Relating a Distribution Transformer's Connection Topology and the Influence of Inductive Disparity to the Observed Frequency Response

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Abstract-This paper uses generic three phase models to illustrate how the frequency response analysis (FRA) data of distribution transformers with different vector group numbers will be effected by inductive disparity. The inductive disparity of a three phase transformer is a well documented property due to variation in the observed core reluctance of transformer windings on different phases. At low frequency this will result in a subtle inductance inequality between the centre and outer limb windings of the transformer core. Understanding the influence of this disparity is important when analysing the frequency response of the transformer. Frequency Response Analysis data taken from different terminals on the same transformer will display significant variation at low frequencies due to the effects of this inductive disparity. Effects include differences in the self resonant frequency and levels of attenuation. This research facilitates a more comprehensive understanding of the observed responses.

I. INTRODUCTION

Frequency Response Analysis (FRA) is a commonly used diagnostic tool that provides a frequency response signature unique to the electrical and mechanical topology of a transformer. Any variation in the electrical and/or mechanical properties will result in a corresponding variation in the frequency response. Generally, testing personnel only look for significant frequency response variation between phases of the same transformer or against historical data [1] to be indicative of a potential problem. The actual physical interpretation of what the changes in the frequency response represent is an area of continued research interest. In fact, the international council on large electric systems (CIGRE) published a report in 2006 [2] which emphasised the need for more research to focus on physical modeling to aid in FRA interpretation. The model must take into account the windings, connection topology and mechanical geometry for a greater understanding of the physical properties behind the observed frequency response.

The disparity in inductance between the phases of a three phase transformer is a well documented property [3] [4] [5] [6]. The effective magnetic path length observed by a winding is inversely proportional to the windings inductance [7]. For a three phase core type transformer this results in the inductance of phase B being moderately larger than that of phase A and phase C, which are approximately equal. Inductive disparity can create significant variation between the observed frequency responses of an FRA test in the low frequency regions.

To detect change, testing personnel compare the FRA waveforms against historical data, sister transformer data or between phases on the same transformer. The latter test typically being the most convenient. It is the practice of interphase comparison that makes understanding the influence of inductive disparity and its relationship with a transformer's connection topology so critical. On this basis, the paper investigates the low frequency response of three different FRA tests on two transformers with different vector group numbers using physically representational models. The research examines how inductive disparity coupled with different connection and measurement topologies can produce a significantly different frequency response. To the authors knowledge, this research has not been discussed in the existing literature and the paper advances the research into the physical interpretation of FRA (as by CIGRE [2]).

II. BACKGROUND

In this paper a practical model is developed in order to explain the observed behaviour in the frequency response for three different FRA tests. So that the models are not phase specific, generic phase references X, Y and Z are used. To minimise the models some fundamental assumptions are made which are justified in the following section.

A. Inductive Disparity

FRA testing currents will generate an alternating magnetic field which will subsequently induce eddy currents in the transformer's core. At high frequencies this effect will cause attenuation in the core's relative permeability which will be proportional to the square root of frequency [7]. This ultimately leads to the core's inductive influence to be considered negligible at high frequencies. As such, the



Fig. 1. Magnetic circuit of a core-type three phase transformer

 \mathfrak{F}_P = Magnetomotive force due to a high voltage (HV) winding \mathfrak{F}_S = Magnetomotive force due to a low voltage (LV) winding \mathfrak{R}_H = Reluctance of a horizontal core section \mathfrak{R}_V = Reluctance of a vertical core section



Fig. 2. Dyn vector group connection

The inductance is then given by,

$$L = \frac{N^2}{\Re} , \qquad (5)$$

where L = Winding inductance,

N = Number of winding turns.

Note that for windings with the same number of turns the inductance of phase B will be greater than that of phases A and C, i.e.

$$L_B > L_A = L_C . (6)$$

The inductive disparity shown by the relationship (6) will be of consequence for both the high voltage and low voltage windings at frequencies where the relative permeability is still significant (e.g. <10kHz). A number of frequency response tests will be discussed in this paper to highlight the impact of this inductive variation on the observed frequency response.

