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Performance Limitations in the Feedback Control of a Class of Resonant Systems[1]

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Abstract

There has been a large literature on the feedback control of flexible and resonant systems. Such systems arise naturally when system weight and or response speed issues push designers toward lighter, faster structures for a range of mechanical systems. Feedback control of such systems is often proposed to ameliorate the effects of the resonance. In this paper, we investigate the extent to which the dynamic structure of a simple class of resonant systems limits the achievable feedback control performance for such systems. It turns out that in the class of systems considered, there is a trade off between three common control objectives, namely: (i) good initial transient response (that is the absence of large overshoot or undershoot in the initial rise time), (ii) fast response, (iii) good settling behaviour (that is, the absence of very slow modes in the step response).

1 Introduction

Physical systems which exhibit resonance, or flexible modes, have been noted in several application areas. Some examples of such applications include: Computer Disk Drives [1], Robotics [2],[6], Spring Mass systems and Noise Cancelling Systems [4], Flexible Structures [8],[12] and Rolling Mills [3].

Many flexible systems inherently contain a number (possibly infinite) of lightly damped resonant modes. Here, we consider a simplified class of systems with a single lightly damped resonant mode. It is our contention that the performance limitations associated with more complex systems considering multiple modes will be at least as restrictive as the limitations we expound for the single resonance case.

Stein and Greene [10] consider a flexible system and study the implications of Bode gain-phase when attempting to 'phase stabilize' (or 'strongly actively damp') a large number of resonant modes. Goodwin et al [3] describe performance limitations in a SISO setting due to resonant zeros using time domain constraints assuming the lightly damped resonant zeros are not cancelled in the controller. Hong and Bernstein [4] consider actuator placement, spillover and other issues in a four block setting, where a scalar input, disturbance, sensor and performance variables are considered. Here we wish to generalize these results, in particular, we wish to consider aspects of sensitivity, minimal achievable linear quadratic and L_∞ costs, and situations where there is a single actuator, yet multiple performance variables in a resonant system. To do this, we first "sharpen" the results of [3], to give the infimal L_∞ error in a SISO system with resonant zeros. This result implicitly assumes that the lightly damped zeros are not cancelled in the controller. Later results interpret [13], [14] & [15] for resonant systems with two performance variables. These results support our earlier analysis without the need to assume zeros aren't cancelled.

In Section 2 we define the class of systems we wish to consider and introduce some preliminary definitions. Following this, in Section 3 we consider the problem of the best achievable L_∞ performance for SISO systems with lightly damped zeros. In Section 4, we turn to the problem of the cheap control tracking for resonant systems. In particular, we show that in a single input two output (SITO) sense, there is a non-trivial lower bound on the achievable L_2 performance for resonant systems. In section 5, we interpret a SITO integral developed in [13] for resonant systems. The particular class of systems we consider here simplifies the integral constraints in [13] and therefore admits more direct interpretations. In section 6, an example is used to illustrate the main conclusions of the paper. Section 7 concludes, by noting that from several perspectives, there is a fundamental tradeoff in the class of resonant systems considered, between the three common control objectives of:

- Keeping transient and sensitivity peaks small
- Achieving a fast, high bandwidth response
- Settling accurately, without long time constants

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2 Problem Formulation

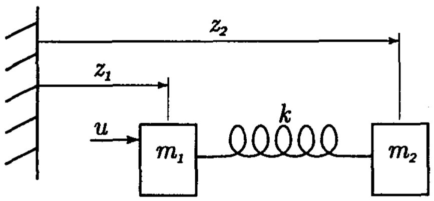

Figure 1: Spring Mass System

To facilitate this discussion, we consider a simplified system as shown in Figure 1. Here we use z_1, z_2, m_1, m_2 to denote (respectively) the actual positions, and masses of the two objects. We also use u and k to denote (respectively) the input force (applied to m_1) and the spring constant of the system. For simplicity here we ignore small but positive damping in the system. We denote by y_1, y_2, n_1, n_2 the measurements of z_1 and z_2, and the sensor noise in these measurements, respectively.

