A novel approach for assessing the interaction of masonry walls under vertical loads

Márcio R.S. Corrêa and Adrian W. Page

Abstract: This paper investigates the behaviour of masonry load-bearing walls subjected to differential vertical load. A new approach for evaluating the interaction of intersecting walls is used, focusing on the mechanism of load transfer and the resulting shear stresses. The study is carried out using finite element modelling. Previous full-scale tests are used to verify the features of the numerical model. Once confirmed, the model is then used to study the phenomenon, varying parameters such as the number of floors and the dimensions of the walls. It is shown that the distance down the wall at which homogenization of the applied loads occurs can be predicted by application of the Saint Venant's Principle. The distribution of shear stresses along the interface can be simulated by a simple parabolic distribution. A simple design procedure is proposed, allowing more realistic, cost-effective designs of load-bearing masonry structures.

Key words: masonry, walls, vertical loads, finite elements, interaction of walls.

Résumé : Cet article étudie le comportement des murs porteurs en maçonnerie soumis à une charge verticale différentielle. Une nouvelle approche pour évaluer l'interaction des murs d'intersection est utilisée, ciblant le mécanisme de transfert de charge et les contraintes de cisaillement qui en résultent. L'étude est effectuée par modélisation par éléments finis. Des essais antérieurs à pleine échelle sont utilisés pour vérifier les caractéristiques du modèle numérique. Une fois confirmé, le modèle est ensuite utilisé pour étudier le phénomène, en variant des paramètres tels que le nombre d'étages et les dimensions des murs. Il est démontré que la distance le long du mur à laquelle survient l'homogénéisation des charges appliquées peut être prédite en appliquant le principe de Saint Venant. La distribution des contraintes de cisaillement le long de l'interface peut être simulée par une simple distribution parabolique. Une procédure de conception simple est proposée, permettant des conceptions plus réalistes et rentables des structures portantes en maçonnerie.

Mots clés : maçonnerie, murs, charges verticales, éléments finis, interaction des murs.

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Introduction

Two intersecting load-bearing walls will interact provided there is some form of structural connection crossing the plane of the interface (steel bars, connectors, or bonded units), resulting in an interface that has sufficient strength to transmit the transferred forces. This interaction will result from loading conditions such as out-of-plane bending or differential compressive loading on different parts of the wall system. Compared to walls subjected to horizontal loads, the differential vertical loading phenomenon has not been widely studied.

It is common for load-bearing walls at the same floor level to be subjected to different levels of vertical load due to their different tributary areas, with internal walls usually supporting higher loads than external walls. The ratio of internal wall load to external wall load can be up to two in residential buildings. Normally the most heavily loaded wall governs the masonry strength, since the same masonry units will normally be used for all walls. In the case of block work it is also possible to use grout to vary the masonry strength, although this can create site control difficulties. If two intersecting walls are subjected to different levels of vertical load, they will shorten differentially, with resulting interaction at the interface. If the interface has sufficient shear strength, the more heavily loaded wall will transfer load to the other, leading to progressive vertical stress homogenization across the complete wall system. This sharing of the load uses the walls more efficiently, thus enabling a more cost-effective design, particularly for tall load-bearing walls. This homogenization of the load is not as important in low-rise structures where load levels are low, but even in this case it is useful to gain a more complete understanding of the mechanism of this interaction.

Stockbridge (1967) carried out one of the first research projects on this subject at the University of Edinburgh. The author monitored wall strains on a five-storey full-scale building and found evidence of the homogenization of

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vertical compressive stresses not only in isolated walls but also in groups of walls. His experimental observations confirmed the existence of interaction forces at the interface of intersecting walls and the influence of the horizontal restraints provided by the floors on the reduction of the in-plane loading eccentricity.

Hendry (1981), dealing with the analysis of brickwork under vertical loads, warns that huge differences can arise in estimated wall stresses, depending on the assumptions made. The usual procedure is to subdivide the floor into tributary areas (triangles and trapeziums) and to allocate loads to the appropriate walls on that basis. Despite the resulting non-uniformity of the load along the wall length, Hendry points out that uniform stresses will be produced farther down the height of the wall, in a manner similar to that suggested by Stockbridge (1967). If the wall stresses are calculated using the first procedure, this inherently disregards the interaction of the walls. Hendry also refers to an alternative approach by Sutherland (1969), where the tributary areas are allocated to groups of walls. Any eccentricity produced by the difference between the centroid of the loaded area and the centroid of the wall group is taken into account, assuming a linear distribution of the vertical normal stresses across the wall. Hendry commends the superiority of this approach because of the consideration of wall interaction.

Sinha and Hendry (1979) reported a large experimental research program on the compressive strength of brick masonry walls with various slenderness ratios stiffened by bonded flanges. The main goal of the study was to establish the influence of the return walls on load-carrying capacity. H-shaped panel tests at full, one-half, and one-third scales were carried out, with slenderness ratios varying from 8 up to 32. Distributed loads were applied in two different ways: uniformly across the top of all the walls, and with only the top section of the wall web being loaded. Unstiffened walls without returns were also tested to allow comparison in performance. In all cases, cracks appeared at the web–flange intersection at high load levels, resulting in the loss of the stiffening effect of the flange at ultimate load. When only the web was loaded, cracks occurred in the web, again near the web–flange interface. When the entire panel was loaded, cracks appeared in both flanges, near the web–flange interface. In all cases the cracking commenced at the top of the wall and propagated to the base. The stress–strain curves for the wall were linear up to 90% of the failure load, confirming that the web and flanges acted together up until this point. The separation of the flanges resulted in strong nonlinearity of response, with subsequent sudden, brittle failure. It is clear from the results that the web and flanges were working together before the interface failed, with linear behaviour at least up to 90% of the compressive failure load. This, of course, is much larger than the usual service loads in a masonry building.

A study of the interaction of walls under vertical loads has been in progress at the University of São Paulo, Brazil, since 1990. Initially, Corrêa and Ramalho (1994, 1998) carried out a series of theoretical studies based on finite element modelling of masonry buildings. These analyses were then complemented by a more recent experimental study. A summary of the testing program is given here, and more details are reported in Capuzzo Neto (2000) and Capuzzo Neto et al. (2000). To verify the interaction of walls and determine the load transfer ratio, full-scale panels were tested in the laboratory of the Structural Engineering Department of the University of São Paulo. All the panels consisted of three bonded hollow masonry walls, built with full bedding, with an H symmetrical plan shape. Figure 1 shows a general view of the panels and their geometrical dimensions. All the joints were 10 mm thick. Panels were laid directly on the strong floor with the same mortar. Only the web of the H section was loaded.

The main findings are summarized as follows. The average values of the failure load of the panels were in good agreement with the expected values assessed without considering the contribution of the flanges; the first visible cracks appeared at the top of the web, close to the interface, spreading downward as the load increased, causing separation of the returns. The loss of linear response coincided with the separation of returns and was more pronounced in the panels without mid-height bond beams. The nonlinear behaviour started at approximately 75% of the failure load, which confirmed the findings of Hendry et al. (1981); the tendency for homogenization, i.e., spreading of load from the loaded web to the flanges, always occurred, being more significant in the lower parts of the walls.

The tendency for homogenization can be seen from Fig. 2*a*, which shows the stress–strain curves for the web and flanges observed in the lower section in one of the panels up to 75% of the failure load. Note that stress has been evaluated using the entire cross-sectional area, i.e., the gross area of the web and the flanges, and the strain is the average of similar points on each side of the wall in the vertical direction (Fig. 2*b*). Linear regression of the data for the flange and web produces a coefficient R^2 of 0.992 and 0.994, respectively, showing that the phenomenon at this phase is nearly linear.

Corrêa and Ramalho (2002) applied the homogenization assumption to analyse the behaviour of a six-storey building. Recently, Capuzzo Neto (2005) also confirmed homogenization for a number of full- and small-scale H-shaped walls of varying dimensions. Andolfato (2004)[2] also confirmed similar behaviour, based on observations of an actual full-scale four-storey building by monitoring the loads induced in foundation piles and in the lower walls of the building.