B. High Voltage Winding to Low Voltage Winding Inductance Ratio for Dyn Connected Transformer

With reference to Figure 2, it is noted that there is a direct coupling of the HV winding line voltage V_{XY} to that of the LV winding phase voltage V_{xn} . This results in a topologically based $\sqrt{3}$ voltage step up due to the transformers Dyn connection [8]. Now,

$$\frac{N_X}{N_x} = \frac{\sqrt{3}V_{XY}}{V_{xy}} , \qquad (7)$$

where V_{XY} and V_{xy} represent line voltages for the HV and LV windings respectively, and N_X and N_x the corresponding number of turns per winding. Both the HV and LV windings under consideration share the same magnetic circuit, hence the circuit reluctance parameter, \Re , will be the same. Therefore, from (5), a generic winding inductance ratio between the high and low voltage windings of a Dyn connected transformer can be found in terms of the turns ratio:

$$\frac{L_X}{L_x} = \frac{N_X^2}{N_x^2} \,. \tag{8}$$

influence of inductive disparity will be most significant at low frequencies (<10kHz) where the relative permeability stays approximately constant at a figure typically much greater than unity ($u_R >> 1$). With reference to Figure 1, the reluctance of a magnetic core section of mean length l and cross sectional area A, with an effective permeability μ is defined as [7]:-

$$\mathfrak{R} = \frac{l}{\mu A} , \qquad (1)$$

where
$$\mu = \mu_0 \mu_R$$
,
 $\mu_0 = 4\pi \ge 10^{-7}$ H/m,
 $\mu_R =$ Relative permeability of core.

It is assumed that the transformer core cross sectional area and permeability are uniform across all core sections. The reluctance observed by individual windings is therefore related to the dimension and layout of the physical magnetic circuit. Hence the reluctance, \Re , is directly proportional to the effective core length. In Figure 1 we note that l_H is the transformer core mean width and l_V the core mean height. Substituting the effective magnetic circuit length into (1), the reluctance for the A and C phase windings is given by:

$$\Re_A = \Re_C = \frac{1}{\mu A} \left[l_H + l_V + \frac{(l_H + l_V)l_V}{(l_H + l_V) + l_V} \right].$$
 (2)

Similarly, for phase B

$$\Re_B = \frac{1}{\mu A} \left[l_V + \frac{(l_H + l_V)(l_H + l_V)}{2(l_H + l_V)} \right].$$
 (3)

In practical transformer design, l_V and l_H will be dimensionally similar hence,

$$\mathfrak{R}_B < \mathfrak{R}_A = \mathfrak{R}_C . \tag{4}$$

Substituting (7) into (8) and noting that both the distribution transformers under test are 11kV-415V:

$$\frac{L_X}{L_x} = 3\left(\frac{V_{XY}}{V_{xy}}\right)^2$$
$$= 3\left(\frac{11000}{415}\right)^2$$
$$L_X = 2108L_x$$
$$\therefore \quad L_X >> L_x . \tag{9}$$

The relationship (9) is necessary for model reduction purposes in the latter sections of the paper due to the fact that at the low frequencies discussed, the low voltage winding inductance can be considered negligible relative to that of the high voltage winding.

C. Self and Mutual Inductance Relationships

The mutual inductance between the low and high voltage windings, on a particular phase, where the coefficient of coupling approaches unity can be defined as:

$$M_{xX} \approx \sqrt{L_x L_X} \ . \tag{10}$$

From (9) we have that

$$M_{xX} \approx \sqrt{2108L_xL_x} \approx 45L_x$$
$$M_{xX} \gg L_x . \tag{11}$$

Therefore, the mutual inductance between the low and high voltage windings on a given phase is of an order of magnitude greater than the self inductance of the low voltage winding. To determine the interphase mutual inductance, the following mutual inductance definition [7] is required,

$$M_{jk} = \frac{\Lambda_{jk}}{I_j} , \qquad (12)$$

 M_{jk} = Mutual inductance between windings j and k Λ_{jk} = Flux linkage between windings j and k I_i = Current in winding j

With reference to Figure 1, at low frequencies where $\mu_R >> 1$, it is observed that the flux generated by a winding on a given phase has a flux linkage on the winding of another phase relative to the magnetic path reluctance ratio. We note that whilst the reluctance of phase B is less than that of phases A and C, as shown in (4), the difference is only modest and as such, the following inequality can be deduced from (12),

$$M_{xX} > M_{xY} . (13)$$

This shows that the mutual inductance between the low and high voltage windings on the same phase is always greater than the mutual inductance between the low and high voltage windings of different phases. Similarly the self inductance of



Fig. 3. 3 phase transformer Dyn vector group topologies: (a) Dyn1, (b) Dyn11

a winding on a particular phase can be assumed to be larger than its mutual inductance with another similar winding of different phase,

$$L_x > M_{xy} . (14)$$

III. FRA TESTING RESULTS

On a three phase transformer, FRA tests are conducted in sets of three such that all phase combinations are included in the testing sequence. Correlation between the model's generic phase (X, Y or Z) and the true phase for a particular test can be made via reference to the appropriate model tables (Tables I, II and III). In addition, to highlight the influence that the connection topology will have on results, testing was conducted on two transformers with different vector group numbers, Dyn1 and Dyn11 (Refer Figure 3).

