2</td>
</tr>
<tr>
<td>1.6</td>
<td>1.6</td>
<td>594</td>
<td>642</td>
<td>1.6</td>
</tr>
<tr>
<td>2.0</td>
<td>2.0</td>
<td>707</td>
<td>798</td>
<td>2.0</td>
</tr>
</tbody>
</table>

Table B2: Calculated upper and lower limits to the ultimate bearing capacity of dense sand over relatively loose sand for top sand friction angle of 42.5°.

<table>
<thead>
<tr>
<th>φ₂</th>
<th>27.5</th>
<th>30.0</th>
<th>32.5</th>
<th>35.0</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>H</td>
<td>Qᵥ (kPa)</td>
<td>Qᵤ (kPa)</td>
<td>H</td>
</tr>
<tr>
<td>H/B</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>0.6</td>
<td>405</td>
<td>423</td>
<td>0.6</td>
</tr>
<tr>
<td>0.3</td>
<td>0.9</td>
<td>495</td>
<td>516</td>
<td>0.9</td>
</tr>
<tr>
<td>0.4</td>
<td>1.2</td>
<td>596</td>
<td>623</td>
<td>1.2</td>
</tr>
<tr>
<td>0.5</td>
<td>1.5</td>
<td>688</td>
<td>719</td>
<td>1.5</td>
</tr>
<tr>
<td>0.6</td>
<td>1.2</td>
<td>528</td>
<td>554</td>
<td>1.2</td>
</tr>
<tr>
<td>0.8</td>
<td>1.6</td>
<td>686</td>
<td>715</td>
<td>1.6</td>
</tr>
<tr>
<td>1.0</td>
<td>2.0</td>
<td>858</td>
<td>896</td>
<td>2.0</td>
</tr>
<tr>
<td>1.2</td>
<td>1.2</td>
<td>530</td>
<td>567</td>
<td>1.2</td>
</tr>
<tr>
<td>1.6</td>
<td>1.6</td>
<td>730</td>
<td>776</td>
<td>1.6</td>
</tr>
<tr>
<td>2.0</td>
<td>2.0</td>
<td>958</td>
<td>1031</td>
<td>2.0</td>
</tr>
<tr>
<td>2.4</td>
<td>2.4</td>
<td>1082</td>
<td>1309</td>
<td>2.4</td>
</tr>
<tr>
<td>2.8</td>
<td>2.8</td>
<td>1139</td>
<td>1296</td>
<td>2.8</td>
</tr>
</tbody>
</table>
Table B3: Calculated upper and lower limits to the ultimate bearing capacity of dense sand over relatively loose sand for top sand friction angle of 45°.

<table>
<thead>
<tr>
<th>Φ₀</th>
<th>27.5</th>
<th>30.0</th>
<th>32.5</th>
<th>35.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>H/B</td>
<td>H</td>
<td>Q₀</td>
<td>kPa</td>
<td>H</td>
</tr>
<tr>
<td>0.2</td>
<td>0.6</td>
<td>427</td>
<td>445</td>
<td>0.6</td>
</tr>
<tr>
<td>0.3</td>
<td>0.9</td>
<td>531</td>
<td>553</td>
<td>0.9</td>
</tr>
<tr>
<td>0.4</td>
<td>1.2</td>
<td>648</td>
<td>675</td>
<td>1.2</td>
</tr>
<tr>
<td>0.5</td>
<td>1.5</td>
<td>757</td>
<td>792</td>
<td>1.5</td>
</tr>
<tr>
<td>0.6</td>
<td>1.2</td>
<td>590</td>
<td>616</td>
<td>1.2</td>
</tr>
<tr>
<td>0.8</td>
<td>1.6</td>
<td>777</td>
<td>810</td>
<td>1.6</td>
</tr>
<tr>
<td>1.0</td>
<td>2.0</td>
<td>985</td>
<td>1031</td>
<td>2.0</td>
</tr>
<tr>
<td>1.2</td>
<td>1.2</td>
<td>617</td>
<td>647</td>
<td>1.2</td>
</tr>
<tr>
<td>1.6</td>
<td>1.6</td>
<td>882</td>
<td>924</td>
<td>1.6</td>
</tr>
<tr>
<td>2.0</td>
<td>2.0</td>
<td>1194</td>
<td>1256</td>
<td>2.0</td>
</tr>
<tr>
<td>2.4</td>
<td>2.4</td>
<td>1528</td>
<td>1631</td>
<td>2.4</td>
</tr>
<tr>
<td>2.8</td>
<td>2.8</td>
<td>1802</td>
<td>2041</td>
<td>2.8</td>
</tr>
</tbody>
</table>

