Maximal Controllability via Reduced Complexity Model Predictive Control

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I hereby certify that the work embodied in this thesis is the result of original research and has not been submitted for a higher degree to any other University or Institution.

(Signed): _____

Adrian Medioli

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> *"Remember – that which does not kill us can only make us stronger."*
> *"And that which *does* kill us leaves us *dead*!"*
>
> – (Terry Pratchett, Carpe Jugulum)

CONTENTS

1 Introduction **1**

2 Model Predictive Control **7**
2.1 Introduction . . . 7
2.2 The Algorithm . . . 7
2.2.1 System Model . . . 7
2.2.2 Constraints . . . 8
2.2.3 Optimisation Problem . . . 8
2.3 Controllable Sets . . . 9
2.4 Main Implementation Issues . . . 11
2.5 Stability . . . 12
2.5.1 Guaranteeing Stability Using Terminal Set Conditions . . . 12
2.5.2 Effect of Stability Criteria . . . 16
2.6 Feasibility . . . 16
2.7 Complexity . . . 17
2.8 Region of Attraction . . . 19
2.9 MPC in Practice . . . 19
2.9.1 Guaranteeing Stability in Practice . . . 19
2.9.2 Active Steady-state Constraints . . . 20
2.10 Summary . . . 20

3 Maximally Controllable Systems **23**
3.1 Introduction . . . 23
3.1.1 Implicit Constraints . . . 24
3.1.2 Continuous-Time MPC . . . 25
3.2 Input Constrained Unstable Systems . . . 28
3.2.1 Systems . . . 28
3.2.2 Control . . . 29
3.2.3 Analysis Elements . . . 34
3.2.4 Properties of the System . . . 39
3.2.5 Properties of the Analysis Regions . . . 41
3.2.6 Maximal Controllability . . . 45
3.2.7 Examples . . . 46
3.3 Active Steady-state Systems . . . 51
3.3.1 Systems . . . 52
3.3.2 Control . . . 53
3.3.3 Analysis Elements . . . 58
3.3.4 Properties of the Analysis Regions . . . 58

3.3.5 Maximal Controllability . . . 60
3.3.6 Example . . . 61
3.4 Summary . . . 62

4 Transfer Function Model Constraints 63
4.1 Introduction . . . 63
4.2 State-space Representation of ARMA Models . . . 64
4.3 Input Constrained Unstable Systems . . . 65
4.3.1 Isolating Unstable Dynamics and Bounds . . . 65
4.3.2 Equivalent Implicit Constraints . . . 66
4.3.3 Examples . . . 67
4.4 Active Steady-state Non-minimum Phase Systems . . . 71
4.4.1 Isolating Unstable Dynamics and Bounds . . . 71
4.4.2 Equivalent Implicit Constraints . . . 75
4.4.3 Example . . . 75
4.5 Summary . . . 77

5 Controllable Set Structure 79
5.1 Introduction . . . 79
5.2 Systems . . . 81
5.3 Controllable Sets . . . 81
5.3.1 Null Controllable Set . . . 81
5.3.2 N-step Controllable Set . . . 83
5.3.3 Controllable Set Representations . . . 85
5.3.4 Reduced Parameterisation . . . 89
5.3.5 Facet Structure of Controllable and Reduced Parameterisation Sets . . . 92
5.3.6 Facet Structure Properties . . . 93
5.4 Summary . . . 104

6 Reduced Parameterisation MPC 107
6.1 Introduction . . . 107
6.2 Implementation . . . 109
6.2.1 Algorithm . . . 109
6.2.2 Determining Set Membership and Reduced Parameterisation . . . 112
6.2.3 Optimisation . . . 136
6.3 Stability . . . 138
6.4 Worst Case Complexity . . . 141
6.4.1 Producing the Test Horizon Sequence . . . 142
6.4.2 Performing a Membership Determination . . . 143
6.4.3 Overall Complexity . . . 145
6.5 Numerical Aspects . . . 149
6.5.1 Exit Facet Determination Near Vertices . . . 150
6.5.2 Effect of Fast Poles . . . 150
6.6 Examples . . . 151