It is obvious from the aforementioned and previous research that the walls interact under vertical loads. This interaction results in the homogenization of the vertical compressive stresses in the walls, with the "smearing" of peak load stresses across the composite wall section, thus enhancing the wall capacity. Provided the design capacity of the wall is limited to a level below which flange–web separation occurs, this homogenization of stress will result in economies of wall design, with consequent savings in building costs.

The study presented in this paper investigates this phenomenon in detail and develops a simple method that

[2] Andolfato, R.P. 2004. Study of the vertical load distribution in structural masonry buildings. Ph.D. thesis, University of São Paulo, São Paulo, Brazil. [In Portuguese, in preparation.]

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Fig. 1. View of the tested panels: (*a*) dimensions (in mm); (*b*) general view of the test.

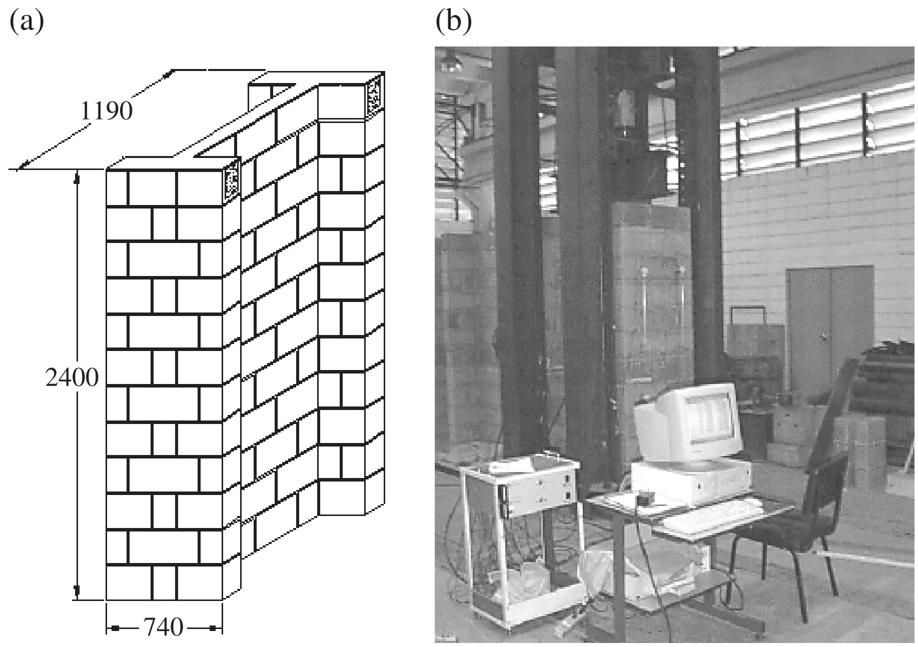
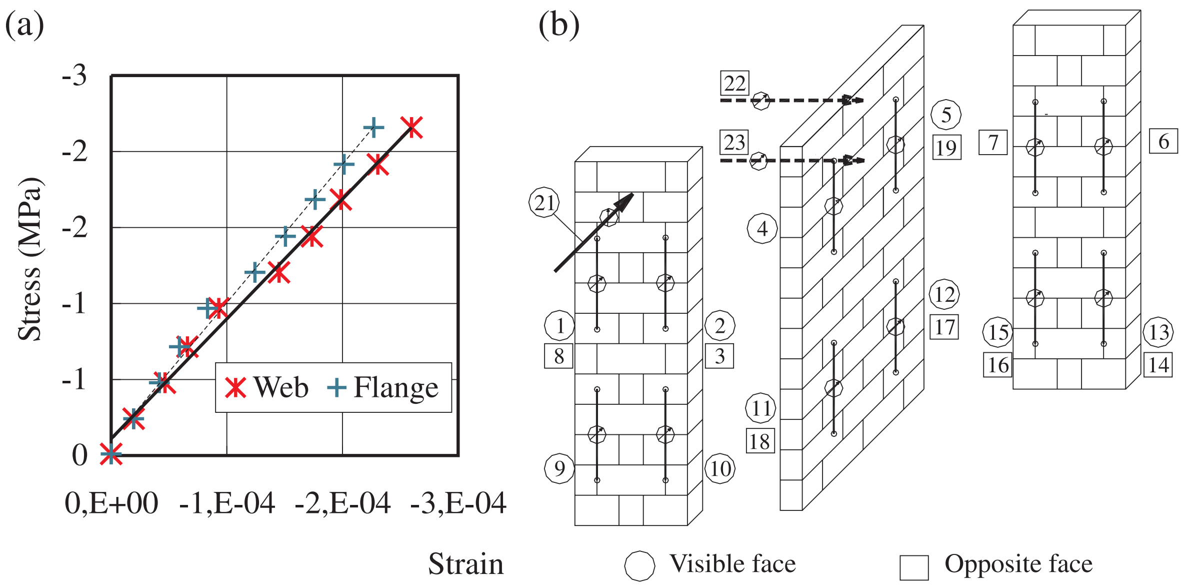


Fig. 2. Lower region of the panel: (*a*) stress–strain curves; (*b*) position of the instruments.



allows this composite behaviour to be considered in design. The present paper covers the case of uniform strip loading on the load-bearing wall; however, it can be easily extended to other loading patterns.

Analytical modelling

When using the finite element method (FEM) for masonry, some initial choices have to be made, depending on the nature of the problem and the goals of the analysis, the element geometry (two-dimensional (2-D) or three-dimensional (3-D) elements to simulate the walls), the analysis type (linear or nonlinear), the modelling type (micromodelling, considering units and mortar separately, or macromodelling, considering them together), and the element type (e.g., shell or membrane elements in a 2-D analysis).

Element geometry

One of the most important simplifying techniques for modelling masonry walls is to represent them by their middle surfaces. Usually this 2-D simulation of wall behaviour is successful, as reported by Lourenço (1996) and Saliba et al. (1996). In some special cases, when 3-D effects are significant, it is necessary to use 3-D elements, as reported by Ganesan and Ramamurthy (1994).

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As the general behaviour of the walls is the main interest of the present work, 2-D elements are sufficient for the numerical analysis. The model still has to be 3-D, however, as intersecting walls in different planes are to be studied. The use of 2-D elements allows important simplifications in a conceptual sense and results in significant reduction of computational effort.

Analysis type (linear or nonlinear)

To provide a level of safety in the structural design of a building, the designer must keep the applied loads well below the failure levels. The compressive stresses at working load levels are typically 20%–30% of the masonry compressive strength. As reported earlier, Sinha and Hendry (1979) observed linear behaviour of H panels up to 90% of the failure load, and Capuzzo Neto (2000) observed linearity up to 75%, consistent with the findings of Hendry et al. (1981). Because of the linear nature of the response for the bulk of the loading range, the use of linear elastic analysis is therefore justified in the present work provided composite behaviour is assumed to be maintained. That fact was confirmed by Peleteiro (2002), who analysed the same tested Brazilian H panels, using both linear and nonlinear micromodelling, and found very little difference in the results, with the overall behaviour being linear. This justifies once more the adoption of the linear hypothesis in this present work.

Model type and element properties

A commercial FEM package, STRAND (G+D Computing Pty Ltd. 2000), was used in conjunction with macromodelling to simulate the behaviour of the H panel. The effectiveness of the model was assessed by comparing the results with those obtained by Peleteiro (2002) using micromodelling in conjunction with ABAQUS (Hibbitt, Karlsson & Sorenson 1996). The meshes used by Peleteiro for the micromodelling of the web and flanges were very dense, with a total of 2520 elements for a half panel.

In the present work the panel was simulated by a macromodel with elastic parameters for masonry evaluated by a homogenization procedure described by Pande et al. (1989). Figure 3*b* shows the STRAND macromodel of half the H panel. The model has much fewer elements than the Peleteiro ABAQUS model, with 408 elements in this case. To obtain maximum simplicity, QUAD4 membrane elements were used, with four nodes, two degrees of freedom each, in a condition of plane stress. The viability of using a macromodel and the effectiveness of the membrane element were then assessed by comparing the results with those from the ABAQUS micromodel.