If we then chose as state variables: $x = \begin{bmatrix} z_1 & \dot{z}_1 & z_2 & \dot{z}_2 \end{bmatrix}^T$ then it is a straightforward exercise in dynamics (see also [4]) to derive the state space equations:

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k}{m_1} & 0 & \frac{k}{m_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k}{m_2} & 0 & -\frac{k}{m_2} & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{1}{m_1} \\ 0 \\ 0 \end{bmatrix} u \quad (1) \\ y &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \end{aligned}$$

This model can also be expressed in transfer function form as:

$$\begin{aligned} Y(s) &= \frac{1}{Den(s)} \begin{bmatrix} (m_2 s^2 + k) \\ k \end{bmatrix} U(s) + N(s) \quad (2) \\ &= P(s)U(s) + N(s) \end{aligned}$$

$$\text{where} \quad D(s) = s^2 \left(s^2 m_1 m_2 + k\left(m_1 + m_2\right)\right) \quad (3)$$

Although this transfer function has been derived for a particular physical system, a very similar dynamic structure arises from other example problems including flexible drive robots [6], crane control (or "pendulum down") type problems [13, Example 2.5.11], and stepper motor damping [5].

Also, in many cases of interest, the plant transfer function, $P(s)$, from input to performance variables is not square. In particular, we are considering an actuator deficient plant, with more performance variables than actuators.

For later use, we define two resonant frequencies, the *free* resonance, ω_f , and the *locked*[3] resonance, ω_l defined by:

$$\omega_f = \sqrt{\frac{k\left(m_1 + m_2\right)}{m_1 m_2}} \quad (4)$$

$$\omega_l = \sqrt{\frac{k}{m_2}} \quad (5)$$

In [15, Sec2.2], the concept of *Feasible Set point Coordinates (FSC)* is introduced. These coordinates define a linear static reparameterization of the outputs which clearly describe the achievable steady state set points. In this case, the outputs in FSC, $\tilde{y} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} y$ can be represented by:

$$\begin{aligned} \tilde{Y}(s) &= \frac{1}{\sqrt{2}D(s)} \begin{bmatrix} m_2 s^2 + 2k \\ m_2 s^2 \end{bmatrix} U(s) + \tilde{N}(s) \quad (6) \\ &= \tilde{P}(s)U(s) + \tilde{N}(s) \end{aligned}$$

Note that here, apart from scaling, FSC corresponds to using the average position, and the difference in positions, rather than the individual positions of the masses.

In the remainder of the paper, we shall also make use of the following "settling" time definition:

Definition 1 *We say that the closed loop system has an* exact settling time T_{es} *if for all* $t > T_{es}$ *the error in the step response is zero.*

Note that in theory, an *exact* settling time does not exist in most systems. However, for brevity here, we will assume that an exact settling time does exist. Note that the results that follow in sections 3 and 4 which use the exact settling time can be generalized to the case where we have an exponential bound on the tail of step response, such as $|e(t)| \le \epsilon e^{\alpha(t-T_s)}$ for all $t > T_s$. Provided ϵ is small, and α is not too small compared with T_s^{-1}, then qualitatively similar results will follow. This is explored in more detail in [3, Section 5.4] for a similar problem. For simplicity, we do not repeat the details of the arguments here, except to note that if α is allowed to be small, that is if we allow slow final settling behaviour, then the results following are relaxed, or possibly even removed.

In the next section, we consider the problem of determining the lowest worst case peaks in the error signal which can occur. These results investigate more thoroughly the results of [3] by using L_1 optimal control theory ([7]).

[3] The term *free* is used to denote the resonant frequency when there are no external forces; the term *locked* is used to denote the resonant frequency when the actuated mass, m_1, is fixed. This term arises in the context of compliant drive systems where m_1 is analogous to the drive inertia, m_2 is analogous to the load inertia, and ω_l can be determined by "locking" the drive.

