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Achievable Performance and Sensitivity Integral Constraints in Preview Control

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Abstract

Preview Control refers to a tracking control scheme whereby a reference trajectory, and a finite 'preview' of future values of the reference trajectory are used in the controller. Such controllers have been explored in the context of linear and nonlinear inverse tracking controllers, where particularly for non stably invertible plants, preview of the reference trajectory gives significant performance advantages. Without the presence of preview, it has been known that lack of a stable inverse imposes inherent limitations on the achievable performance of a feedback control loop. In this paper, we explore the extension of these performance limitations to the preview control case. In particular, we consider the infimal achievable $\mathcal{H}_\infty$ performance, and Poisson sensitivity integral results for the finite preview case.

1 Introduction

Consider a situation where we have a single input single output linear time invariant plant, where the control objective is to track a trajectory. If the plant is minimum phase (that is, it possesses a marginally stable inverse), and in the absence of other constraints, then appropriate feedforward and high gain feedback control techniques may be used to give high performance tracking of the system output. However, in the case where the plant is non-minimum phase (that is, it lacks a stable inverse), such tracking is not, in general, possible. Further research (see for example [5], [11], [12] and [4]) has shown that in situations where the reference trajectory is known in advance, improved tracking performance can be attained; indeed, with unlimited advance knowledge of the trajectory, perfect asymptotic tracking of any bounded reference trajectory can be attained. This method of control, whereby advance knowledge of the reference trajectory is utilised, is sometimes referred to as 'Preview Control'. We shall further differentiate control schemes which require complete knowledge of the reference trajectory ahead of time, from those where at time t the reference trajectory is known up until time $t+T$, where T is a fixed positive time, which we denote as the 'Preview Horizon'. This latter class we denote as 'finite preview control' and it is this class which is our primary focus in this paper.

One of the aspects of this technique of preview control which has received little attention until recently is the sensitivity and robustness properties of such control schemes. In [14], some aspects of sensitivity and robustness are discussed, mainly from the point of view of resolving an apparent paradox with time domain integral constraints espoused in [13]. Recent work in [5] considers the achievable H_2 tracking performance in a preview setting.

The main aims of this paper are twofold. First, we consider the problem of optimal achievable H_∞ tracking performance in a finite preview control setting. In particular, we show that there is a non trivial lower bound on this achievable performance which is dictated by the cost function weighting, the plant non minimum phase zeros, and the preview horizon, T. Secondly, we shall examine frequency domain interpolation constraints, and their consequent implications on a Poisson logarithmic sensitivity integral, in the spirit of those developed in [6] and [15]. From both perspectives we shall see that increasing the preview horizon permits improved tracking performance in the preview control scheme.

2 Preliminaries

We denote by ORHP and CRHP respectively, the open and closed right half complex planes. We consider linear time invariant scalar dynamic systems, with rational transfer functions. We use the notation $\mathcal{Z}_T$ and $\mathcal{P}_T$ to denote respectively the sets of ORHP zeros (NMP zeros) and poles of a transfer function $T(s)$. We say that $T(s)$ is 'stable' if none of it's poles is in the CRHP, and minimum phase if none of it's zeros is in the ORHP. A transfer function, $T(s)$ is said to be 'proper' if $\sup_{s\to\infty, s\in\mathbb{C}_+} |T(s)|$ exists and is finite. For any stable, proper transfer function $T(s)$ the $\mathcal{H}_\infty$ norm is defined as $\|T(s)\|_\infty = \sup_\omega |T(j\omega)|$.