The vertical stresses are the primary stresses to be checked in a wall design and are also used to assess the homogenization process. Figure 3 shows the contour curves of vertical normal stresses in the two alternative models corresponding to a load of 425 kN uniformly distributed along the top of the web. The two solutions have the same pattern, despite the extreme simplicity of the macromodel. It is significant to note the homogenization of the stresses at mid-height in the two models. The maximum values of vertical stress occur at the mid-point at the top of the web. They were –7.2 MPa in the micromodel and –6.3 MPa in the macromodel.

The two models also produced the same distribution patterns for the shear stresses, as reported by Corrêa and Page (2001). The maximum values occurred at the top of the web near the flange, the same place where cracking started in the experimental tests. The stress at this location was 3.4 MPa in the STRAND model and 3.1 MPa in the micromodel. With respect to the maximum vertical displacement, which occurs at the middle point of the top of the web, the micromodel predicted a displacement of 0.85 mm, and the macromodel a displacement of 0.88 mm.

The previous comparisons show that for analysing the wall interaction phenomenon it is reasonable to use a simple finite element model incorporating linear elastic behaviour, macromodelling, and 2-D membrane elements. The simple analytical model was therefore used to study the phenomenon, varying parameters such as the number of floors, the wall dimensions, and the lack of symmetry.

Analysis using the simplified model

The numerical model described in the previous section was used to investigate the way in which forces spread through the intersecting walls and the resulting stress distributions produced along the intersecting wall planes with a view to developing a simplified model for the structural design of masonry buildings

H panel

The H panel was the basic structural unit investigated, with the panels having the same dimensions as those in the previously described Brazilian tests. To generalize the analysis, the number of floors, the nature of the horizontal restraints, and the horizontal dimensions were also varied. The walls were modelled without bond beams, the typical practice for solid brick masonry.

A vertical applied load of 460 kN was chosen because it was the theoretical failure load of the panel. The load was applied to the top of the web of the H section as a pressure of 6.26 MPa on the membrane elements (Fig. 4). The thickness of the elements was 70 mm to maintain a ratio of net area to gross area of 50% (typical for hollow clay blocks). An important value for further comparisons is an average vertical normal stress of 2.58 MPa, corresponding to that for total homogenization of the load. The same mesh as shown in Fig. 4*a* was used. All the base nodes were totally restrained, and only half the panel was modelled because of symmetry.

Figure 4*b* shows the stress distribution for the vertical normal stresses in the panel. It is significant to note the complete homogenization of stress throughout a significant proportion of the height, with the stresses being virtually uniform below mid-height.

Figure 5 shows the distribution of the same stresses at the base, for the flanges and the web, together with the average value that would be expected with total homogenization. It can be seen that the stresses are uniform and close to the average value, except at the ends (at those points, even if the loading were uniform across the section, there would be

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Fig. 3. Normal vertical stresses (MPa): (*a*) micromodel; (*b*) macromodel.

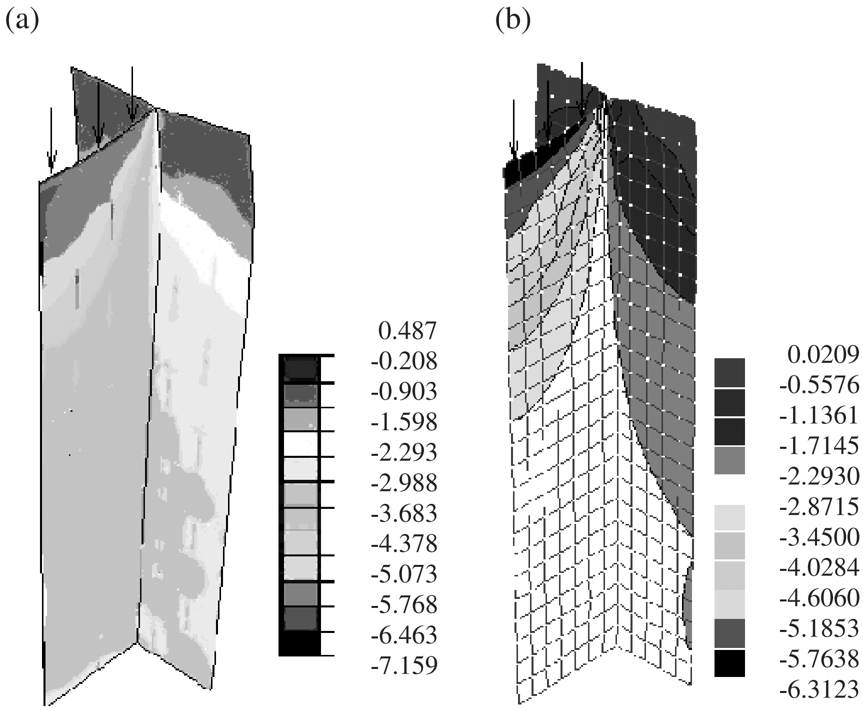
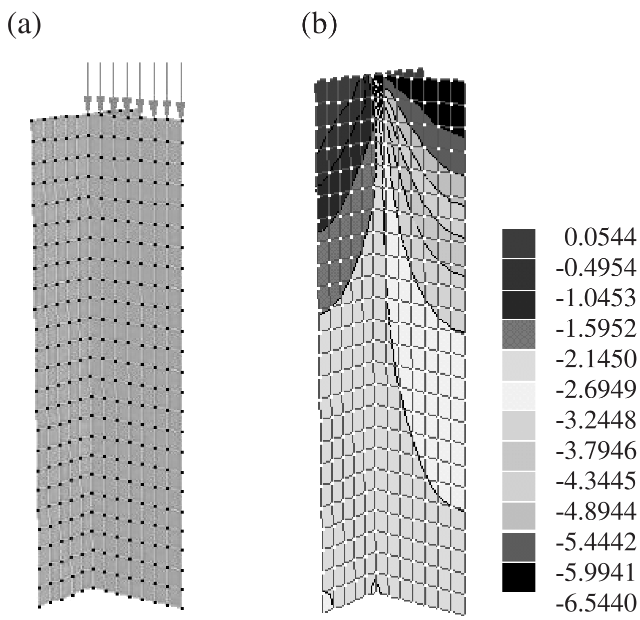


Fig. 4. H section analysis using macromodelling: (*a*) mesh and load pattern (arrows); (*b*) vertical normal stresses (MPa).



some differences because of restraint conditions). The integration of the vertical stress distribution at the base gives the values of the vertical reactions for the flange and for the half web. Table 1 shows these values and those expected with total homogenization, i.e., corresponding to a uniform stress across the whole area. The percent differences relative to the finite element results are very low, indicating that the assumption of total homogenization is a very good approximation in this case.

Figure 6 shows the distribution of shear stresses down the intersecting plane. The distribution is presented in Fig. 6*a* as contour curves and in Fig. 6*b* by the distribution at the interface. The plotted results are for stresses in the web, 37.5 mm from the interface, corresponding to the position of the centroids of the closest elements. Shear stresses decrease rapidly from the top to the base, consistent with the rapid homogenization of the vertical stresses.

The easiest design assumption would be to consider the shear stresses at the interface to be uniform. This would be erroneous, however, since the shear stress variation down the wall is large. In the present example, for full homogenization a total force of 138 kN is transferred from the web to the flange. The average shear stress on the interface area, a strip 70 mm thick and 2400 mm in height, resulting from this load is 0.82 MPa, which is much less than the maximum value (Fig. 6*b*). The other important point is that an average design value would have to be based on experimental strength values for the entire panel, thus eliminating the possibility of using tests on small specimens to determine the basic masonry parameters. The inherent variation in distribution of the shear stress must therefore be considered in any design method.

Similar behaviour was observed when load was applied on the flange, as reported by Corrêa and Page (2001).