A. High Voltage Terminal FRA Tests for Dyn1 and Dyn11 Vector Group Transformers

The high voltage terminal FRA test records the frequency response between two of the three high voltage transformer terminals. The third high voltage terminal and the low voltage terminals are left open circuit. This test is repeated for all three high voltage terminal combinations. From a modeling perspective, with reference to (9) for the two transformers under consideration, the influence of the disconnected low voltage winding can be considered negligible. At low frequencies a model of a Dyn connected transformer can be reduced to a series combination of two windings in parallel with the third as shown in Figure 4. To consider all three phase combinations for this test, it is convenient to present the model in a generic form.

Analysing the simplified model and noting (6) and (14), it can be shown that the equivalent inductance between the input and output terminals is larger when the generic winding X represents phase B. The test terminal combination for X to represent the phase B high voltage winding, is dependent upon the vector group of the transformer and is highlighted in Table I. The influence of the larger equivalent inductance



Fig. 4. Simplified generic model of a high voltage terminal FRA test of a Dyn connected transformer

 $T_{IN} = FRA \text{ Input / Transformer HV terminal}$ $T_{OUT} = FRA \text{ Output / Transformer HV terminal}$ $L_X, L_Y, L_Z = Self \text{ inductance of delta connected HV windings}$ $M_{XY}, M_{XZ}, M_{YZ} = Mutual \text{ inductance between HV windings}$

 TABLE I

 High voltage terminal fra test: Generic phase reference for

 Figure 4

Vector Group	$T_{IN} T_{OUT}$	Х	Y	Ζ
Dyn1	AC	Α	C	В
	BA	В	A	С
	CB	C	В	Α
Dyn11	AB	Α	В	С
	BC	B	C	Α
	CA	C	A	В

at low frequencies for tests between terminals B to A for a Dyn1 connection, or terminals B to C for a Dyn11 connection, is observed in the frequency responses of Figures 5 and 6. One effect that this inductive disparity has is for test BA in Figure 5 (or test BC in Figure 6) to have a greater attenuation in the magnitude response for frequencies ranging between 100Hz and 2kHz. This region where the roll off is 20dB/dec, is referred to as the inductive roll off. Another variation in the frequency response can be observed at the self resonant frequency which marks the transition between the inductive and capacitive regions of the frequency response [9]. The larger effective inductance results in a lower self resonant frequency which is clearly visible for test BA in Figure 5 and test BC in Figure 6.

B. High to Low Voltage Terminal FRA Tests for Dyn1 and Dyn11 Vector Group Transformers

This test measures the frequency response between a high and low voltage terminal on a particular phase. In this test the remaining high and low voltage terminals are left open circuit. Again the test is repeated for all three high to low voltage terminal phase combinations. A low frequency generic model for this test is presented in Figure 7. From (9), the low



Fig. 5. FRA test using a HP89410A Vector Analyser on a 1MVA 11kV/415V Dyn1 transformer between high voltage terminals A to C, B to A and C to B



Fig. 6. FRA test using a HP89410A Vector Analyser on a 200kVA 11kV/415V Dyn11 transformer between high voltage terminals A to B, B to C and C to A.

voltage winding inductance is negligible at low frequency with respect to the high voltage winding and approximates a short circuit. Neglecting for the moment the inductive disparity, if the distributed high voltage winding inductance is assumed equal for all phases, then the total voltage across the phase X winding is equal to that of the phase Z winding:

$$V_{LX(k)} \approx V_{LZ(N-k+1)}$$
 where $k = 1$ to N . (15)

Therefore the current that flows into either end of winding Y is,

$$I_{Y1} \approx I_{YN} . \tag{16}$$

It then follows that the voltage across winding Y will tend to zero, i.e.

$$V_{LY} \longrightarrow 0$$
. (17)

From (17), the high voltage windings can be approximated to be a parallel combination of L_X and L_Z . When either



Fig. 7. Generic low frequency model representing FRA high voltage to low voltage terminal test of a Dyn connected transformer