Consider a linear feedback system as indicated in Figure 1 where $P_o(s)$ and $C(s)$ denote respectively the linear time invariant open loop plant and controller; $r(t+T)$, $u(t)$ and $y(t)$ are scalar functions of time denoted, respectively, as the reference signal, with preview of T, the plant input and the plant output. Our control objective is to make the plant output $y(t)$ track the reference $r(t)$ accurately. The controller,

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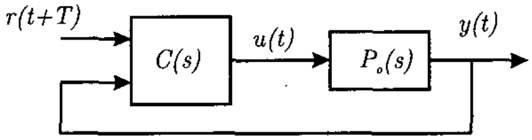

Figure 1: Preview Control Block Diagram

$$C(s) = c(s)\left[\begin{array}{cc} H(s) & C_{FB}(s) \end{array}\right] \tag{1}$$

is assumed to be rational, and proper. We shall assume that the feedback loop in Figure 1 is internally stable, and therefore can be rearranged as shown in Figure 2 where $e(t)$ is now the performance variable, and $P(s)$ is the stable closed loop plant

$$P(s) = \frac{c(s)\,P_o(s)}{1 + c(s)\,C_{FB}(s)\,P_o(s)} \tag{2}$$

Note that due to the requirement of internal stability,

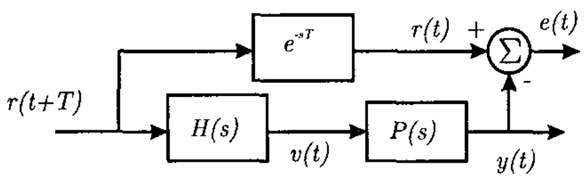

Figure 2: Feedforward only equivalent system

$P(s)$ inherits all NMP zeros of $P_o(s)$. Indeed, it can be shown that

$$\mathcal{Z}_P = \mathcal{Z}_{P_o} \cup \mathcal{Z}_c \cup \mathcal{P}_{C_{FB}} \tag{3}$$

For simplicity, we make the following assumption.

Assumption 1 *Both $\mathcal{P}_{C_{FB}}$ and $\mathcal{Z}_c$ are empty, that is $C_{FB}(s)$ is stable, and $c(s)$ is minimum phase.*

Note that Assumption 1 can always be achieved by partitioning the feedback transfer function $(c(s)\,C_{FB}(s))$ appropriately.

Under Assumption 1, it is clear that $P(s)$ is a stable transfer function, with precisely the same set of NMP zeros as the original open loop plant, $P_o(s)$. We then define the command tracking sensitivity function:

$$\begin{aligned} S_C(s) &= e^{-sT} - H(s)\,P(s) \\ &= e^{-sT}\frac{E(s)}{R(s)} \end{aligned} \tag{4}$$

The use of a two degree of freedom control architecture implies that unstable open loop poles do not impose interpolation constraints upon the command tracking sensitivity. This effect does not depend on the use of preview control. Indeed, suppose that the architecture of Figure 1 is applied without preview. Then the command tracking sensitivity reduces to:

$$\begin{aligned} S_C(s) &= 1 - H(s)\,P(s) \\ &= 1 - H(s)\left(\frac{c(s)\,P_o(s)}{1 + c(s)\,C_{FB}(s)\,P_o(s)}\right) \end{aligned} \tag{5}$$

Therefore, at any unstable open loop plant pole, p_o, we have

$$S_C(p_o) = 1 - \frac{H(p_o)}{C_{FB}(p_o)} \tag{6}$$

and there is no 'fundamental' interpolation condition required in this two degree of freedom controller structure. In other words, the value of S_C at an open loop plant pole is not fixed, but is determined by the controller applied. This phenomena may be understood by noting that poles are altered by feedback.

Conversely, it follows from (5) that interpolation constraints due to plant NMP zeros cannot be removed by the two degree of freedom architecture of Figure 1 with no command preview. As may be seen from (12), the use of preview control does alter the interpolation constraints, and we shall investigate the design implications of this below.

In the following sections we shall give a more detailed examination of properties of the command tracking sensitivity function. In particular, in Section 3 we describe the limiting achievable $\mathcal{H}_\infty$ tracking performance with finite preview, followed by examining frequency domain Poisson integral inequalities in Section 4.

