Frequency Domain Analysis of Sampled-Data Control Systems

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I hereby certify that the work embodied in this thesis is the result of original research and has not been submitted for a higher degree to any other University or Institution.

Julio H. Braslavsky

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[1] Frank — Richard Harris in the excellent movie Wrestling Ernest Hemingway (1993).

[2] I forgive him; he also introduced me to the enchanting art of boomeranging.

[3] Hopefully, some of them will also remember a fisherman that once put a handkerchief in his mouth.

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[4]Bueh, este de trabajo mucho no. . . pero buena joda.

[5]Juan Carlos caso extremo: se vino entero.

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A Encarnación.

Contents

Abstract

This thesis is aimed at analysis of sampled-data feedback systems. Our approach is in the frequency-domain, and stresses the study of sensitivity and complementary sensitivity operators. Frequency-domain methods have proven very successful in the analysis and design of linear time-invariant control systems, for which the importance and utility of sensitivity operators is well-recognized. The extension of these methods to sampled-data systems, however, is not straightforward, since they are inherently time-varying due to the intrinsic sample and hold operations.

In this thesis we present a systematic frequency-domain framework to describe sampled-data systems considering full-time information. Using this framework, we develop a theory of design limitations for sampled-data systems. This theory allows us to quantify the essential constraints in design imposed by inherent open-loop characteristics of the analog plant. Our results show that: (i) sampled-data systems inherit the difficulty imposed upon analog feedback design by the plant's non-minimum phase zeros, unstable poles, and time-delays, independently of the type of hold used; (ii) sampled-data systems are subject to additional design limitations imposed by potential non-minimum phase zeros of the hold device; and (iii) sampled-data systems, unlike analog systems, are subject to limits upon the ability of high compensator gain to achieve disturbance rejection. As an application, we quantitatively analyze the sensitivity and robustness characteristics of digital control schemes that rely on the use of generalized sampled-data hold functions, whose frequency-response properties we describe in detail.

In addition, we derive closed-form expressions to compute the L_2-induced norms of the sampled-data sensitivity and complementary sensitivity operators. These expressions are important both in analysis and design, particularly when uncertainty in the model of the plant is considered. Our methods provide some interesting interpretations in terms of signal spaces, and admit straightforward implementation in a numerically reliable fashion.

1

Introduction

This thesis deals with frequency-domain properties and essential design limitations in linear sampled-data feedback control systems.

A sampled-data system combines both continuous and discrete-time dynamic subsystems. Because of this inherent mixture of time domains, we shall also refer to a sampled-data system as a *hybrid* system, understanding both terms as synonyms. A typical hybrid feedback control configuration is shown in Figure 1.1. Although the plant is usually a continuous-time, or *analog*, system, the controller is a discrete-time device in most practical applications. This is mainly due to the numerous advantages that digital equipments offer over their analog counterparts. With the great advances in computer technology, today digital controllers are more compact, reliable, flexible and often less expensive than analog ones.

There is a fundamental operational difference between digital and analog controllers: the digital system acts on *samples* of the measured plant output rather than on the continuous-time signal. A practical implication of this difference is that a digital controller requires special interfaces that link it to the analog world.

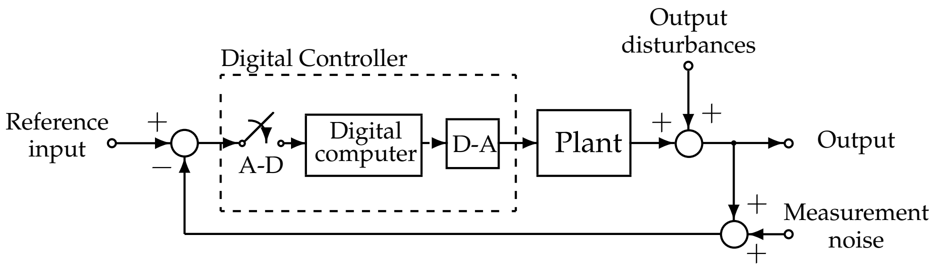

Figure 1.1: Typical sampled-data feedback configuration.

A digital controller can be idealized as consisting of three main elements: the analog-to-digital (A-D) interface, the digital computer, and the digital-to-analog (D-A) interface. The A-D interface, or *sampler*, acts on a physical variable, normally an electric voltage, and converts it into a sequence of binary numbers, which represent the values of the variable at the sampling instants. These numbers are then processed by the digital computer, which generates a new sequence of binary numbers that correspond to the discrete control signal. This control signal is finally converted into an analog voltage by the D-A interface, also called the *hold device*.

The digital computer implements the control algorithm as a set of difference equations, which represent a dynamic system in the discrete-time domain. We shall refer to this system as the *discrete controller*. In general, the discrete controller will include nonlinearities and varying parameters in it; our discussion here is restricted to linear time-invariant controllers, which nevertheless constitute an useful and important case in analysis and design.