6.6.1 Systems with One Unstable Mode . . . 151
6.6.2 Systems with Two Unstable Modes . . . 151
6.6.3 Systems with Three Unstable Modes . . . 154
6.6.4 Average Computational Requirements . . . 155
6.7 Summary . . . 160

7 Conclusions 163

A Chapter 3 Appendix 171
A.1 Properties of Stabilising Gains K . . . 171
A.2 Proof of Lemma 3.1 . . . 173
A.2.1 Preliminary Results . . . 173
A.2.2 Proof of Lemma 3.1 . . . 175
A.3 Zero-dynamics Form . . . 180
A.3.1 Second Order Zero-dynamics Form . . . 181
A.4 Hurwitz Conditions for Systems in Section 3.3.1 . . . 182
A.5 Selection of ε for the Controller of Section 3.3.2 . . . 183

B Chapter 5 Appendix 185
B.1 Proof of Theorem 5.1 . . . 185

C Chapter 6 Appendix 189
C.1 Controllable Set Decomposition . . . 189
C.2 Limits of a Geometric Series . . . 190

Bibliography 191

Index 196

ABSTRACT

This dissertation presents some new approaches to addressing the main issues encountered by practitioners in the implementation of linear model predictive control (MPC), namely, stability, feasibility, complexity and the size of the region of attraction. When stability guaranteeing techniques are applied nominal feasibility is also guaranteed. The most common technique for guaranteeing stability is to apply a special weighting to the terminal state of the MPC formulation and to constrain the state to a terminal region where certain properties hold. However, the combination of terminal state constraints and the complexity of the MPC algorithm result in regions of attraction that are relatively small. Small regions of attraction are a major problem for practitioners.

The main approaches used to address this issue are either via the reduction of complexity or the enlargement of the terminal region. Although the ultimate goal is to enlarge the region of attraction, none of these techniques explicitly consider the upper bound of this region. Ideally the goal is to achieve the largest possible region of attraction which for constrained systems is the null controllable set.

For the case of systems with a single unstable pole or a single non-minimum phase zero their null controllable sets are defined by simple bounds which can be thought of as implicit constraints. We show in this thesis that adding implicit constraints to MPC can produce maximally controllable systems, that is, systems whose region of attraction is the null controllable set.

For higher dimensional open-loop unstable systems with more than one real unstable mode, the null controllable sets belong to a class of polytopes called zonotopes. In this thesis, the properties of these highly structured polytopes are used to implement a new variant of MPC, which we term reduced parameterisation MPC (RP MPC). The proposed new strategy dynamically determines a set of contractive positively invariant sets that require only a small number of parameters for the optimisation problem posed by MPC.

The worst case complexity of the RP MPC strategy is polylogarithmic with respect to the prediction horizon. This outperforms the most efficient on-line implementations of MPC which have a worst case complexity that is linear in the horizon. Hence, the reduced complexity allows the resulting closed-loop system to have a region of attraction approaching the null controllable set and thus the goal of maximal controllability.

1

Introduction

The focus of this thesis revolves around the linear variant of an advanced control technique known as *model predictive control* (MPC).

MPC is a discrete-time, model based, optimal control strategy. In brief, a linear model of the process is used to determine a control sequence that is optimal with respect to a chosen cost function and satisfying a set of constraints, that takes the process from some initial state to a desired target state or target set. Once the optimal control sequence is determined, the first control in the sequence is applied to the process. This procedure is repeated at the next control time interval using updated measurements from the process.