Influence of the number of floors

For walls that form part of a multistorey load-bearing building, one obvious important aspect of the study is the influence of the number of floors. As has already been shown by Capuzzo Neto et al. (2001), the influence is beneficial, since the vertical load is applied at many levels, thus

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Fig. 5. Compressive vertical stresses at the base.

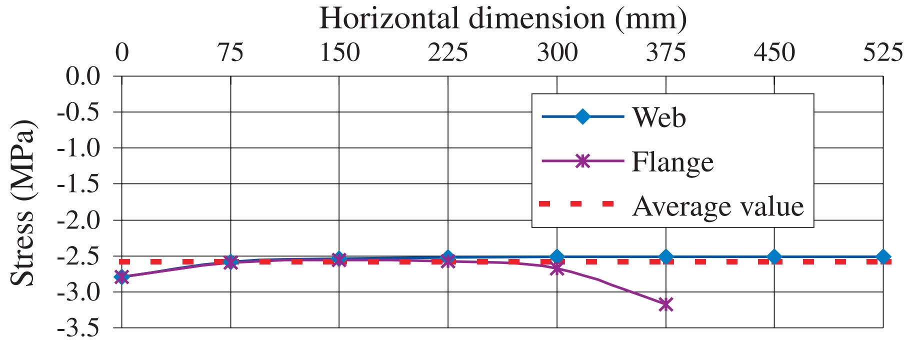

Table 1. Vertical reactions at the base.

Feature	Half web	Flange
Integration of finite element results (kN)	92	138
Total homogenization (kN)	95	135
Difference (%)[a]	+3	–2

[a]Relative to the FEM values.

reducing the local maximum stress and helping the homogenization process.

Four additional finite element models were developed to study this phenomenon, simulating the presence of two, three, four, and five floors. They had the same features as the previous model, with the height always being a multiple of the basic storey height of 2.40 m. Initially only the web was loaded at the floor levels, with the total force in the whole panel being maintained at 460 kN, thus allowing easy comparisons to be made. Note that the load of 460 kN was distributed equally among the floors in each case.

Only the main results are shown here. More detailed results can be found in the report by Corrêa and Page (2001). The homogenization process is practically completed within only one floor height. This pattern is repeated with two floors, as seen in Fig. 7, which shows the vertical normal stresses distribution between the base and the second floor. Stress distributions at the base are also shown.

Figures 4*b* and 7*a* show that for the single- and two-floor models, respectively, shear transfer between the wall components allows homogenization of the vertical stress to occur progressively as the vertical load is applied at each floor level. As a consequence, the peak vertical stresses immediately beneath a floor level are produced predominantly by the load applied at that floor. In this particular comparison, the maximum stress for the single-floor case is approximately twice that for the two-floor case, even though the total applied loads are the same, as expected. In general, the stress differences will be influenced by the wall geometry and the storey height, but the trend will be consistent. In tests on panels with only one floor, the flange typically separates from the web, resulting in a failure load equal to that of the strip wall corresponding to the web. As a result, some engineers usually separate walls a priori to distribute vertical loads, keeping them composite only for distributing horizontal loads in the building. From the present study it is apparent that in a multistorey building it is possible to maintain the wall interaction under vertical loads because the stress concentration is much smaller than that in any one-floor panel with the same total load. For design purposes, if composite action is assumed, it is obviously essential to verify the capability of the interface to transfer the interacting forces between the web and the flanges. The pattern of this stress distribution is therefore important.

Figure 8 shows the shear stress distribution at the interface for models of one to five floors. As mentioned previously, the stresses are those on a vertical line in the web, 37.5 mm from the web–flange intersection. The plots show that there are three distinct sections in the curves: between the base of the wall and level 1, between each intermediary floor level, and between the last floor and the roof. The maximum values are at the top, being inversely proportional to the number of floors, because of the way load was applied (with the same total load in each case). At the top of each wall there is a perturbation in the results, related to local numerical effects. To obtain overall trends this should be ignored. At the base, the values for the five models are close and small. At intermediate floor levels there are consistent values of shear stress, corresponding to nearly equal transferred forces in different levels. Those values are very close to those of the top floors if the local stress instability is ignored. Curves between adjacent levels approximate a quadratic parabola; the fit of a parabola in the section of the five-floor curve, between the fourth and fifth floors, leads to a correlation coefficient of $R^2 = 0.94$.

The parabolic shape of the curve between two adjacent levels can be used to find a simple way of assessing the maximum shear stress. The end values can be considered the same, with a zero value at midpoint (note that midpoint values are very small). To enable the total force to be transferred through the interface, the end values must be three times the average value. Table 2 shows the values obtained for the end stresses at intermediate floor levels for the FEM and the simplified parabolic assumption. When the transferred force at the interface was assessed assuming the total homogenization of the normal stresses, the value obtained was 135 kN. This value is used to generate results for the simplified model in Table 2. The results are in good agreement even in the case of only one floor where the curve is nonsymmetrical between the two ends. This case does not appear in multistorey buildings and is therefore of less importance in the present study.

For the sake of completeness, an additional load case was considered in each model, considering the same total force of 460 kN, applied to the flanges (230 kN per flange in the models). A parabolic distribution of stress was again apparent, as reported by Corrêa and Page (2001). The

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Fig. 6. Shear stresses: (*a*) overall distribution (MPa); (*b*) distribution at the interface.

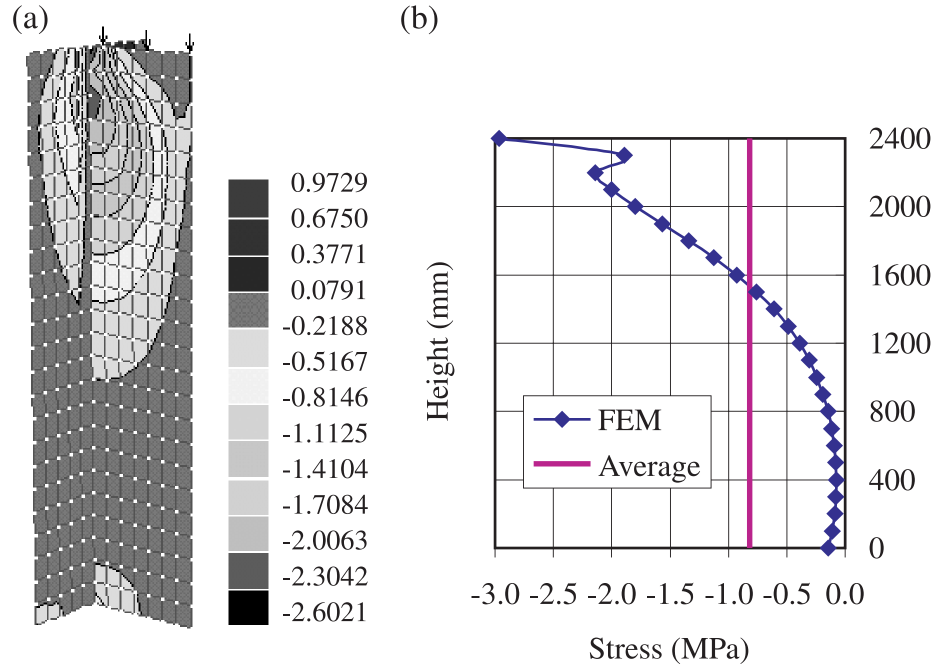
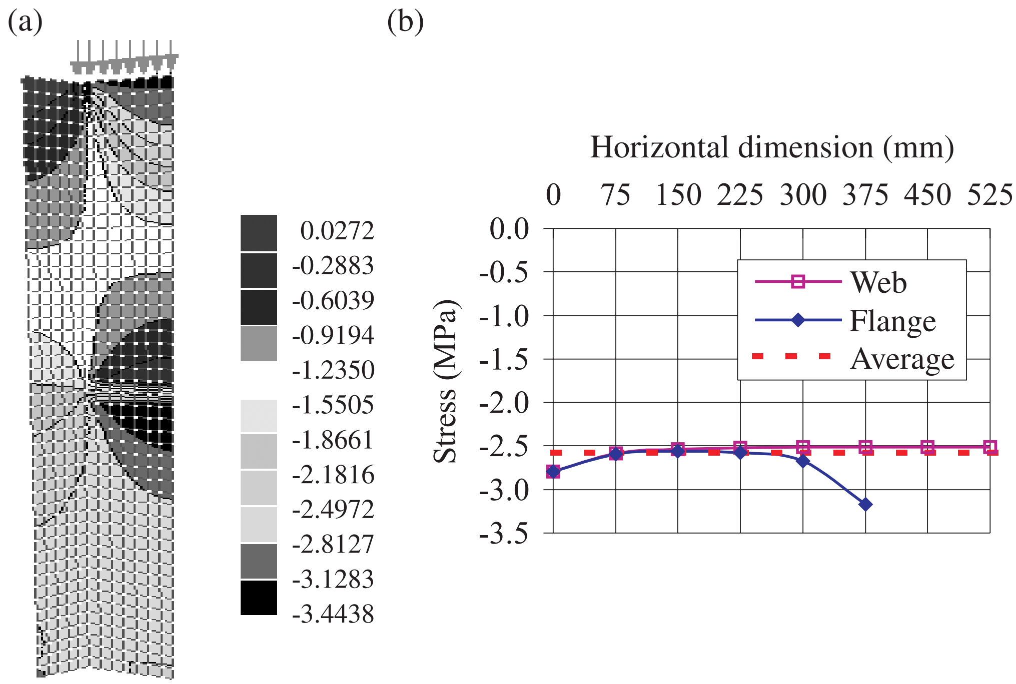