3 $\mathcal{H}_\infty$ Tracking Performance

Consider the infimal achievable weighted $\mathcal{H}_\infty$ norm :

$$\gamma_{\min} := \inf_{H \text{ stable}} \{\|w(s)S_C(s)\|_\infty\} \tag{7}$$

where $w(s)$ is a real rational function. $w(s)$ is assumed to be minimum phase and stable, with no loss of generality. Furthermore, we shall assume that $w(s)$ is strictly proper. This assumption of a strictly proper weighting is required for technical reasons (see for example Lemma 3 below). In addition for any strictly proper plant, note that without such an assumption on $w(s)$ regardless of the amount of preview, $\gamma_{\min} \geq w(\infty)$ since $S_C(\infty) = 1$. As we shall see below, this is in contrast to the strictly proper case where arbitrarily good infimal $\mathcal{H}_\infty$ tracking performance may be achieved by increasing the amount of preview.

The minimal $\mathcal{H}_\infty$ norm $\gamma_{\min}$ may in general be interpreted as a worst-case performance measure of tracking error, i.e., the best possible tracking error quantified under the integral square criterion, in response to all possible energy-bounded reference input signals. Thus, it serves as a benchmark complementary to the usual $\mathcal{L}_2$ type criterion. Tracking with reference preview is predominantly effective in the low frequency range, and is generally performed on a prefiltered signal. In this sense, $w(s)$ may be interpreted as the transfer function of a lowpass prefilter, or the spectrum of the reference signal whose essential frequency component is in the low frequency range.

The main technical tool to be used in our development will be the theory of analytic function interpolation. The following preliminary lemma is standard in this theory (see, e.g., [1]), which gives a necessary and sufficient condition to the classical Nevanlinna-Pick interpolation problem.

Lemma 2 *Consider two sets of complex numbers* $(z_i,\ u_i)$, $i = 1,\ \cdots,\ k$, *where* z_i *are distinct. There exists a rational function* $F(s)$ *such that (i)* $F(s)$ *is stable, (ii)* $\|F(s)\|_\infty \leq \gamma$, *and (iii)* $F(s)$ *satisfies the conditions*

$$F(z_i) = u_i, \qquad i = 1,\ \cdots,\ k, \tag{8}$$

if and only if

$$Q - \frac{1}{\gamma^2} U Q U^H \geq 0. \tag{9}$$

where $U = diag(u_1,\ u_2,\ \cdots,\ u_k)$, *and*

$$Q := \begin{bmatrix} \frac{1}{z_1+\bar{z}_1} & \frac{1}{z_1+\bar{z}_2} & \cdots & \frac{1}{z_1+\bar{z}_k} \\ \frac{1}{z_2+\bar{z}_1} & \frac{1}{z_2+\bar{z}_2} & \cdots & \frac{1}{z_2+\bar{z}_k} \\ \vdots & \vdots & \cdots & \vdots \\ \frac{1}{z_k+\bar{z}_1} & \frac{1}{z_k+\bar{z}_2} & \cdots & \frac{1}{z_k+\bar{z}_k} \end{bmatrix}. \tag{10}$$

Based upon Lemma 2, we shall pose and solve the above minimization problem as one of Nevanlinna-Pick interpolation. We note that interpolation results are available for more general functions that need not be rational. However, such circumstances will generally require more delicate treatment [8]. We choose to base our development on Lemma 2 for technical simplicity. For this purpose, we shall also need rational approximants of complex exponential functions. The lemma given below follows directly from [2, pp 67], and is found in [3].

Lemma 3 *Let*

$$F_n(s) := \left(\frac{1 - \frac{sT}{2n}}{1 + \frac{sT}{2n}} \right)^n. \tag{11}$$

Then $F_n(s) \to e^{-sT}$ *uniformly on any compact set as* $n \to \infty$.

For any stable function $F(s)$, define next the transfer function

$$S_{C,F}(s) := F(s) - H(s)P(s), \tag{12}$$

It is clear that $S_C(s) = S_{C,e^{-sT}}(s)$. The following interpolation constraints on $S_{C,F}(s)$ follow immediately.