 $\begin{array}{l} T_{IN} = \mbox{FRA Input / Transformer HV terminal} \\ T_{OUT} = \mbox{FRA Output / Transformer LV terminal} \\ N = \mbox{Number of distributed winding sections} \\ L_X, L_Y, L_Z = \mbox{Distributed self inductance of HV windings} \\ L_x, L_y, L_z = \mbox{Distributed self inductance of LV windings} \\ M_{XY}, M_{XZ}, M_{YZ} = \mbox{Mutual inductance between HV windings} \\ C_{PS} = \mbox{Distributed HV to LV winding capacitance} \\ i_{Y1} = \mbox{Current into winding node 1 of HV winding Y} \\ i_{Y(N)} = \mbox{Current into winding node N of HV winding Y} \end{array}$

TABLE II HIGH VOLTAGE TO LOW VOLTAGE TERMINAL FRA TEST: GENERIC PHASE REFERENCE FOR FIGURE 7

Vector Group	$T_{IN} T_{OUT}$	X	Y	Ζ
Dyn1	Aa	Α	C	В
	Bb	В	A	C
	Cc	C	В	Α
Dyn11	Aa	Α	В	С
	Bb	В	C	Α
	Cc	C	A	В

of these two generic windings represent the B phase high voltage winding, the effective inductance will be larger then the parallel combination of the outer limb windings (phases A and C). The larger effective inductance combined with the interwinding capacitance will result in a lower resonant frequency. This effect is observed in Figure 8 for the high to low voltage terminal test Aa and Bb of the Dyn1 connection and in Figure 9 for the high to low voltage terminal test Bb and Cc on the Dyn11 connection. These results are highlighted in Table II.

C. Low Voltage to Neutral FRA Tests on Dyn1 and Dyn11 Vector Group Transformers

This FRA test records the frequency response between each of the low voltage terminals and the neutral connection. During the test the remaining two low voltage terminals and three high voltage terminals are left open circuit. The test is repeated for each phase. The FRA testing circuit is of low impedance and as such, at relatively low frequencies, the low voltage star



Fig. 8. High to Low Voltage FRA test using a HP89410A Vector Analyser on a 1MVA 11kV/415V Dyn1 Transformer between the respective high voltage and low voltage terminals A to a, B to b and C to c.



Fig. 9. Transfer Function FRA test using a HP89410A Vector Analyser on a 200kVA 11kV/415V Dyn11 Transformer between the respective high voltage and low voltage terminals A to a, B to b and C to c.

connected windings which have their terminals unconnected, can be neglected from the model. The generic low frequency Dyn based transformer model is presented in Figure 10.

With reference to (6), (11) and (13) it is noted that the equivalent low frequency inductance between the testing terminals is dominated by the windings on phase X, and that the most significant contribution will be made by the high voltage winding as per (11). Based on this result the B phase to neutral test (Bn) on both the Dyn1 and Dyn11 connections will have a larger magnitude attenuation in the inductive roll off region of the frequency response when compared to the An and Cn tests. The Bn test will also have a lower self resonant frequency. These results can be observed in Figures 11 and 12. Table III presents the generic model phase reference for this test.



Fig. 10. Generic low frequency model of a low voltage to neutral terminal FRA test of a Dyn connected transformer

 $T_{IN} = \text{FRA Input / Transformer LV terminal}$ $T_{OUT} = n = \text{FRA Output / Neutral terminal}$ $L_X, L_Y, L_Z = \text{Self inductance of delta connected HV windings}$ $L_x = \text{Self inductance of phase x - Star connected LV winding}$ $M_{XY}, M_{XZ}, M_{YZ} = \text{Mutual inductance between HV windings}$ $M_{x\Psi} = \text{Mutual inductance between LV winding x and HV winding } \Psi$ (X, Y or Z)

TABLE III Low voltage to neutral terminal fra test: Generic phase reference for Figure 10

Vector Group	$T_{IN} T_{OUT}$	X	Y	Z
Dyn1	an	A	C	В
	bn	В	A	C
	cn	C	В	A
Dyn11	an	A	В	С
	bn	B	C	A
	cn	C	A	В



Fig. 11. FRA test using a HP89410A Vector Analyser on a 1MVA 11kV/415V Dyn1 Transformer between the low voltage terminal and the neutral; a to n, b to n and c to n.

IV. CONCLUSION

This paper developed generic models for the physical interpretation of low frequency FRA results for distribution trans-



Fig. 12. FRA test using a Doble M5300 SFRA test set on a 200kVA 11kV/415V Dyn11 Transformer between the low voltage terminal and the neutral; a to n, b to n and c to n.

formers with different vector group numbers. It has shown that the influence of inductive disparity on FRA is dependent upon the transformer connection and testing topology. More specifically, the paper explains the FRA response differences between a Dyn1 and a Dyn11 connected transformer, contributing to the research area of transformer frequency response interpretation.

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