Fig. 7. Vertical normal stresses for two floors: (*a*) distribution between base and second floor (MPa); (*b*) stress distributions at the base.



transferred force at interface was 95 kN, based on total homogenization. Table 3 compares the results for the FEM and the simplified method for the maximum values in the five models. The values are very close, confirming the accuracy of the simplified procedure.

Varying the panel dimensions

Five new macromodels were developed to evaluate the influence of the horizontal dimensions of the panel. The web length was increased to 3.150 m (1.575 m in the models for half panel). To maintain the same applied pressure on the web, the total vertical load was also tripled to 1380 kN for the entire panel. All the other features were the same as those for the former models, including the option of two load cases, namely web or flange loaded.

Compared to the former models, there are two important differences to be noted. The length of the web is now very different from that of the flanges, and the plan dimensions are larger than the distance between two adjacent floors. This feature is relevant in relation to the application of the Saint Venant's Principle.

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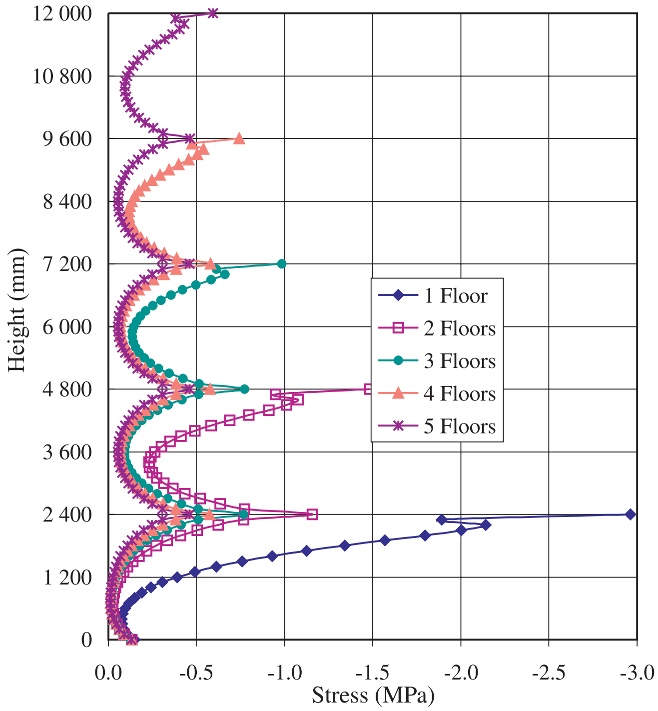

Fig. 8. Shear stresses at the interface in the five models.

Table 2. Shear stresses at the interface for load on the web.

	Shear stress (MPa)		
No. of floors	FEM	Simplified model	Difference (%)[a]
1	2.48	2.41	–3
2	1.16	1.21	+4
3	0.77	0.80	+4
4	0.58	0.60	+4
5	0.46	0.48	+4

[a]Relative to the FEM values.

Table 3. Shear stresses at the interface for load on the flange.

	Shear stress (MPa)		
No. of floors	FEM	Simplified model	Difference (%)[a]
1	1.75	1.70	–3
2	0.83	0.85	+2
3	0.56	0.57	+2
4	0.42	0.43	+2
5	0.33	0.34	+2

[a]Relative to the FEM values.

The degree of homogenization is investigated first. Figure 9 shows the contour curves of the vertical normal stresses for the models with one and two floors. Homogenization of the normal stresses is slower to develop than in the short panels, especially for the second loading case with the loads on the flange. Considering the web and flange together, the average value of the compressive stress is 4.24 MPa, a value that is not close to that obtained in the one-floor model. For the flange-loaded case, the lack of uniformity is more evident. Another important feature is the large difference between the maximum stresses at the top and at the base of the wall, with the results for two floors having lower differences. With an increasing number of floors, the differences diminish. Note that for both load cases, homogenization is already complete within two floors.

Figure 10 shows the development of homogenization as the number of floors increases, with reference to the vertical reaction at the base of the web and the flange. For the sake of comparison, the reactions corresponding to total homogenization are also included: 223 kN for the flange and 467 kN for half the web. It is apparent that the most important transition in relation to the homogenization occurs when the number of floors increases from one to two. After that, the differences become negligible.

The basic difference between the two load cases is the way load is distributed throughout the cross-sectional area. In case 1 the load is distributed across 68% of the area (on the web), whereas in case 2 it is applied to only 32% (on the flange). This explains why in the second case there are greater differences between the finite element compressive stresses at the base and the average values, which are related to total homogenization (Fig. 10). These differences practically disappear, however, with two floors with a total height of 4.80 m. When the panel had shorter dimensions (see previous section), the most important part of the homogenization process occurred within one level, i.e., within the height of 2.40 m. The diameter of the circle that circumscribes the cross-sectional area of the total section in plan view can be used to compare its dimensions with the distance between two adjacent floors. In the former panel the diameter is 1.29 m, smaller than 2.40 m. In the present panel the diameter equals 3.24 m, which is larger than 2.40 m and smaller than 4.80 m. This confirms the aforementioned principle, indicating that in the first case homogenization is completed in one floor height. On the other hand, in the second case, two floors are needed to set up the homogenization. Thus a criterion can be used to assess the degree of homogenization of the vertical stresses, using a simple application of the Saint Venant's Principle. The only question that remains is whether the same criterion can be used when there is no symmetry of cross-sectional area dimensions and (or) loading. This is investigated in the next section.

To complete the analysis, it is necessary to evaluate the shear stress distribution at the interface. Figure 11 shows the values on the web, 37.5 mm from the intersection of web and flange for the first load case, i.e., load on the web. The characteristics of the plotted curves are similar to those of the former short panels (Fig. 8). Only two differences are apparent: the stresses at the base are not as low, and the values at mid-height of each floor are not as close to zero as the earlier ones. The stress distributions are again of parabolic shape.

The simplified model, derived from the parabolic shape of the curves, can be used here once more to evaluate its capability of accurately assessing the shear stresses. As the middle values are not as near zero as they were in the first models, the triple and double average stresses are used in the comparisons (Fig. 12).