The origins of optimal control theory can be traced back to the work of Kalman (1960) in the form of *linear quadratic Gaussian* (LQG) control, and although used extensively to solve control problems in many areas, the process industry did not adopt this technique. Some of the more significant reasons cited for this, included LQG's inability to handle constraints, process non-linearities, model uncertainty, specific performance criteria and various cultural issues (e.g., education). Industry however implemented their own version of optimal control beginning with an algorithm by Richalet, Rault, Testud, and Papon (1978) named dynamic matrix control (DMC) and also referred to as *identify and command* (IDCOM) (Qin and Badgwell, 2003). Due to industry's rapid adoption of IDCOM, many researchers began to analyse the fundamental properties of this control strategy, providing rigorous mathematical results in the areas of stability, feasibility, optimality and complexity. Contributors in the area of stability include Muske and Rawlings (1993), Camacho (1993), de Oliviera and Biegler (1994), Scokaert, Mayne, and Rawlings (1999), Mayne, Rawlings, Rao, and Scokaert (2000). Results in the area of feasibility include Ricker, Subrahmanian, and Sim (1988), Zheng and Morari (1995), Scokaert and Rawlings (1999) and Vada, Slupphaug, Johansen, and Foss (2001). Contributions to the reduction of complexity can be found through a number of different approaches. For example, explicit solutions to MPC are presented in Bemporad, Morari, Dua, and Pistikopoulos (2000), Seron, De Doná, and Goodwin (2000), Tøndel and Johansen (2002) and Johansen, Petersen, and Slupphaug (2002), increased terminal regions appear in De Doná, Seron, Goodwin, and Mayne (2002) and Limon, Alamo, and Camacho (2003), sub-optimal solutions in Henriksson and Åkesson (2004), and simplifications to solving quadratic programming problems appear in Rao, Wright, and Rawlings (1998) and Cannon, Liao, and

Kouvaritakis (2006). Many of these significant contributions, amongst others, have been collected in various reviews, for example, Garcia, Prett, and Morari (1989), Morari and Lee (1997), Rawlings (1998), Mayne et al. (2000) and several comprehensive books specifically on MPC, for example, Camacho and Bordons (1999), Maciejowski (2002), Rossiter (2003) have been published.

The combination of industrial and academic research has enabled the evolution of MPC from its humble beginnings. Today, the process industry has in general adopted MPC as the control strategy of choice for complex processes, due largely to MPC's inherent simplicity for incorporating constraints in its formulation. The excellent survey paper by Qin and Badgwell (2003), details the evolution of MPC and confirms its growing popularity among many process industries.

Constraints are a reality of any real-world process, and fall into the two distinct categories, physical and operational. Physical constraints are the result of physical limitations of the process itself such as valve actuation ranges and tank levels, whereas operational constraints are those that are introduced to satisfy operational requirements such as ensuring optimal process yield, desirable operating ranges for equipment longevity and environmental considerations. Hence, as identified previously it is of fundamental importance that any control strategy employed to control such processes must allow the integration of constraints in its formulation. The incorporation of constraints, in conjunction with the implementation requirements of MPC algorithms, has introduced significant problems in the areas of feasibility and stability. Both of these areas have been addressed by researchers, but due to the close interaction between these areas, the resulting solutions in many cases introduce significant trade-offs. For example, the introduction of stability and implementation requirements reduce the feasible operating range of MPC.

Infeasibility of an optimisation problem occurs when no solution satisfying all of the constraints exists. For MPC, infeasibility occurs when an admissible control sequence that steers a process from some initial state to a final state, does not exist. This condition can occur due to many factors including modelling errors and disturbances, however there is a class of infeasibility conditions that results from ignoring the interaction between model dynamics and imposed constraints. This class of infeasibility conditions are in many cases avoidable. In general infeasibility is a major issue for practitioners and many authors have proposed methods for dealing with infeasibility. The paper by Scokaert and Rawlings (1999), summarises the main approaches to handling infeasibility. The main method of dealing with infeasibility, is by the removal or relaxation of what are termed *soft* constraints. Soft constraints are those constraints that can be relaxed or removed temporarily with little consequence. Various techniques for removing or relaxing constraints exist and include assigning priority levels to constraints to determine the order of relaxation as described by Vada et al. (2001). In the event that the problem is infeasible after the relaxation of all possible soft constraints, then the prediction horizon of MPC is increased or the problem is deemed to be not controllable. In most cases the relaxation of soft constraints allows the algorithm to contend with infeasibility conditions without incident, but in some cases this action leads to a

derogatory effect on the system and, as shown in Medioli, Seron, and Middleton (2005), could introduce instability.