3 SISO Time Domain Constraints - Infimal L_∞norm

We begin by providing a result from [7] and [11] about the structure of the error signal with minimum L_∞ norm. We then show how this result may be interpreted for our example system.

Proposition 1 *Given constants $T \in \mathbf{R}^+$, $A_i \in \mathbf{R}$ and linearly independent analytic functions $C_i : \mathbf{R}^+ \to \mathbf{R}$, $i = 1..n$ define the n integral constraints on a function $v(t)$:*

$$\int_0^T v(t)C_i(t)dt = A_i \tag{7}$$

Suppose there exist real constants α_i (not all zero), $i = 1..n$ and c such that with

$$v^*(t) = c \text{ sign} \left(\sum_{i=1}^{n} \alpha_i C_i(t) \right) \tag{8}$$

(where sign denotes the signum function), $v^*(t)$ satisfies the constraints of (7). Then

$$\min_{\substack{v(.) \\ v(.) \text{ satisfies (7)}}} \|v(.)\|_{L_\infty} = \|v^*(.)\|_{L_\infty} \tag{9}$$

Proof Outline: (see also [7] or [11]).
Note that the result is trivial for $c = 0$. For $c \neq 0$, consider any $v(t)$ which satisfies the constraints, (7). Then since $v^*(t)$ also satisfies the constraints:

$$\int_0^T (v(t) - v^*(t))\, C_i(t)dt = 0, \qquad i = 1 \ldots n \tag{10}$$

which implies:

$$\int_0^T (v(t) - v^*(t)) \sum_{i=1}^{n} \alpha_i C_i(t)dt = 0 \tag{11}$$

Since the integral in (11) is zero either the integrand is zero (except on sets of measure zero), or the integrand must alternate signs. If the integrand is zero, then since C_i are linearly independent and analytic, then $v(t) = v^*(t)$ (except on a set of measure zero), and (9) is satisfied with equality. Otherwise, the integrand must alternate signs on a non-zero measure set and so there exists a set $S \subset [0, T)$ of non-zero measure such that for all $t_0 \in S$

$$(v(t_0) - v^*(t_0))\, \text{sign}(v^*(t_0)) > 0 \tag{12}$$

(8) and (12) directly imply that for all $t_0 \in S$

$$|v(t_0)| > |v^*(t_0)| = |c| \tag{13}$$

$$\text{and so} \qquad \|v(.)\|_{L_\infty} > \|v^*(.)\|_{L_\infty} \tag{14}$$

□

We now show how this result can be applied to systems with imaginary axis zeros.

In [3] it was shown that there are constraints on the achievable performance due to resonant zeros. In this case, if we consider simply the SISO system relating u to y_1 :

$$P_1(s) = \frac{m_2 \left(s^2 + \omega_l^2\right)}{m_1 m_2 s^2 \left(s^2 + \omega_f^2\right)} \tag{15}$$

and from [3] we see that the integral constraints for (unit) step output disturbance[4] performance with exact settling time T_{es} are given by:

$$\begin{cases} \int_0^{T_{es}} e(t) \cos(\omega_l t)\, dt &= 0 \\ \int_0^{T_{es}} e(t) \sin(\omega_l t)\, dt &= \frac{1}{\omega_l} \end{cases} \tag{16}$$

and we know from Proposition 1 that the min L_∞ error signal, subject to (16), will be of the form:

$$\begin{cases} c\ sign\,(\alpha_1 \cos(\omega_l t) + \alpha_2 \sin(\omega_l t)) & 0 \leqslant t \leqslant T_{es} \\ 0 & t > T_{es} \end{cases} \tag{17}$$