Figure 13 shows the most significant values of the shear stresses at the interface for load on the flange (case 2). It includes the average value and double and triple the average value to enable comparisons to be made. With total homoge-

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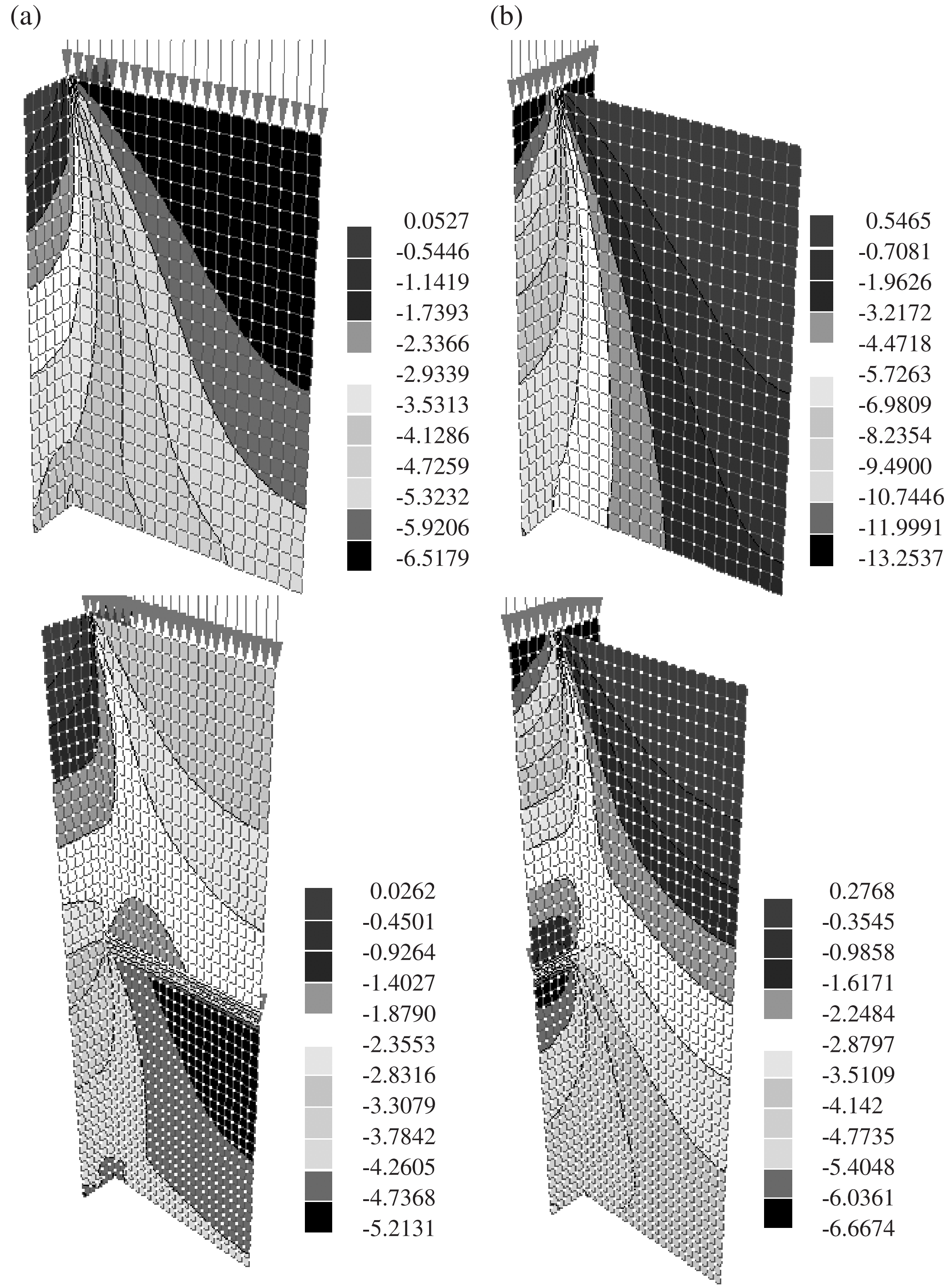
Fig. 9. Vertical normal stresses (MPa) for models with one and two floors: (*a*) load case 1; (*b*) load case 2.



nization, the interface transfers 467 kN in load case 2. The average shear stresses have been calculated with that force, distributing it uniformly along the height of the entire panel. Note that Fig. 13 shows finite element results at the base and at the top, as well as the intermediary values, although the latter are the most important. In the present case the maximum values are much closer to double the average values. This fact can be seen in Table 4, which shows results for the second load case. Despite that, for design purposes the use of triple the average value is recommended because of the need to generate safe values, particularly with shorter dimensions such as in the previous section. Applying the double average model for that panel, the values of the differences in Tables 2 and 3 would grow in absolute values and be on the unsafe side, varying between –35% and –15%, which is undesirable.

Nonsymmetric section and (or) loads

To generalize the study it is necessary to consider the nonsymmetric case, which can result from asymmetric cross-sectional geometry or applied load. This asymmetry

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Fig. 10. Vertical reactions at the base of the panel for the five models: (*a*) web; (*b*) flange.

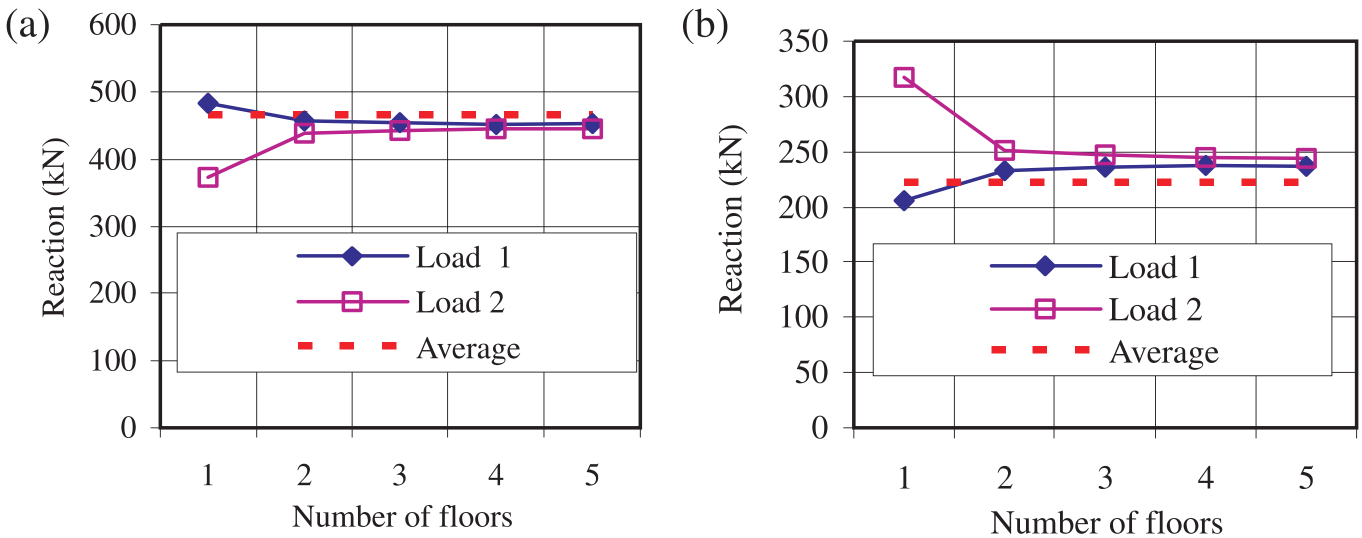


Fig. 11. Shear stresses at the interface for load on the web.