Essentially, two classic approaches are taken in engineering practice for the design of a discrete controller. The first technique, referred to as *emulation* [Franklin et al., 1990], is the most widely applied in industry. Emulation consists in first designing an analog controller such that the closed-loop system has satisfactory properties, and then translating the analog design into a discrete one using a suitable discretization method (see Keller and Anderson [1992] for a recent approach). This technique has the advantage that the synthesis is done in continuous-time, where the design goals are typically specified, and where most of the designer's experience and intuition resides. Also, the system's analog performance will in general be recovered for fast sampling. Yet, the hybrid performance cannot be expected to be better than the analog, and there may be a serious degradation if the sampling is not sufficiently fast. This is an important drawback, since the sampling rate is a critical constraint in many applications.

The second traditional technique consists in discretizing the plant and performing a *discrete design*. The main benefit of this approach is that the synthesis procedure is again simplified, since the discretized plant is linear time-invariant (LTI) in the discrete-time domain. However, a serious limitation of discrete design is that it is generally difficult to translate the analog specifications into discrete. Furthermore, the simple models obtained by discretization fail to represent the full response of the system, since *intersample behavior* is inherently lost or hidden[1].

In particular, neither of these approaches offers an adequate framework for analysis of the continuous-time time-varying hybrid system. Emulation is purely a method of synthesis, whereas discrete design gives a partial answer, since only the *sampled* behavior can be studied in the discretized model. On the other hand, the analysis of the hybrid system requires the consideration of both sampled *and* intersample behavior. This is crucial especially when considering robustness and sensitivity properties of the system, since *analog* uncertainties, disturbances and noise are frequently the issues of practical significance.

1.1 Recent Developments in Sampled-data Systems

Naturally, in view of the technological appeal of digital implementations, sampled-data systems have been the subject of many research works in recent years. Two

[1]Some intersample information can still be handled in a discrete model by using the *modified Z-transform* introduced by Jury [1958]. However, this line of work seems to have been largely abandoned.

research directions in particular have generated much activity. First, various optimal control problems have been stated and solved for hybrid systems using frameworks that incorporate intersample behavior [e.g., Chen and Francis, 1991, Bamieh and Pearson, 1992, Dullerud and Francis, 1992, Tadmor, 1992, Kabamba and Hara, 1993, Bamieh et al., 1993]. Second, several researchers have explored the potential ability of nonstandard hold functions, periodic digital controllers, and multirate sampling to circumvent design limitations inherent to LTI systems [e.g., Khargonekar et al., 1985, Kabamba, 1987, Francis and Georgiou, 1988, Hagiwara and Araki, 1988, Das and Rajagopalan, 1992, Yan et al., 1994]. Within these two research avenues, we shall restrict the discussion here to optimal H_∞ sampled-data control, and control techniques using *generalized sampled-data hold functions* (GSHFs).

The earliest efforts to extend H_∞ control methods to sampled-data systems focused on the computation of the induced L_2-norm. The L_2-induced norm measures the maximum gain of an operator acting on spaces of square integrable, or "finite energy", signals. For a LTI system, the optimization of the L_2-induced norm is equivalent to the minimization of the H_∞-norm of its transfer matrix. This is not trivial to extend to sampled-data systems, since they are time-varying due to the presence of the sampler, and hence we cannot describe their input-output behavior with ordinary transfer matrices. Therefore, special procedures have been developed. For example, Thompson et al. [1983], and Thompson et al. [1986] provided the first bounds for the norm of open-loop hybrid systems using conic sector techniques. Exact expressions of the L_2-induced norms were later on obtained by Chen and Francis [1990] via frequency-domain methods. In 1991, Leung et al. derived a formula for sampled-data feedback systems with band-limited signals.

In recent years, different general frameworks to handle intersample behavior appeared on the scene, and led the way to the solution of certain hybrid optimal H_∞ control problems[2]. These frameworks include *lifting techniques* [Bamieh et al., 1991, Toivonen, 1992, Bamieh and Pearson, 1992, Yamamoto, 1993, 1994], *descriptor system techniques* [Kabamba and Hara, 1993], and techniques based on *linear systems with jumps* [Sun et al., 1993, Sivashankar and Khargonekar, 1994]. More specifically, the lifting technique consists on transforming the original sampled-data system into an equivalent LTI discrete-time system with infinite-dimensional input-output signal spaces. Then, the L_2-induced norm of the sampled-data system is shown to be less than one if and only if the H_∞-norm of this equivalent discrete system is less than one. In the descriptor system approach, on the other hand, the system is represented by a hybrid state-space model, from which the descriptor system is formulated. The solution of the H_∞ sampled-data problem is then characterized by the solution of certain associated Hamiltonian equation. In contrast with these procedures, the theory of linear systems with jumps allows a direct characterization of the problem in similar — although more involved — terms to those of standard LTI H_∞-control problems, and leads to a pair of Riccati equations. Despite the procedural differences in all these approaches, the results

[2] Yet, as pointed out by Glover [1995], practical design guidelines are still under development.

obtained are mathematically equivalent.