If we restrict ourselves to $T_{es} \leqslant \frac{\pi}{\omega_l}$, then $sign\,(\alpha_1 \cos(\omega_l t) + \alpha_2 \sin(\omega_l t))$ will contain only one sign change. Let τ be the time where this switch occurs, so the constraints can be rewritten as:

$$\begin{cases} c\left[\int_0^{\tau} \cos(\omega_l t)dt - \int_{\tau}^{T_{es}} \cos(\omega_l t)dt\right] &= 0 \\ c\left[\int_0^{\tau} \sin(\omega_l t)dt - \int_{\tau}^{T_{es}} \cos(\omega_l t)dt\right] &= \frac{1}{\omega_l} \end{cases} \tag{18}$$

which are solved to give:

$$\|e_{min}(t)\|_{L_\infty} = |c| = \left| \frac{1}{1 + \cos\omega_l T_{es} - 2\cos\sin^{-1}\left(\frac{\sin\omega_l T_{es}}{2}\right)} \right| \quad \text{for } T_{es} \leqslant \frac{\pi}{\omega_l} \tag{19}$$

This is shown (solid) in Figure 2 along with (dashed)the approximate bound derived in [3]. This figure clearly shows that it is not possible to have rapid 'exact' settling and small transient peaks in L_∞. Note also that as the spring becomes stiffer, ω_l becomes larger and therefore the lower bound on the L_∞ norm is relaxed.

4 Cheap Linear Quadratic Cost

In this section, we apply the results of [15] to the problem defined in section 2. In particular, the main result of this section is the following:

Proposition 2 *For any stabilizing controller for the plant (1), the integral squared error for a unit feasible setpoint change must satisfy:*

$$\int_0^\infty e^T(t)e(t)dt \geq 0.64\omega_l^{-1} \tag{20}$$

[4] or equivalently, a negative unit step reference

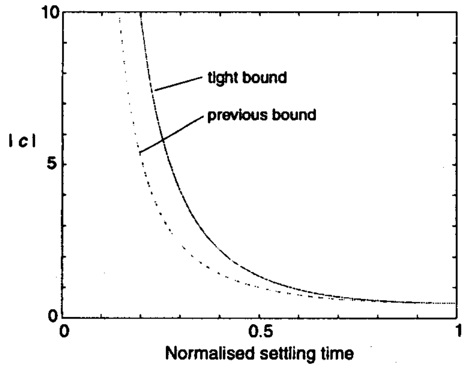

Figure 2: Plot of $|c|$ vs $\frac{\omega_l T_{es}}{\pi}$

Proof: Note that for the plant given in (6), the plant numerator in FSC is:

$$\tilde{P}_N(s) = \begin{bmatrix} m_2 s^2 + 2k \\ m_2 s^2 \end{bmatrix} \tag{21}$$

We note from (21) that $\tilde{P}_{N_1}(s)$ and $\tilde{P}_{N_2}(s)$ have all zeros on the imaginary axis. Therefore, for the cheap control tracking problem, we primarily need to consider the term Δ, defined:

$$\begin{aligned} \Delta(s) &= \tilde{P}_N^T(-s)\tilde{P}_N(s) \\ &= 2\left(m_2^2 s^4 + 2m_2 s^2 k + 2k^2\right) \end{aligned} \tag{22}$$

From (22) it is easy to show that $\Delta(s)$ has exactly two zeros, $\delta_{1,2}$, with positive real parts, namely:

$$\delta_{1,2} = \sqrt{\frac{k}{m_2}}\left(\sqrt{\frac{\sqrt{2}-1}{2}} \pm i\frac{\sqrt{\sqrt{2}+1}}{2}\right) \tag{23}$$