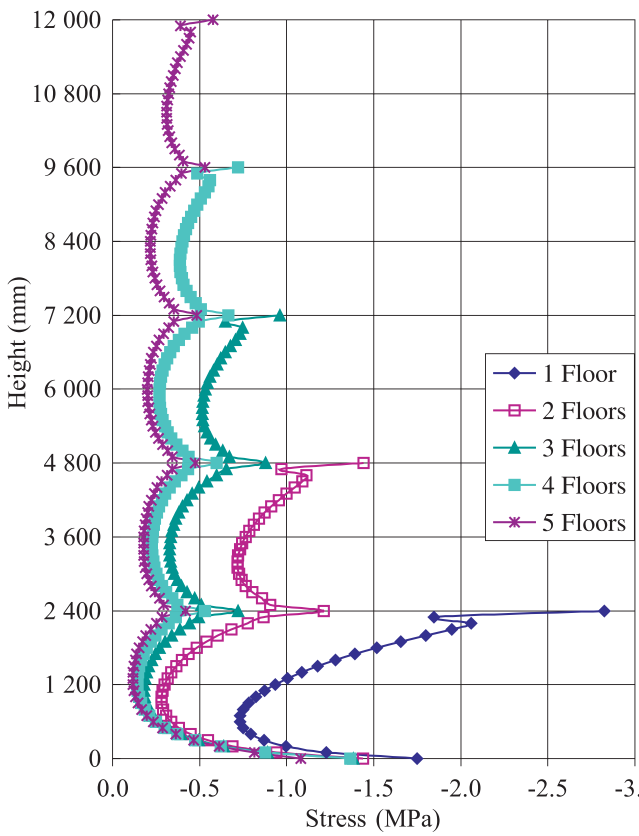
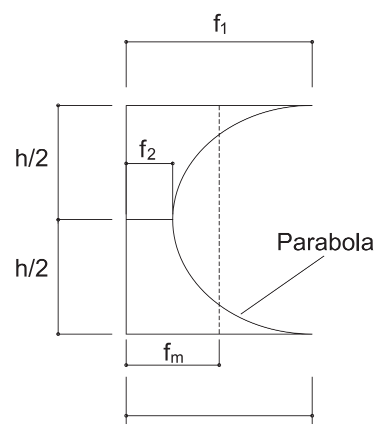


Fig. 12. Quadratic parabola, showing mean values (f_m): $f_m = f_1/3 + 2f_2/3$, where $f_1 = 3f_m$ when $f_2 = 0$ and $f_1 < 3f_m$ when $f_2 > 0$. *h*, height of the floor.



produces in-plane bending of the wall and induces horizontal forces in the wall at each floor level. For this study, a new panel was simulated in the finite element analysis, with half the area of the first panel, but without the consideration of symmetry (Fig. 14). This panel is, in fact, similar to the first short panel with the elimination of the restraints used to simulate symmetry and the addition of horizontal restraints at the floor levels. The same load cases were considered for the wall: distributed forces on the web and on the flange. Note that in both cases, because of the cross-sectional geometry, the loading is applied eccentrically in relation to the centroid of the cross section. As for the earlier investigation, the number of floors considered ranged from one to five.

The first aspect investigated was the development of homogenization of vertical stress in the presence of asymmetry. As the plan dimensions are small, the diameter of the circumscribed circle is therefore smaller than the distance between two adjacent floors. In contrast with the concentric case, however, homogenization of the vertical stresses will not occur within that distance because of the load eccentricity and resulting non-uniform stress distribution from the induced moment. Nevertheless, as can be seen in Fig. 15, the horizontal restraints at floor levels are sufficient to enable homogenization of vertical stresses at the lower levels to occur for the models with two floors or greater. Figure 15 shows the vertical reactions at the base of the walls for a varying number of floors. The most important part of the homogenization process occurs with the transition from one to two floors, in both load cases.

Another point of interest is the shear stress distribution at the interface. Similar patterns were observed for the load on the flange as reported by Corrêa and Page (2001). Curves

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Fig. 13. Shear stresses at the interface for load case 2 (load on the flange).

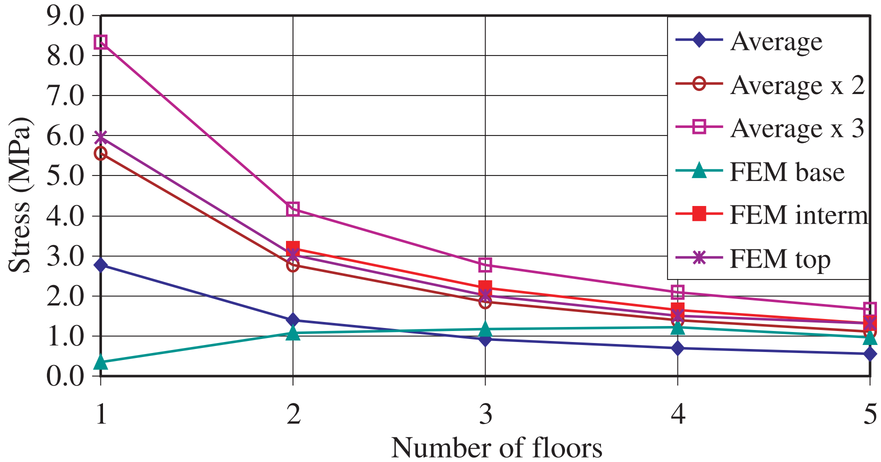
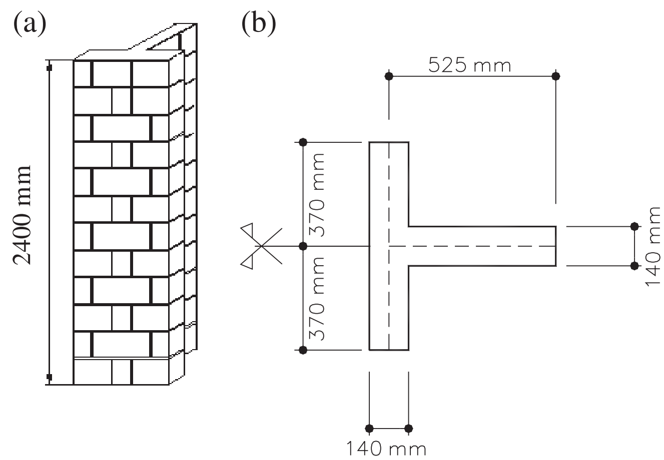


Table 4. Shear stresses at the interface for load on the flange.

No. of floors	Shear stress (MPa) FEM	Avg. × 2	Avg. × 3	Difference (%)[a] FEM vs. Avg. × 2	FEM vs. Avg. × 3
1	5.95	5.56	8.34	–7	+40
2	3.18	2.78	4.17	–13	+31
3	2.20	1.85	2.78	–16	+27
4	1.65	1.39	2.08	–16	+27
5	1.32	1.11	1.67	–16	+27

[a]Relative to the FEM values.

Fig. 14. Panel for nonsymmetrical loading: (*a*) perspective view; (*b*) area dimensions.



similar to those described in previous sections were observed, except for a new "flattening" in the central regions. Figure 16 shows comparisons of the finite element results with the double and triple average values for the first load case (load on the web). Note that the average shear stresses, assuming total homogenization, correspond to a total force of 135 kN being transferred at the interface. The triple average values are again the best choice, as observed in previous sections. Those values are close to the top values and are upper limits for the intermediary shear stresses.

Suggested design approach

The common current design procedure is to disregard the interaction of walls subjected to vertical loads and consider each wall separately. This is based on the inherent assumption that the interface will fail, separating the intersecting walls. On the other hand, when designing for horizontal loads, usually the interaction is considered by taking into account the effect of flanges. In this case, codes often provide guidelines on the maximum flange widths that can be assumed during this interaction. It is obviously inconsistent to consider the contribution of the flanges for horizontal loads and to disregard them for vertical loads, particularly if a check on the potential separation of the web and flange is carried out. Besides this improvement in consistency, there is an additional economic benefit. Provided there is sufficient strength at the wall interface for effective shear transfer, it is possible to develop more cost-effective designs, since the wall interaction and the homogenization of loads are considered. This results in lower predicted stresses at a given building level, as shown by Corrêa and Ramalho (1998). Higher wall capacities can thus be realistically achieved.

The following steps summarize a simple approach to the structural design of load-bearing walls subjected to vertical loading allowing for the effect of the homogenization:

(1) Assess the load on each component of the wall at each load level from the relevant tributary area.