Lemma 4 *Let* $z \in \mathcal{Z}_P$ *be a NMP zero of* $P(s)$. *Then for any stable proper rational functions* $F(s)$ *and* $H(s)$

$$S_{C,F(s)}(z) = F(z). \tag{13}$$

We now state our main result for this section.

Theorem 5 *Let* $P(s)$ *have simple NMP zeros* $z_i \in \mathcal{Z}_P$, $i = 1,\ \cdots,\ k$. *Suppose also that* $P(s)$ *has no zero on the imaginary axis, and that* $w(s)$ *is stable and minimum phase. Then*

$$\begin{aligned} \gamma_{\min} &= \inf\left\{\gamma:\ Q - \frac{1}{\gamma^2}(W\Lambda)Q(W\Lambda)^H \geq 0\right\} \\ &= \bar{\sigma}\left(Q^{-\frac{1}{2}} W \Lambda Q^{\frac{1}{2}}\right) \end{aligned} \tag{14}$$

where

$$\begin{aligned} W &:= diag(w(z_1),\ \cdots,\ w(z_k)) \\ \Lambda &:= diag\left(e^{-z_1 T},\ \cdots,\ e^{-z_k T}\right). \end{aligned} \tag{15}$$

and Q *is formed as in (10) from the NMP zeros of* $P(s)$.

Proof. We shall first evaluate

$$\gamma^{(n)} := \inf_{H \text{ stable}} \left\{\|w(s) S_{C,F_n}(s)\|_\infty\right\}. \tag{16}$$

Define

$$\Phi_n := \text{diag}\left(F_n(z_1),\ \cdots,\ F_n(z_k)\right). \tag{17}$$

In light of Lemma 2 and Lemma 4, we obtain

$$\begin{aligned} \gamma^{(n)} &= \inf_{H \text{ stable}} \left\{\gamma:\ \|w(s)S_{C,F_n}(s)\|_\infty \leq \gamma\right\} \\ &= \inf_{S_{C,F_n}(z_i) = F_n(z_i)} \left\{\gamma:\ \|w(s)S_{C,F_n}(s)\|_\infty \leq \gamma\right\} \\ &= \inf\left\{\gamma:\ Q - \frac{1}{\gamma^2}(W\Phi_n)Q(W\Phi_n)^H \geq 0\right\} \\ &= \inf\left\{\gamma:\ \gamma^2 I \geq Q^{-\frac{1}{2}}(W\Phi_n)Q(W\Phi_n)^H Q^{-\frac{H}{2}}\right\} \\ &= \bar{\sigma}\left(Q^{-\frac{1}{2}} W \Phi_n Q^{\frac{1}{2}}\right). \end{aligned} \tag{18}$$

Next, we show that

$$\lim_{n\to\infty} \gamma^{(n)} = \gamma_{\min}. \tag{19}$$

Indeed, since $w(s)$ is strictly proper, for any arbitrarily small $\delta > 0$, one can find a sufficiently large $r > 0$ such that

$$\sup_{\omega\in[r,\ \infty)} |w(j\omega)\left(S_C(j\omega) - S_{C,F_n}(j\omega)\right)| \le \delta.$$

Furthermore, since $F_n(s)$ converges to e^{-sT} uniformly on any compact set, for any arbitrarily small $\delta > 0$, there exists an N such that for $n > N$,

$$\sup_{\omega\in[0,\ r)} |w(j\omega)\left(S_C(j\omega) - S_{C,F_n}(j\omega)\right)| = \sup_{\omega\in[0,\ r)} \left|w(j\omega)\left(e^{-j\omega T} - F_n(j\omega)\right)\right| \le \delta. \tag{20}$$