In particular, we note from [15, Thm 3.1] that for a setpoint tracking problem, where the setpoint is the (feasible) unit setpoint: $\tilde{y}^* = \begin{bmatrix} 1 & 0 \end{bmatrix}^T$, the cheap control cost is given by[5]:

$$\lim_{\varepsilon \to 0}\left(\int_0^\infty e^T(t)e(t) + \varepsilon^2 u^2(t)dt\right) = Var(P) \tag{24}$$

where

$$\begin{aligned} Var(P) &= \sum \delta_{1,2}^{-1} \\ &= \sqrt{\frac{k}{m_2}}\sqrt{\sqrt{2}-1} \\ &\approx 0.64\omega_l^{-1} \end{aligned} \tag{25}$$

and the result follows. □

One interpretation of this result is the following corollary:

[5]Note that in this case, since the plant includes an integrator (in fact it includes two), then in [15] $\bar{u}$ is zero, and therefore, $u = u_e$.

Corollary 1 *Consider the system [2], and assume that the closed loop has an exact settling time of T_{es}, then the rms error, $e_{rms}^{T_{es}}$ over the interval $[0, T_{es})$ defined by:*

$$e_{rms}^{T_{es}} = \sqrt{\frac{1}{T_{es}}\int_0^{T_{es}} e^T(t)e(t)dt} \tag{26}$$

$$\textit{satisfies} \quad e_{rms}^{T_{es}} \geq \frac{0.8}{\sqrt{\omega_l T_{es}}} \tag{27}$$

Proof: Follows directly from Proposition 2 and the definition of exact settling time. □

It follows directly from the above corollary, that if the product of the locked resonance and the exact settling time is small, then necessarily, the step response will have a large r.m.s. error during the initial phase, $t \in [0, T_{es})$. In particular, small spring constants (that is very flexible systems) give small locked resonant frequencies and are therefore difficult to control with both small r.m.s. errors, and small exact settling times.

5 Frequency Domain Integral Sensitivity Results

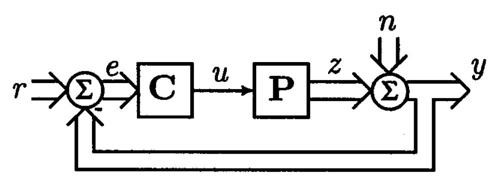

Figure 3: SITO feedback Control

Here, we review some of the main results from [13, Chapter 3], highlighting their interpretation for resonant systems. We first define $\tilde{T}_O$ the output complementary sensitivity function, in FSC, as:

$$\tilde{T}_O(s) = P(s)C(s)\left(I + P(s)C(s)\right)^{-1} \tag{28}$$

where $C(s)$ is the controller transfer function as in Figure 3. Note that $\tilde{T}_O$ is in fact the transfer function from output noise to the output, or equivalently, from the reference signal to the output. We will also define $\tilde{S}_O$ as the output sensitivity, which is the transfer function from output disturbances to the output. $\tilde{S}_O$ also obeys the identity: $\tilde{S}_O + \tilde{T}_O = I$.

Since the only feasible steady state outputs are those where $\tilde{y}_2$ is zero, it is of interest to consider the response to a unit feasible setpoint, $\tilde{r} = [1\ 0]^T$. This is given by the first column of $\tilde{T}_O$ which we denote by $\tilde{T}_{O\bullet 1}$. We then have the following result:

Proposition 3 *Consider any internally stablilizing controller for the system (6). Then:*

$$\int_0^\infty \ln\left(\|\tilde{T}_{O\bullet 1}(j\omega)\|\right)\frac{d\omega}{\omega^2} \geqslant \frac{\pi}{2}Var(P) = 1.005\omega_l^{-1} \tag{29}$$

Proof This follows as a special case of [13, Proposition 3.1.7], upon noting that for the system (6), there are only complex zeros in $\tilde{P}_N(s)$, and therefore, several terms in [13, Proposition 3.1.7] are zero. Also, since there is a double integrator in the plant and $\tilde{P}_{N_2}(s)$ contains a double zero at $s = 0$, then further terms in [13, Proposition 3.1.7] are zero, leaving (29) as the result. □

This proposition shows that the weighted average of $\ln\left(\left\|\tilde{T}_{O\bullet 1}(j\omega)\right\|\right)$ is a strictly positive quantity, which in turn implies that $\left\|\tilde{T}_{O\bullet 1}(j\omega)\right\|$ must exceed unity over a range of frequencies.