The combination of process dynamics and constraints, results necessarily in the reduction of the region in the state-space for which any control strategy is able to control the process from some initial state to a desired final state. For the case of regulation the desired final state is the origin. In some cases it is possible to explicitly characterise the largest set of initial states that can be controlled to the origin. This set of initial states is commonly referred to as the *null controllable set or region*. Many authors have characterised the null controllable region of unstable systems and have provided control strategies that allow the resulting closed-loop system to operate over the entire region; see for example Lasserre (1993), Teel (1999), Hu, Miller, and Qiu (2002) and Favez, Mullhaupt, Shivasan, and Bonvin (2004). These closed-loop systems are often referred to as semi-globally stable systems. None of this research to the authors knowledge has addressed the null controllable region in the context of the MPC control strategy.

Most tractable implementations of MPC have a finite length prediction horizon and in general short horizons are needed to reduce the computational requirements of the algorithm also termed *complexity*. Using a finite length prediction horizon, necessarily introduces some additional constraints to the MPC problem. These additional constraints result in only a subset of the null controllable region being available to the MPC. The largest subset in which MPC is always feasible is termed its *feasibility region*. The distinction between a null controllable region and the feasibility region is that the former is dependent only on the model of the process and its associated constraints, while the latter is in addition dependent on the constraints introduced by the control strategy selected.

Unfortunately, optimality over a finite prediction horizon and in the presence of constraints, does not provide any stability guarantees. Hence, stability for the MPC algorithm can only be guaranteed if additional conditions are placed on the optimisation problem. Research in the area of stability for MPC has progressed to the point where the problem is well understood and the solutions available have been summarised elegantly by the survey paper Mayne et al. (2000). The introduction of stability conditions further restricts the feasibility region for an MPC formulation.

In concert, null controllable regions, finite prediction horizon and stability requirements significantly reduce the *region of attraction* of the MPC formulation, defined as the set of states from which asymptotic stability of the origin can be guaranteed for the MPC controlled system. This in turn results in potentially avoidable infeasibility conditions that have in part resulted in infeasibility handling algorithms. Further, by the recent survey conducted by Qin and Badgwell (2003), stability requirements have not been added to the major commercial implementations of MPC for the process industry.

The results presented in this thesis, provide a characterisation of null controllable sets and formulate variations of the MPC algorithm that leverage this information to produce closed-loop systems that have a region of attraction that is ar-

bitrarily close to the null controllable region. In addition, this is all achieved in the context of greatly reduced complexity, thereby allowing the use of such MPC algorithms in high speed applications.

The main contributions of this thesis are:

1. New insights on the null controllable region of the system to be controlled and the understanding of the significant benefits of incorporating this knowledge into the MPC formulation. For example, for systems with simple null controllable set representations, the boundaries of these sets can be thought of as implicit constraints.

2. The identification of an important, and practically relevant, class of systems, which by the application of MPC-like control formulated with implicit constraints, result in semi-globally asymptotically stable or maximally controllable closed-loop systems. That is, they have a region of attraction arbitrarily close to the null controllable region of the system.

3. A technique for introducing implicit constraints into MPC frameworks that are based on ARMA models.

4. A detailed analysis of the null controllable sets of input constrained, open-loop unstable systems with multiple unstable modes, which results in a comprehensive understanding of their structure and properties.

5. The development of a new variant of MPC called reduced parameterisation MPC (RP MPC), formulated using the results of the detailed analysis of input constrained, open-loop unstable systems. This new algorithm has a worst case polylogarithmic complexity with respect to the horizon length. Since, the best, worst case complexity for the on-line solution of MPC is linear with respect to horizon length, this constitutes a significant improvement. The reduced complexity allows increased horizon lengths and therefore expands the region of attraction closer to the null controllable set and maximal controllability.