Table B4: Calculated upper and lower limits to the ultimate bearing capacity of dense sand over relatively loose sand for top sand friction angle of 47.5°.

<table>
<thead>
<tr>
<th>Φ₀</th>
<th>27.5</th>
<th>30.0</th>
<th>32.5</th>
<th>35.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>H/B</td>
<td>H</td>
<td>Q₀</td>
<td>kPa</td>
<td>H</td>
</tr>
<tr>
<td>0.2</td>
<td>0.6</td>
<td>448</td>
<td>467</td>
<td>0.6</td>
</tr>
<tr>
<td>0.3</td>
<td>0.9</td>
<td>564</td>
<td>590</td>
<td>0.9</td>
</tr>
<tr>
<td>0.4</td>
<td>1.2</td>
<td>699</td>
<td>730</td>
<td>1.2</td>
</tr>
<tr>
<td>0.5</td>
<td>1.5</td>
<td>827</td>
<td>865</td>
<td>1.5</td>
</tr>
<tr>
<td>0.6</td>
<td>1.2</td>
<td>648</td>
<td>681</td>
<td>1.2</td>
</tr>
<tr>
<td>0.8</td>
<td>1.6</td>
<td>865</td>
<td>910</td>
<td>1.6</td>
</tr>
<tr>
<td>1.0</td>
<td>2.0</td>
<td>1116</td>
<td>1171</td>
<td>2.0</td>
</tr>
<tr>
<td>1.2</td>
<td>1.2</td>
<td>705</td>
<td>744</td>
<td>1.2</td>
</tr>
<tr>
<td>1.6</td>
<td>1.6</td>
<td>1018</td>
<td>1077</td>
<td>1.6</td>
</tr>
<tr>
<td>2.0</td>
<td>2.0</td>
<td>1412</td>
<td>1486</td>
<td>2.0</td>
</tr>
<tr>
<td>2.4</td>
<td>2.4</td>
<td>1846</td>
<td>1957</td>
<td>2.4</td>
</tr>
<tr>
<td>2.8</td>
<td>2.8</td>
<td>2339</td>
<td>2488</td>
<td>2.8</td>
</tr>
</tbody>
</table>
Table B5: Calculated upper and lower limits to the ultimate bearing capacity of dense sand over relatively loose sand for top sand friction angle of 50°.

<table>
<thead>
<tr>
<th>φ</th>
<th>H/B</th>
<th>27.5</th>
<th>30.0</th>
<th>32.5</th>
<th>35.0</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>H</td>
<td>Q_U (kPa)</td>
<td>Q_L (kPa)</td>
<td>Q_U (kPa)</td>
<td>Q_L (kPa)</td>
</tr>
<tr>
<td>0.2</td>
<td>0.6</td>
<td>465</td>
<td>489</td>
<td>0.6</td>
<td>638</td>
</tr>
<tr>
<td>0.3</td>
<td>0.9</td>
<td>596</td>
<td>626</td>
<td>0.9</td>
<td>794</td>
</tr>
<tr>
<td>0.4</td>
<td>1.2</td>
<td>748</td>
<td>785</td>
<td>1.2</td>
<td>985</td>
</tr>
<tr>
<td>0.5</td>
<td>1.5</td>
<td>899</td>
<td>941</td>
<td>1.5</td>
<td>1154</td>
</tr>
<tr>
<td>0.6</td>
<td>1.2</td>
<td>706</td>
<td>746</td>
<td>1.2</td>
<td>905</td>
</tr>
<tr>
<td>0.8</td>
<td>1.6</td>
<td>960</td>
<td>1013</td>
<td>1.6</td>
<td>1205</td>
</tr>
<tr>
<td>1.0</td>
<td>2.0</td>
<td>1245</td>
<td>1319</td>
<td>2.0</td>
<td>1537</td>
</tr>
<tr>
<td>1.2</td>
<td>1.2</td>
<td>801</td>
<td>848</td>
<td>1.2</td>
<td>976</td>
</tr>
<tr>
<td>1.6</td>
<td>1.6</td>
<td>1168</td>
<td>1239</td>
<td>1.6</td>
<td>1394</td>
</tr>
<tr>
<td>2.0</td>
<td>2.0</td>
<td>1631</td>
<td>1724</td>
<td>2.0</td>
<td>1930</td>
</tr>
<tr>
<td>2.4</td>
<td>2.4</td>
<td>2161</td>
<td>2292</td>
<td>2.4</td>
<td>2522</td>
</tr>
<tr>
<td>2.8</td>
<td>2.8</td>
<td>2773</td>
<td>2942</td>
<td>2.8</td>
<td>3179</td>
</tr>
</tbody>
</table>