Note that since the plant (2) contains a double integrator, and we are in feasible set point coordinates, any internally stabilizing controller yields at low frequencies:

$$\tilde{S}_{O11}(s) = O\left(s^2\right) \tag{30}$$

Therefore, there will exist positive constants c_2 and ω_0 such that

$$\left|\tilde{S}_{O11}(j\omega)\right| \leqslant c_2 \frac{\omega^2}{\omega_0^2}, \qquad \forall \omega < \omega_0 \tag{31}$$

Proposition 4 *If we define the* low frequency L_2 energy in the error signal *as:*

$$E_{LF}^2(\omega_0) = \int_0^{\omega_0} \|E(j\omega)\|^2 d\omega \tag{32}$$

where $E(s)$ is the Laplace transform of the error in the step response, then:

$$\sup_{\omega > \omega_0} \ln\left(\|\tilde{T}_{O\bullet 1}(j\omega)\|\right) \geqslant 1.005\frac{\omega_0}{\omega_l} - c_2 - \frac{\omega_0}{2} E_{LF}^2(\omega_0) \tag{33}$$

Proof Using (31) in (29) we can show that:

$$\sup_{\omega > \omega_0} \ln\left(\|\tilde{T}_{O\bullet 1}(j\omega)\|\right) \geqslant \omega_0 \int_{\omega_0}^{\infty} \ln\left(\|\tilde{T}_{O\bullet 1}(j\omega)\|\right) \frac{d\omega}{\omega^2} \tag{34}$$

Splitting the interval of intergration in (33) to the difference between $\int_0^\infty$ and $\int_0^{\omega_0}$ and using (29) gives:

$$\sup_{\omega > \omega_0} \ln\left(\left\|\tilde{T}_{O\bullet 1}(j\omega)\right\|\right) \geqslant 1.005\frac{\omega_0}{\omega_l} - \frac{\omega_0}{2}\int_0^{\omega_0} \ln\left[\left(\begin{bmatrix}1\\0\end{bmatrix} - \tilde{S}_{O\bullet 1}(-j\omega)\right)^T \left(\begin{bmatrix}1\\0\end{bmatrix} - \tilde{S}_{O\bullet 1}(j\omega)\right)\right] d\omega \tag{35}$$

From (35), using the fact that $\ln(1+x) \leq x$ and $E(s) = \frac{1}{s}\tilde{S}_{O\bullet 1}(s)$ gives:

$$\begin{aligned}
\sup_{\omega > \omega_0} \ln\left(\left\|\tilde{T}_{O\bullet 1}(j\omega)\right\|\right) &\geqslant 1.005\frac{\omega_0}{\omega_l} - \frac{\omega_0}{2}\int_0^{\omega_0}\left(-2\Re e(\tilde{S}_{O11}(j\omega)) + \|\tilde{S}_{O\bullet 1}(j\omega)\|^2\right)\frac{d\omega}{\omega^2} \\
&\geqslant 1.005\frac{\omega_0}{\omega_l} - \omega_0\int_0^{\omega_0} c_2\frac{\omega^2}{\omega_0^2}\frac{d\omega}{\omega^2} - \frac{\omega_0}{2}\int_0^{\omega_0}\|E(j\omega)\|^2 d\omega \\
&= 1.005\frac{\omega_0}{\omega_l} - c_2 - \frac{\omega_0}{2}E_{LF}^2(\omega_0)
\end{aligned} \tag{36}$$

as required. □

Note that $E_{LF}^2(\omega_0)$, the low frequency L_2 energy in the error signal, is indicative of the long term (i.e. settling) behaviour of the error signal. Clearly, from equation (33) there is a tradeoff between the objectives of:
(i) keeping E_{LF}^2 and c_2 small (for good settling and low frequency disturbance rejection);
(ii) keeping ω_0 for fast control; and
(iii) keeping peaks in $\tilde{T}_{O\bullet 1}$ small (to avoid undesirable transient and sensitivity peaks).