This suggests that for any arbitrarily small $\delta > 0$, there exists an N such that for $n > N$,

$$\left|\|w(s)S_C(s)\|_\infty - \|w(s)S_{C,F_n}(s)\|_\infty\right| \le \delta$$

for any stable H. Consequently, we have proved (5). The proof may then be completed by evaluating the limit $\lim_{n\to\infty} \gamma^{(n)}$. In light of Lemma 3, it is clear that

$$\lim_{n\to\infty} \Lambda_n = \Lambda. \tag{21}$$

This yields (14) as required. ∎

Remark 6 *For a minimum phase $P(s)$, it is evident that $\gamma_{\min} = 0$; that is, it is possible to achieve perfect tracking even without preview. This is no longer true of NMP plants, though preview can be used to improve the tracking performance. To illustrate, it is instructive to examine certain limiting cases. Suppose that $P(s)$ has only one NMP zero z, for which Theorem 5 gives rise to*

$$\gamma_{\min} = |w(z)|e^{-\operatorname{Re}(z)T}. \tag{22}$$

Thus, $\gamma_{\min}$ may still be kept small by selecting a large T, and in the limit it approaches zero when $T \to \infty$. This error may be substantially smaller than $|w(z)|$, i.e., the minimal error achievable in the absence of preview ($T = 0$).

It is worth noting that while in general it does not seem possible to obtain an explicit expression such as (22) for $\gamma_{\min}$, the qualitative statement remains valid. This is seen from the following corollary.

Corollary 7 *Let $P(s)$ have simple NMP zeros $z_i \in \mathcal{Z}_P$, $i = 1, \cdots, k$. Suppose also that $P(s)$ has no zero on the imaginary axis, and that $w(s)$ is stable and minimum phase. Then:*
(i) $\gamma_{\min}(T)$ is a continuous, monotonically non-increasing function of T.
(ii)

$$\gamma_{\min} \le \kappa^{\frac{1}{2}}(Q) \max_{1\le i\le k} |w(z_i)|e^{-\operatorname{Re}(z_i)T}. \tag{23}$$

where $\kappa(Q)$ denotes the condition number of Q.
(iii)

$$\inf_{T\in[0,\ \infty)} \gamma_{\min} = 0. \tag{24}$$

(iv)

$$\gamma_{\min} \ge \max_{1\le i\le k} |w(z_i)|e^{-\operatorname{Re}(z_i)T} \tag{25}$$

(v) Suppose that $0 < \operatorname{Re}(z_1) < \operatorname{Re}(z_2) \ldots..$ Then for T large,

$$\gamma_{\min} \approx |w(z_1)|e^{-\operatorname{Re}(z_1)T}\sqrt{\left(Q^{-1}\right)_{11} Q_{11}}$$

where Q_{11} denotes the 1,1 element of the matrix Q; and
(vi)

$$\max_{T\in[0,\ \infty)} \gamma_{\min} = \bar{\sigma}\left(Q^{-\frac{1}{2}}WQ^{\frac{1}{2}}\right) \tag{26}$$

Proof. The proof utilises standard linear algebraic techniques applied to (14). See a full version of this paper for details. ∎

Corollary 7 gives important qualitative information about the behaviour of the infimal $\mathcal{H}_\infty$ performance under fairly general circumstances. It is monotonically, and exponentially decreasing in the preview horizon, T, relative to the 'time constant for the zeros exponential envelope', namely $\frac{1}{\operatorname{Re}(z_i)}$. This is illustrated in the following example.

Example 8 *Suppose that the weighting function is given by*

$$w(s) = \frac{1}{1 + s\tau_w} \tag{27}$$

where $\tau_w = 0.3$. Suppose also that two NMP plant zeros are located at $s = 1$ and $s = 2$. Then Figure 3 shows the achievable $\mathcal{H}_\infty$ performance calculated using Theorem 5. Note from Figure 3 that as expected from Corollary 7, $\gamma_{\min}$ is a monotonically decreasing function of T. In addition, for T large, $\gamma_{\min}$ is approximately exponential in T and drops by a factor of approximately 2.7 for every second of increase in T.