Note: φ = 50°, γ1 = 18.0 KN/m³, (γ2/γ1) = 1.2
### Table B6: Measured horizontal distance between corners of actual and effective footings and corresponding calculated $\theta$, for top sand friction angle of 40°.

<table>
<thead>
<tr>
<th>$\phi_2$</th>
<th>27.5</th>
<th>30.0</th>
<th>32.5</th>
<th>35.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>H/B</td>
<td>$\Delta X(m)$</td>
<td>$\theta(\circ)$</td>
<td>$\Delta X(m)$</td>
<td>$\theta(\circ)$</td>
</tr>
<tr>
<td>0.2</td>
<td>0.6</td>
<td>0.50</td>
<td>39.8</td>
<td>0.6</td>
</tr>
<tr>
<td>0.3</td>
<td>0.9</td>
<td>0.70</td>
<td>37.9</td>
<td>0.9</td>
</tr>
<tr>
<td>0.4</td>
<td>1.2</td>
<td>0.84</td>
<td>35.0</td>
<td>1.2</td>
</tr>
<tr>
<td>0.5</td>
<td>1.5</td>
<td>0.91</td>
<td>31.3</td>
<td>1.5</td>
</tr>
<tr>
<td>0.6</td>
<td>1.2</td>
<td>0.69</td>
<td>29.9</td>
<td>1.2</td>
</tr>
<tr>
<td>0.8</td>
<td>1.6</td>
<td>0.85</td>
<td>28.0</td>
<td>1.6</td>
</tr>
<tr>
<td>1.0</td>
<td>2.0</td>
<td>0.97</td>
<td>25.9</td>
<td>2.0</td>
</tr>
<tr>
<td>1.2</td>
<td>1.2</td>
<td>0.50</td>
<td>24.7</td>
<td>1.2</td>
</tr>
<tr>
<td>1.6</td>
<td>1.6</td>
<td>0.50</td>
<td>22.5</td>
<td>1.6</td>
</tr>
<tr>
<td>2.0</td>
<td>2.0</td>
<td>0.50</td>
<td>21.0</td>
<td>2.0</td>
</tr>
</tbody>
</table>

### Table B7: Measured horizontal distance between corners of actual and effective footings and corresponding calculated $\theta$, for top sand friction angle of 42.5°.