6 Examples

In this section we will look at typical responses achievable at the outputs of the spring-mass system. Additional constraints due to factors such as input bandwidth, saturations, slew rate limits or measurement noise will further restrict the achievable response but are not considered here.

From (2) we see that the system has no multivariable zeros and no right half plane poles[6]. In fact, if we use state feedback, then the closed loop response at the output is given by:

$$Y_{cl}(s) = \frac{1}{d_{cl}(s)}\begin{bmatrix} m_2 s^2 + k \\ k \end{bmatrix} r \tag{37}$$

where $d_{cl}(s)$ has degree 4 and the system poles (in d_{cl}) can be arbitrarily chosen.

The following figures illustrate typical reference step responses for the normalized system ($1 = m_1 = m_2 = k$) under state feedback: $u = -K\left(x - [1\ 0\ 1\ 0]^T r\right)$. These parameter values put the system zeros at $\pm j$.

From the results in the preceding sections it is clear that there are restrictions on the responses available. These will generally take one of the following forms:

Case (a) Fast response, with rapid settling (and therefore poor transients). This type response is shown in signals Y1A & Y2A of Figure 4 where fast settling has occurred, but at the expense of a bad initial transient. State feedback gains for this example are: $K = [\ 30.46 \quad 9.00 \quad 8.28 \quad 46.80\ ]$.

Case (b) Fast response, but with a long slow settling. Signals Y1B & Y2B in Figure 4 represent a

[6] we ignore imaginary axis poles since they don't effect the "costs" discussed in sections 3-5

different trade-off which results in good (fast) initial transients, but poor settling characteristics. In this case the state feedback gain was chosen as: $K = [\ 0.150 \quad 1.80 \quad 0.674 \quad 0.0082\ ]$. The controller has poles near the imaginary axis zeros which trades off a better y_1 response against y_2 performance.

Case (c) Slow response, with good overall behaviour. A typical response of this type is shown in Figure 4 with signals Y1C & Y2C, where we have traded both speed of initial response & the final settling time for a much better although slower response. State feedback gains are: $K = [\ 2.38 \quad 3.40 \quad -1.64 \quad -0.72\ ]$.

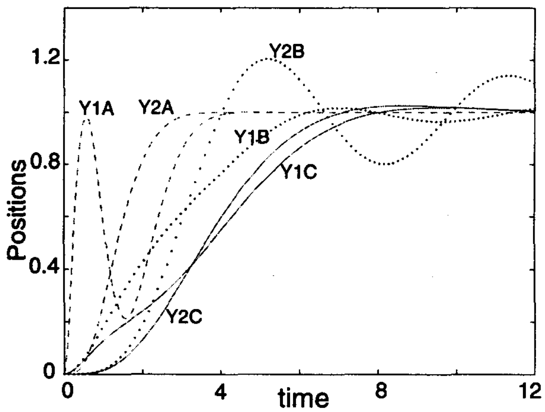

Figure 4: Reference Step Responses

7 Conclusions

We have examined the control of a class of resonant system with more performance variables than actuators. In the class of systems considered, there is a feedback control performance limitation which may be expressed in various ways, including, a non-trivial minimal integral squared error; a non-trivial L_1 performance cost whenever there is a limit on the closed loop response time, and a frequency domain sensitivity integral. All of these factors point to a limitation which can be described qualitatively as a trade-off between response time (that is bandwidth) and sensitivity/transient performance limitations.

Further work in this area is aimed at understanding further issues in actuator and sensor placement, and looking at multiple modes (e.g. flexible structure) systems.

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