Remark 9 *Theorem 5 can be generalized to situations where $P(s)$ may have zeros on the imaginary axis. This is accomplished in a straightforward manner by an appeal to a limiting argument, one typically found in boundary Nevanlinna-Pick interpolation problems. This leads to the following extension of Theorem 5 which for brevity we state without proof.*

Theorem 10 *Let $P(s)$ have simple zeros $z_i \in ORHP$, $i = 1, \cdots, k$, and simple zeros $j\omega_i$, $i = 1, \cdots, l$.*

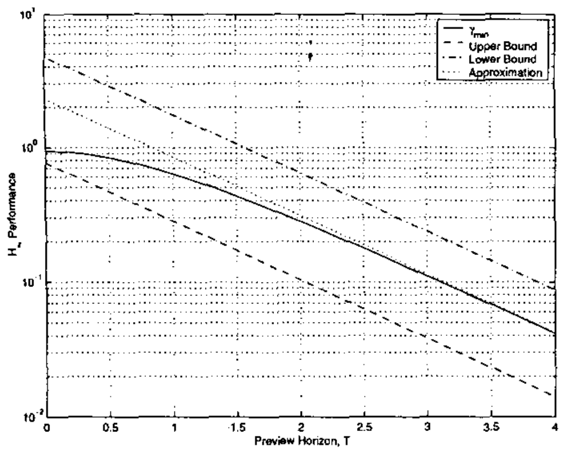

Figure 3: Example achievable performance for various preview horizons.

Furthermore, suppose that $w(s)$ is stable and minimum phase. Then,

$$\gamma_{\min} = \max\left\{|w(j\omega_i)| : i = 1 \ldots \ell,\ \bar{\sigma}\left(Q^{-\frac{1}{2}} W \Lambda Q^{\frac{1}{2}}\right)\right\}. \tag{28}$$

where Q, W and Λ are formed from the (strictly) NMP zeros only.

From Theorem 10, it is clear that in cases where the performance is dominated by $j\omega$ axis zeros, namely when

$$\bar{\sigma}\left(Q^{-\frac{1}{2}} W \Lambda Q^{\frac{1}{2}}\right) \leq \max\left\{|w(j\omega_1)|,\ \cdots,\ |w(j\omega_l)|\right\},$$

then the achievable H_∞ performance is independent of the preview. This can also be understood by considering the results in Corollary 7, whereby the bounds are related to $|w(z_i)|\,|e^{z_i T}|$ and we note that as $z_i \to j\omega_i$ the bounds depend on $|w(z_i)|$ only. In this case preview control is rendered less effective, when $|w(j\omega_i)|$ is comparatively large. Intuitively, in these cases, performance is dominated by the response to narrowband signals, which are perfectly predictable. Therefore, for the signals of interest in the worst case analysis, preview does not add any additional information which is not available without preview. It is therefore reasonable to conjecture that the preview would not improve the performance in such cases.

4 Frequency Domain Integral Inequalities

We now turn to interpretations and derivations of logarithmic sensitivity integrals, in the spirit of [6], [15] and others. These integrals arise due to the interpolation constraints expounded in Lemma 4. The following Poisson type integral can be established for $S_C(s)$.

Theorem 11 *Consider the closed loop system in Figure 2, where $H(s)$ and $P(s)$ are stable rational transfer functions, and neither is identically zero. Then for any NMP zero $z = x_0 + jy_0 \in \mathbb{C}_+$ of $P(s)$,*

$$\frac{1}{\pi}\int_0^\infty \log|S_C(j\omega)| W_z(\omega) d\omega = -x_0 T + \log\left|B_s^{-1}(z)\right| \geq -x_0 T \tag{29}$$

where

$$B_s(s) := \prod_{z_i \in \mathcal{Z}_{S_C}} \left(\frac{z_i - s}{\bar{z}_i + s}\right)$$
$$W_z(\omega) := \frac{2x_0}{x_0^2 + (\omega - y_0)^2}.$$