<table>
<thead>
<tr>
<th>$\phi_2$</th>
<th>27.5</th>
<th>30.0</th>
<th>32.5</th>
<th>35.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>H/B</td>
<td>$\Delta X(m)$</td>
<td>$\theta(\circ)$</td>
<td>$\Delta X(m)$</td>
<td>$\theta(\circ)$</td>
</tr>
<tr>
<td>0.2</td>
<td>0.6</td>
<td>0.42</td>
<td>35.0</td>
<td>0.6</td>
</tr>
<tr>
<td>0.3</td>
<td>0.9</td>
<td>0.55</td>
<td>31.4</td>
<td>0.9</td>
</tr>
<tr>
<td>0.4</td>
<td>1.2</td>
<td>0.68</td>
<td>29.6</td>
<td>1.2</td>
</tr>
<tr>
<td>0.5</td>
<td>1.5</td>
<td>0.82</td>
<td>28.7</td>
<td>1.5</td>
</tr>
<tr>
<td>0.6</td>
<td>1.2</td>
<td>0.62</td>
<td>27.3</td>
<td>1.2</td>
</tr>
<tr>
<td>0.8</td>
<td>1.6</td>
<td>0.74</td>
<td>24.8</td>
<td>1.6</td>
</tr>
<tr>
<td>1.0</td>
<td>2.0</td>
<td>0.84</td>
<td>22.8</td>
<td>2.0</td>
</tr>
<tr>
<td>1.2</td>
<td>1.2</td>
<td>0.47</td>
<td>21.4</td>
<td>1.2</td>
</tr>
<tr>
<td>1.6</td>
<td>1.6</td>
<td>0.50</td>
<td>19.9</td>
<td>1.6</td>
</tr>
<tr>
<td>2.0</td>
<td>2.0</td>
<td>0.50</td>
<td>18.8</td>
<td>2.0</td>
</tr>
</tbody>
</table>
# APPENDIX B NUMERICAL RESULTS USED IN CHAPTER 8

Table B8: Measured horizontal distance between corners of actual and effective footings and corresponding calculated $\theta$. for top sand friction angle of 45˚.

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>$\gamma_1 = 18.0 \text{ KN/m}^3$</th>
<th>$\gamma_1 = 18.0 \text{ KN/m}^3$, ($\gamma_2/\gamma_1$) = 1.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>H/B</td>
<td>$\Delta X(m)$</td>
<td>$\theta(\degree)$</td>
</tr>
<tr>
<td>0.2</td>
<td>0.6</td>
<td>0.41</td>
</tr>
<tr>
<td>0.3</td>
<td>0.9</td>
<td>0.54</td>
</tr>
<tr>
<td>0.4</td>
<td>1.2</td>
<td>0.65</td>
</tr>
<tr>
<td>0.5</td>
<td>1.5</td>
<td>0.74</td>
</tr>
<tr>
<td>0.6</td>
<td>1.2</td>
<td>0.48</td>
</tr>
<tr>
<td>0.8</td>
<td>1.6</td>
<td>0.65</td>
</tr>
<tr>
<td>1.0</td>
<td>2.0</td>
<td>0.75</td>
</tr>
<tr>
<td>1.2</td>
<td>1.2</td>
<td>0.41</td>
</tr>
<tr>
<td>1.6</td>
<td>1.6</td>
<td>0.50</td>
</tr>
<tr>
<td>2.0</td>
<td>2.0</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Table B9: Measured horizontal distance between corners of actual and effective footings and corresponding calculated $\theta$. for top sand friction angle of 47.5˚.

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>$\gamma_1 = 18.0 \text{ KN/m}^3$, ($\gamma_2/\gamma_1$) = 1.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>H/B</td>
<td>$\Delta X(m)$</td>
</tr>
<tr>
<td>0.2</td>
<td>0.6</td>
</tr>
<tr>
<td>0.3</td>
<td>0.9</td>
</tr>
<tr>
<td>0.4</td>
<td>1.2</td>
</tr>
<tr>
<td>0.5</td>
<td>1.5</td>
</tr>
<tr>
<td>0.6</td>
<td>1.2</td>
</tr>
<tr>
<td>0.8</td>
<td>1.6</td>
</tr>
<tr>
<td>1.0</td>
<td>2.0</td>
</tr>
<tr>
<td>1.2</td>
<td>1.2</td>
</tr>
<tr>
<td>1.6</td>
<td>1.6</td>
</tr>
<tr>
<td>2.0</td>
<td>2.0</td>
</tr>
</tbody>
</table>
Table B10: Measured horizontal distance between corners of actual and effective footings and corresponding calculated $\theta$ for top sand friction angle of 50°.