On the other hand, new control schemes using GSHFs were introduced to approach various problems that are insoluble with LTI control schemes. A GSHF reconstructs an analog signal from a discrete sequence of values, but instead of holding these values constant along the sample period — as it is the case of a classic *zero-order hold* (ZOH) — a GSHF scales a fixed suitable waveform. In particular, by selecting this waveform it is possible to assign the zeros of the discretized plant, and hence, e.g., convert a non-minimum phase (NMP) analog plant into a minimum phase discrete plant [Bai and Dasgupta, 1990]. This is the key technique of several applications of GSHFs. For example, Kabamba [1987] obtained simultaneous pole-assignment of an arbitrary finite number of plants using a single GSHF; and Yan et al. [1994] proposed the combination of a discrete controller with a GSHF to achieve arbitrary gain-margin improvement of continuous-time NMP linear systems. Other applications of GSHFs include decoupling, exact model-matching, and exact discrete loop transfer recovery of NMP plants [Liu et al., 1992, Paraskevopoulos and Arvanitis, 1994, Er and Anderson, 1994].

Besides the benefits offered by GSHFs, some authors have pointed out the existence of intersample difficulties and serious robustness and sensitivity problems associated with the use of these devices [Araki, 1993, Feuer and Goodwin, 1994, Zhang and Zhang, 1994]. For example, Feuer and Goodwin [1994] have argued that GSHF control relies on the generation of high-frequency harmonics, which tend to make the system more sensitive to high-frequency plant uncertainty, disturbances and noise. As a consequence, the potential utility of GSHFs in overcoming LTI design limitations seems still to be a matter of debate.

Despite these advances in synthesis, there is as yet no well-developed theory of inherent design limitations for hybrid feedback systems. For analog feedback systems, on the other hand, many results on design limitations are available. Bode first stated the sensitivity integral theorem in 1945, whose importance for feedback control was emphasized by Horowitz [1963]. Later extensions were obtained by several researchers; of particular relevance to the present discussion are the results of Freudenberg and Looze [1985] and Middleton [1991]. Briefly, the theory describes how plant properties such as NMP zeros, unstable poles, and time delays limit the achievable performance of a feedback system consisting of a LTI plant and a continuous-time controller. These limitations manifest themselves as tradeoffs between desirable system properties in different frequency ranges, and are expressed mathematically using Bode and Poisson integrals.

A parallel theory of inherent design limitations for purely discrete-time feedback systems is also available [Sung and Hara, 1988, Middleton and Goodwin, 1990, Mohtadi, 1990, Middleton, 1991]. Unfortunately, this theory is insufficient to describe fundamental limitations in hybrid systems. Indeed, discrete-time results do not consider intersample behavior, and therefore do not tell us the whole story (in particular, good sampled behavior is necessary but not sufficient for good overall behavior). The development of an equivalent theory for sampled-data systems is one of the main goals of this thesis.

1.2 Contributions of this Thesis

This thesis is aimed at analysis of sampled-data feedback systems. Our approach is in the frequency-domain, and stresses the study of sensitivity and complementary sensitivity operators. Our main contributions may be summarized as follows:

(i) We expound a systematic frequency-domain framework to describe sampled-data systems considering full-time information. This framework allows us to study important properties of the system in a way that appears to be simpler than in alternative state-space approaches. There are two reasons why we believe the frequency-domain approach to be simpler. First, this frequency-domain setting has better links with classical frequency-domain analysis for analog control systems, in which a large heuristic knowledge is available. Second, the mathematics involved seems easier to understand and relate to the original plant model.

(ii) We develop a theory of design limitations for sampled-data systems. This theory allows us to quantify the essential constraints imposed by NMP zeros of the hold function, and NMP zeros and unstable poles of the analog plant and discrete controller. As an application, we quantitatively analyze the sensitivity and robustness properties of control schemes that rely on GSHF discrete zero-shifting capabilities.

(iii) We derive closed-form expressions to compute the L_2-induced norms of the hybrid sensitivity and complementary sensitivity operators. These expressions have interesting interpretations in terms of signal spaces associated with the hold, the plant and the anti-aliasing filter. All our formulas admit straightforward implementation in a numerically reliable fashion.

(iv) We study the frequency-domain properties of GSHFs, providing results that describe in detail their zero-distribution, and some integral relations that their frequency response must satisfy. In particular, these results show the source of some of the difficulties associated with the use of GSHFs.