This thesis is divided into seven chapters that detail both the background and the original contributions which in addition to this chapter are summarised as follows:

Chapter 2: This chapter provides background to the MPC algorithm to gain a better perspective on the features of the control strategy. The addition of constraints to the MPC formulation is paramount to any real-world implementation of MPC, but we note that some of the implications of constraints are not fully explored in the current literature and this fact leads to unexpected consequences. These consequences occur mainly in the context of feasibility for which many authors have supplied additional techniques for addressing this situation. These techniques are far from made redundant by the work presented here however some interesting observations are made in the

presence of our work. We also note that the optimality of MPC does not immediately result in stability guarantees in the presence of constraints and as such the additional stability requirements proposed by other authors are investigated. Closing the chapter is a discussion of the main issues that affect the practical implementation of MPC.

Chapter 3: This chapter analyses two classes of systems with relatively simple null controllable set representations, namely, input constrained systems with one unstable pole and output constrained second-order systems with one non-minimum phase zero. Analysis of the null controllable sets for these classes of systems shows that they can be equivalently represented by what we term *implicit constraints*. An MPC-like control technique, that includes implicit constraints, is formulated and applied to the two classes of systems. The resulting closed-loop systems are analysed and found to have a region of attraction arbitrarily close to the null controllable set or maximally controllable. Further, maximal controllability is achieved, from the MPC-like formulation, with a horizon length of one.

Chapter 4: All of the analysis presented in previous chapters is performed in state-space as this provides a logical framework which yields some important insights. We however note that in practice state-space models in MPC are far less prevalent. Model identification is a critical component to the formulation of MPC and its natural formulation is in terms of transfer function models. To show that the concepts developed in Chapter 3 are applicable in the prevalent implementation framework, we present a conversion technique for the application of implicit constraints to transfer function models.

Chapter 5: Having identified the relatively simple structure of unstable systems with a single unstable mode, this chapter investigates the characteristics of the null controllable sets of unstable systems with a higher number of unstable modes. The analysis develops important aspects and properties of these higher complexity null controllable sets, that are then used in the subsequent chapter.

Chapter 6: We propose a new algorithm termed *reduced parameterisation MPC* (RP MPC) that addresses avoidable infeasibility and reduces the complexity of MPC to allow the implementation of high speed MPC when operating near the controllable extremes of a system. This algorithm results directly from observations in previous chapters with respect to maximally controllable systems and their region of attraction.

Chapter 7: Summarises the work presented, details the conclusions gained from the research and addresses the direction of future research.

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INDEX

Active steady-state constraints, 20
    ARMA models, 71
    non-minimum phase system, 51

Complexity, *see* MPC or RP MPC

Facets
    basis, 87
    composite, *see* Reduced parameterisation
    core, *see* Reduced parameterisation
    exposed, *see* Test polytopes
    Minkowski sum, 86
    offset, 87
    support hyperplane, 87

Generators, 85
    composite, 90
    core, 85
    set ordering, 86

H-polytope
    normals, 88
    offsets, 88

Implicit constraints, 23
    ARMA models, 63

Maximally controllable, 10
    systems, 23
Membership determination, 115
    composite generator partitioning, 120
    exposed facet expansion, 119
    non-exposed facet expansion, 121
    test ray, 117
Minkowski sum, 85
MPC, 7
    continuous-time, 25
    feasibility, 16
    stability, 12
    worst case complexity, 17, 141

Nominal infeasibility, 11
N-step controllable set, 83
    feasible control sequence, 95
    H-polytope representation, 88
    Minkowski sum, 86
Null controllable set, 10
    multiple unstable poles, 81
    single non-minimum phase zero, 53
    single unstable pole, 29

Orthogonal complement, 87

Positively invariant set, 10

Reduced parameterisation, 89
    composite facet, 93
    core facet, 98
    generators, 90
    H-polytope representation, 91
    index sequence, 90
    maximum parameters, 96
    Minkowski sum, 90
Region of attraction, 10
RP MPC, 107
    algorithm, 109
    stability, 138
    worst case complexity, 145

Test polytopes, 116
    corresponding reduced parameterisation, 116
    exposed facets, 119
    facet sets, 118
    H-polytope representation, 116
    Minkowski sum representation, 116

Zonotopes, 85
    facets, 86
    support hyperplane, 87