$$\phi_1=50^\circ, \gamma_1 = 18.0 \text{ KN/m}^3, (\gamma_2/\gamma_1)=1.2$$

| H/B | $\phi_2$ | 27.5 | | 30.0 | | 32.5 | | 35.0 | | 45° | | 50° |
|---|---|---|---|---|---|---|---|---|---|---|---|
| H | $\Delta x (m)$ | $\theta (\circ)$ | H | $\Delta x (m)$ | $\theta (\circ)$ | H | $\Delta x (m)$ | $\theta (\circ)$ | H | $\Delta x (m)$ | $\theta (\circ)$ |
| 0.2 | 0.6 | 0.30 | 26.6 | 0.6 | 0.31 | 28.8 | 0.6 | 0.35 | 30.3 | 0.6 | 0.43 | 35.6 |
| 0.3 | 0.9 | 0.42 | 25.0 | 0.9 | 0.44 | 26.1 | 0.9 | 0.47 | 27.6 | 0.9 | 0.55 | 31.4 |
| 0.4 | 1.2 | 0.50 | 22.6 | 1.2 | 0.52 | 23.4 | 1.2 | 0.56 | 25.0 | 1.2 | 0.66 | 28.8 |
| 0.5 | 1.5 | 0.58 | 21.1 | 1.5 | 0.62 | 22.5 | 1.5 | 0.64 | 23.1 | 1.5 | 0.74 | 26.3 |
| 0.6 | 1.2 | 0.41 | 18.9 | 1.2 | 0.43 | 20.7 | 1.2 | 0.47 | 22.0 | 1.2 | 0.52 | 24.4 |
| 0.8 | 1.6 | 0.49 | 17.0 | 1.6 | 0.53 | 18.3 | 1.6 | 0.57 | 19.6 | 1.6 | 0.63 | 21.5 |
| 1.0 | 2.0 | 0.57 | 15.9 | 2.0 | 0.61 | 17.0 | 2.0 | 0.66 | 18.3 | 2.0 | 0.72 | 19.8 |
| 1.2 | 1.2 | 0.31 | 14.5 | 1.2 | 0.34 | 15.8 | 1.2 | 0.36 | 16.7 | 1.2 | 0.90 | 18 |
| 1.6 | 1.6 | 0.36 | 12.7 | 1.6 | 0.41 | 14.4 | 1.6 | 0.44 | 15.4 | 1.6 | 0.53 | 16.7 |
| 2.0 | 2.0 | 0.41 | 11.6 | 2.0 | 0.46 | 13.0 | 2.0 | 0.50 | 14.0 | 2.0 | 0.50 | 15.9 |
| 2.4 | 2.4 | 0.44 | 10.4 | 2.4 | - | - | 2.4 | - | - | 2.4 | - | - |
APPENDIX C- DETAILED RESPONSE TO REVIEWER’S COMMENTS
PREFACE

To investigate the effect of assuming non-associated flow rule for sand on the predicted collapse load of a footing on sand-over-clay, a comparison is made between the experimental test results of Okamura et al. (1998) and ultimate bearing capacities estimated from:

- FELA simulations, using the Mohr-Coulomb failure criterion and assuming associated flow (as in all analyses presented in the main thesis).
- FELA simulations using the Mohr-Coulomb failure criterion with equivalent strength parameters for the top sand layer (Eqs. 2-30 and 2-31, see section 2-6-1).
- Displacement Finite Element simulations with Plaxis, using the Mohr-Coulomb failure criterion and non-associated flow.

In addition to the above, the effect of the sand dilation angle on the inclination of slip surfaces in the top layer ($\theta$) is assessed by comparing the failure mechanisms interpreted from FELA analyses, and Finite Element analyses with Plaxis using the Mohr-Coulomb failure criterion with non-associative flow to describe sand behaviour.

EFFECT OF NON-ASSOCIATIVITY ON FOOTING COLLAPSE LOAD

Assuming associated flow rule for frictional soils results in overestimation of the limit loads in stability problems. It is known that the degree of overestimation becomes more significant for higher friction angles (generally, greater than 35°, see Loukidis and Salgado 2009). Here we consider a series of tests on sand with relatively high sand friction angle of 47°, to better reflect the effect of non-associativity on the predicted footing collapse loads.