The framework of (i), and the results of (iii) are valid for multiple-input multiple-output (MIMO) systems. The results in (ii) and (iv) are restricted to the single-input single-output (SISO) case. Due to the issue of directionality, the generalization of these results to multivariable is difficult — for (ii) this is so even in the analog case — and hence we have not pursued it here.

Many of the results referred to in (ii) have been developed in collaboration, and published in Freudenberg et al. [1995] and Freudenberg et al. [1994], with significant input from the first author. The results in (i) and (iii) have been partially communicated in Braslavsky et al. [1995b], while some of the results in (iv) will appear in Braslavsky et al. [1995a].

We now give an overview of the rest of the thesis.

Chapter 2: This chapter introduces most of our notation, main assumptions, and the basic preliminary results upon which the rest of the chapters will be developed. Here, we define the mathematical representations of the A-D and D-A interfaces, the sampler, and the hold device. A distinctive feature of our approach is that the hold device is not restricted to the ZOH. Indeed, we shall consider that the hold is a GSHF of the type introduced by Kabamba [1987], which will allows us to develop a comprehensive framework to study sampled-data systems. We also present in this chapter a basic but key sampling formula concerning the Laplace transform of a sampled signal. This relation will be the starting point of our discussion on the frequency response of hybrid systems in the following chapters. We conclude with a review of two important results concerning the closed-loop stabilizability properties of sampled-data systems.

Chapter 3: The focus of this chapter is the frequency response of a GSHF. As opposed to that of a ZOH, the frequency response of a GSHF may have large high-frequency peaks that compromise the robustness properties of the system. It is also known that GSHFs may have zeros off the $j\omega$-axis that pose discrete stabilizability difficulties. In this chapter we go deeper into the analysis of these issues by studying fundamental properties of the frequency response of GSHFs. Specifically, we describe their zero-distribution and the constraints that these zeros impose on the values on the $j\omega$-axis. One of the main results of this chapter is that GSHFs with "asymmetric" pulse response function will necessarily have zeros off the $j\omega$-axis.

Chapter 4: In this chapter, we study the frequency response of a sampled-data system, and develop a theory of design limitations wherein we consider the response of the analog system output. To do this, we use the fact that the steady-state response of a hybrid feedback system to a sinusoidal input consists of a fundamental component at the frequency of the input together with infinitely many harmonics, located at frequencies spaced integer multiples of the sampling frequency away from the fundamental. This fact allows us to define fundamental sensitivity and complementary sensitivity functions that relate the fundamental component of the response to the input signal. These sensitivity and complementary sensitivity functions must satisfy integral relations analogous to the Bode and Poisson integrals for purely analog systems. The relations show, for example, that design limitations due to NMP zeros of the analog plant constrain the response of the sampled-data feedback system regardless of whether the discretized system is minimum phase, and independently of the choice of hold function.

Chapter 5: This chapter deals with the analysis and computation of the L_2-induced norm of operators in sampled-data systems. We first expound a *frequency-domain lifting* technique to derive "closed-form" expressions for the frequency gains of hybrid sensitivity operators in a MIMO setup. We show that these frequency-gains can be characterized by the maximum eigenvalue of certain finite-dimensional discrete transfer matrices; even in the case of the sensitivity operator, which — since it is known to be non-compact — presents extra difficulties for the analysis. The L_2-induced norm is then computed by searching the maximum of this eigenvalue over a finite range of frequencies. At the end of the chapter, we provide expressions from which the generation of numerical algorithms to compute these norms is straightforward.

Chapter 6: This chapter is about stability robustness of sampled-data systems. Dullerud and Glover [1993] have derived necessary and sufficient conditions for robust stability of hybrid systems against multiplicative perturbations in the analog system. These authors have used a frequency-domain formulation based on state-space lifting techniques. We show in this chapter that the same type of result may be obtained in a simpler way when the problem is directly formulated in the frequency-domain. We do this by using the frequency-domain lifting framework introduced in Chapter 5. We also give both necessary conditions and sufficient conditions for robust stability as simple expressions that emphasize the role played by the fundamental and harmonic sensitivity functions defined in Chapter 4. We conclude the chapter by showing that the same framework may be used to approach the problem of robust stability against divisive perturbations.

Chapter 7: As an application of the preceding results, in this chapter we study the difficulties associated with the zero-shifting capabilities of GSHFs. Many GSHF-based proposed schemes rely on zero-shifting, since this appears to circumvent fundamental limitations imposed by analog NMP zeros. We show that if the plant has a NMP zero with significant phase lag within the desired closed-loop bandwidth of the system, then zero-shifting will necessarily lead to serious robustness and sensitivity problems in both analog and discrete performances of the system.

Chapter 8: In this chapter we summarize the main results of the thesis, and give some concluding remarks and directions for future research.