Proof. The result is immediate by application of the well-known Poisson integral [6], [15] to $G(s) := S_C(s) B_s^{-1}(s)$, yielding

$$\frac{1}{\pi}\int_0^\infty \log|G(j\omega)| W_z(\omega) d\omega = \log|G(z)|.$$

Since $|G(j\omega)| = |S_C(j\omega)|$, and

$$\begin{aligned} |G(z)| &= \left|S_C(z) B_s^{-1}(z)\right| \\ &= \left|e^{-zT} B_s^{-1}(z)\right| = e^{-\operatorname{Re}(z)T}\left|B_s^{-1}(z)\right|, \end{aligned}$$

the result follows. ■

Note that in the absence of preview, that is with $T = 0$, it is possible to design $H(s)$ such that $H(s)P(s) \neq 1$ for all $s \in \mathbb{C}_+$, in other words, $\mathcal{Z}_{S_C} = \mathcal{Z}_{(1-HP)}$ is empty For example, if $\|H(s)P(s)\|_\infty \leq 1$ then this condition will be satisfied. Under this circumstance, the integral (29) becomes the 'normal' unity feedback result, except with the effect of open loop unstable poles eliminated:

$$\frac{1}{\pi}\int_0^\infty \log|S_C(j\omega)| W_z(\omega) d\omega = 0$$

This elimination of the effect of open loop unstable poles in the command tracking Poisson sensitivity integral is achieved as a result of using a two degree of freedom controller structure, as in Figure 1.

With preview in effect, the situation is more complicated, however, we can make the following statement.

Corollary 12 *Suppose that the plant $P(s)$ has a single real NMP zero at $s = z$. Then the inequality in (29) is tight in the sense that:*

$$\inf_{H(s)} \frac{1}{\pi} \int_0^\infty \log |S_C(j\omega)| W_z(\omega) d\omega = -zT \tag{30}$$

Proof. (Outline) Clearly from (29) the infimum cannot be less than $-zT$. It therefore remains to demonstrate that by suitable choice of $H(s)$ we can approach this value from above.
Let r be the relative degree of $P(s)$. For $\delta > 0$ and any n select

$$H(s) = \frac{1}{(1+s\delta)^r} P^{-1}(s) \left(F_n(s) - F_n(z)\right) \tag{31}$$

where $F_n(s)$ is the time delay approximation in (11). It follows by some lengthy algebra (see full version of this paper) that indeed the choice of $H(s)$ in (31) is stable, proper and gives a cost which may be made arbitrarily close to $-zT$. ∎

Note, however, that in the more general case where there are multiple NMP plant zeros, the equivalent discussion is significantly more complicated. In particular, it would seem that in general, with multiple NMP plant zeros and strictly proper H and P, there will always be NMP zeros of $S_C(s)$.This can be established by the result from [15, Lemma A.11.1(ii), pp 323] as follows:

Lemma 13 *For any stable, proper $H(s)$, $T > 0$ and strictly proper $P(s)$, $S_C(s)$ has NMP zeros.*

Note that Lemma 13 applies even in the case where we have a single real NMP plant zero, as in Corollary 12. This does not necessarily contradict the tightness inherent in Corollary 12, since it may be possible to produce $H(s)$ such that the contribution of the zeros of S_C to (29) is small. A more detailed understanding of cases where there are multiple zeros is a topic of continuing research.

5 Conclusions

This paper has considered a general class of finite preview linear tracking control systems for NMP plants. Under fairly mild assumptions on the class of plants, we show that the weighted $\mathcal{H}_\infty$ performance achievable with preview T is approximately (for large T) proportional to $e^{-\alpha T}$ where α is the smallest real part of any NMP zero. In addition, we have been able to exhibit a Poisson sensitivity integral inequality, with weighted log tracking sensitivity lower bounded by $-\alpha T$. In both cases, it is seen that the possible tracking performance is improved by the use of sufficient preview. In particular, a preview horizon T which is significantly longer than the 'time constant' of the slowest NMP zero is desirable.

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