(2) Determine the number of floors needed for homogenization to occur for each group of walls by comparing the plan dimensions of the wall group with the storey height.

(3) Assess the vertical stresses at the base of the walls at the level where homogenization is complete.

(4) If more than two floors are needed for homogenization to occur, (*i*) for each wall component, calculate the vertical reaction at the level where homogenization is complete; (*ii*) assess the difference between that reaction and the original load; and (*iii*) to estimate the vertical reaction at intermediate levels related to that load, distribute this difference evenly among the floor levels between the loaded and the homogenized levels.

(5) Determine the final load at each floor level by adding the load applied at that level to the loads from above, taking

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Fig. 15. Vertical reactions at the base: (*a*) flange; (*b*) web.

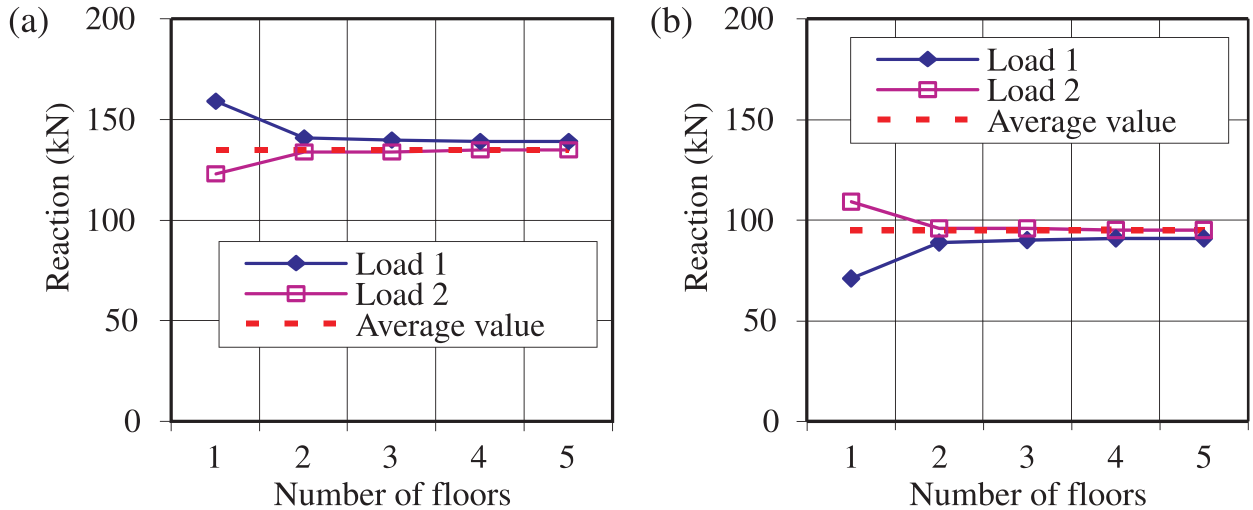
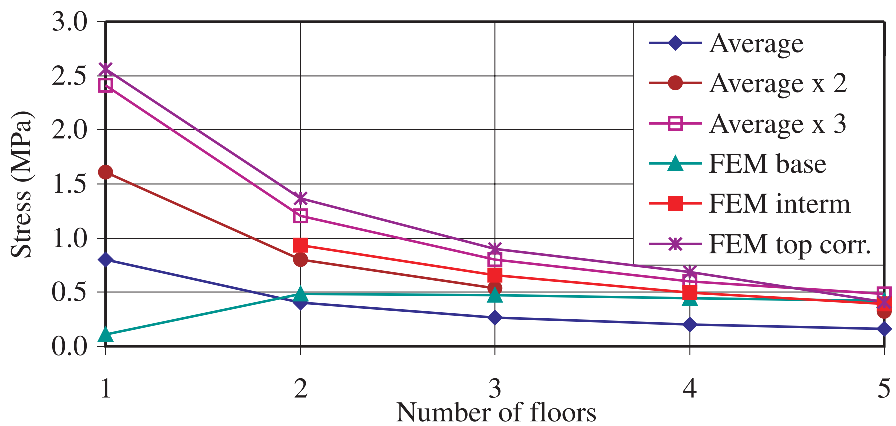


Fig. 16. Shear stresses at the interface for load on the web.



into account the full and (or) partial homogenization of those loads.

(6) For each storey height, evaluate the induced shear force at the interface from the difference in vertical loads in the same wall component between two adjacent floors and calculate the corresponding shear stresses.

(7) Confirm the capacity of the interface to transmit these stresses: (*i*) if yes, the procedure is valid; (*ii*) if no, design wall components separately, ignoring composite action, or limit the load levels to those governed by the shear stresses.

Note that the proposed procedure is appropriate for the design of load-bearing walls at the ultimate strength limit state, as the method checks the two limiting loads related to crushing and shear failure. For the effective application of the method, there is a need for research to develop a test to establish the shear strength of the interface of walls, as no current representative test exists.

A suitable shear test to satisfy the aforementioned requirements should involve the use of small specimens to simplify the experimental procedure and to avoid the shear-lag effects present in a larger specimen such as a storey-high wall. Complementary studies on the development of a suitable test have been carried out by Capuzzo Neto (2004) and Bosiljkov et al. (2004). In both cases a shear test on a five-high, H-shaped section has been found to be suitable. In the absence of a test, the designer can use code provisions to predict the shear strength of the interface, such as those provided by Section 11.6 of the Canadian Standards Association Standard CSA S304.1 (CSA 1994), Section 25 of the British Standards Institution Standard BS-5628 (BSI 1978), or Section 3.3.4 of the Standards Association of Australia Standard AS 3700 (SAA 1998). The Australian provisions have been confirmed to be conservative by Bosiljkov et al. (2004).

Another important aspect related to the method of wall analysis is the design of the foundations, as the calculated vertical forces applied to them will differ depending on whether or not composite wall action has been assumed. If isolated behaviour of each wall is assumed, it is implicit that every wall interface must fail before the capacity of any foundation element is reached, since they will be designed for vertical loads consistent with that assumption. If this distribution of load cannot be guaranteed, the foundation design is potentially unsafe. The proposed design procedure avoids this possibility, as the condition of each interface is evaluated. The assumption of composite action is only maintained for those interconnected walls in which the interface has sufficient strength to transfer the induced shear forces.

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The final loads on the foundations are therefore consistent with all the interface conditions and will more realistically represent the true foundation load distribution.

Summary and conclusions

A new design approach has been developed to allow for the interaction of intersecting walls subjected to vertical loads. Two major factors form the basis of the new approach: the homogenization of loads across all the components of the cross section, and the distribution of the shear stresses at the interface induced by this homogenization process.

It has been shown that the Saint Venant's Principle can be used to predict the homogenization process in the case of symmetric cross sections and symmetric loading. The vertical distance needed to reach homogenization must be larger than the diameter of the circle that circumscribes the intersecting walls in plan view. Each diameter should be compared with the distance between two adjacent floors to determine the minimum number of floors needed to obtain uniformity of the vertical normal stresses in each group of walls. In the nonsymmetric case, the investigation has shown that a similar assumption can be used in design (even though the Saint Venant's Principle cannot be used to justify the homogenization). It has been shown that for homogenization at least two storeys are needed because of the influence of the horizontal restraint provided by the floors. Note that this homogenization process only applies to wall systems whose components are unequally loaded. It also inherently assumes that the transfer of the appropriate shear stresses can occur at the interfaces of the wall components.

In each wall, the shear force to be transferred through the interface can be assessed from the differences between the vertical reactions for each of the relevant components at the two adjacent floors. The shear stress distribution down the interface can then be approximated using a parabolic pattern, with peak values at floor levels. The peak value between two floors can be estimated in a practical and safe way using three times the average value of the shear stress, with the average value calculated by dividing the transferred load by the interface area. Note that when the building is also subjected to horizontal loads, there will be a need to superimpose these effects to obtain the design values of forces and stresses.

The simplified design process presented allows composite action to be assessed and included, with consequent economies and design of more realistic predictions of building behaviour.

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