The experiments of Okamura et al. (1998) on surface strip footings on dense sand overlying soft clay were simulated with Plaxis. The FELA method described in Chapter 6 was also used to predict the collapse loads considering the soil properties reported by Okamura et al. (1998). Another set of results was also obtained by considering the equivalent sand friction angle $\phi^*$ in the FELA model (Eq. 2-30). The dilation angle adopted for the Plaxis analyses considering non-associated flow was 20°. The results are summarised in Figure C1.

The results depicted in Figure C1 suggest that the collapse loads estimated from FELA analyses using the friction angle reported by Okamura et al. (1998) are closer to the experimental measurements for all cases. Displacement finite element analyses with Plaxis, considering non-associated flow for sand, provide collapse loads about 20% to 30% lower than those predicted by FELA. The collapse loads predicted from FELA analyses with the equivalent sand friction angle $\phi^*$ are still higher than Plaxis results, but the difference is less than 7% in all cases. Therefore, for the problem at hand the FELA analyses with associated flow and the actual measured sand friction angle provide the best match to the
experimental results. However, this conclusion cannot be generalised, as it depends on the degree of kinematic constraint of the analysed problem.

Figure C1: Effect of flow rule on estimates of the collapse load, and comparison with experimental results.

**DEPENDENCY OF INCLINATION OF SLIP SURFACES ON DILATION ANGLE**

The inclination of slip surfaces in sand $\theta$ was interpreted from the results of Plaxis displacement finite element analyses for typical problem parameters. As above, sand is modelled as non-associated material with a dilation angle of 20°. These values are compared with $\theta$ values calculated from Equation 21 (Chapter 6), which was derived while assuming associated flow for sand. Typical collapse mechanisms are also presented in Figure C2, using contours of incremental strains and power dissipation intensities to visualise the collapse mechanisms in Plaxis and FELA, respectively.
Four analyses were performed considering a strip footing resting on dense sand overlying very soft to stiff clay. Table C1 presents the parameters used in the considered analyses.

Table C1: Material properties used in the displacement finite element models.

<table>
<thead>
<tr>
<th>Material</th>
<th>$c_u$ (kPa)</th>
<th>$\phi_p$ ($^\circ$)</th>
<th>$\phi_l$ ($^\circ$)</th>
<th>$\psi$ ($^\circ$)</th>
<th>$\nu$</th>
<th>$E$ (MPa)</th>
<th>$\gamma$ (kN/m$^3$)</th>
<th>$c_u/\gamma H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand</td>
<td>-</td>
<td>45</td>
<td>29</td>
<td>20</td>
<td>0.33</td>
<td>40</td>
<td>18</td>
<td>-</td>
</tr>
<tr>
<td>Clay#1</td>
<td>13.5</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.495</td>
<td>2.5</td>
<td>16</td>
<td>0.5</td>
</tr>
<tr>
<td>Clay#2</td>
<td>27</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.495</td>
<td>5</td>
<td>16</td>
<td>1.0</td>
</tr>
<tr>
<td>Clay#3</td>
<td>54</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.495</td>
<td>10</td>
<td>16</td>
<td>2.0</td>
</tr>
<tr>
<td>Clay#4</td>
<td>81</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.495</td>
<td>20</td>
<td>16</td>
<td>3.0</td>
</tr>
</tbody>
</table>

Figure C2: Observed failure mechanisms (a), Plaxis model, (b), FELA upper bound analysis.
Figure C3: Inclination of failure surface in sand obtained from displacement finite element analyses (non-associated flow) and FEL (associated flow). Negative values of $\theta$ refer to inward inclination of the slip surfaces.

Figure C3 indicates that $\theta$ values estimated from Plaxis results are in general, lower than those measured from FELA. In other words, higher dilation angles result in wider effective footing widths (please refer to Chapter 6 for the definition of effective footing width). However, the trend is the same viz. the inclination angle decreases and becomes negative as $c_u/\gamma H$ increases, suggesting that the geometry of the failure mechanism indeed varies with the relative strength of materials, as discussed before. Notice also that the difference becomes less significant as the strength of the clay layer increases relative to the sand and the mechanism approaches that of general shear failure.